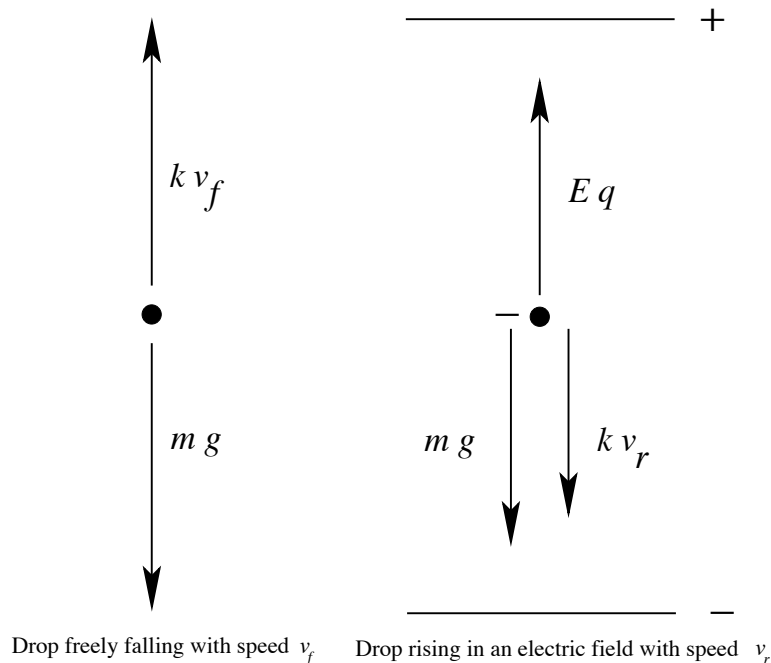


Millikan Oil Drop Derivation ··· Seth Hopper ··· 4/3/06

Our goal is to empirically determine the charge of an electron. Start by considering a small drop of oil, with mass m , falling freely through air at terminal velocity, v_f , as shown. If the local acceleration of gravity is g , then Newton's second law gives

$$kv_f - mg = ma = 0, \quad (1)$$

since the acceleration is zero at terminal velocity. We're assuming the drag force from the



atmosphere is directly proportional to the velocity for some proportionality constant k . This turns out to be a valid assumption.

Now, let's put the same drop in a static electric field, as would be produced by a capacitor. Then, if the drop has charge $-q$ it will feel a force Eq in the direction of the positive plate, as is also depicted. Assuming the field is strong enough, the drop will travel upward, so the drag force will be pointed down, and at terminal velocity, v_r , we have

$$Eq - mg - kv_r = 0. \quad (2)$$

This provides us with a system of two equations. Eliminating k and solving for q gives

$$q = \frac{mg(v_f + v_r)}{Ev_f}. \quad (3)$$

All these quantities can be measured directly except for the mass of the drop. To find it we write the mass in terms of the density of the oil, ρ , as

$$m = V\rho = \frac{4}{3}\pi a^3\rho, \quad (4)$$

where we've assume the drop is a sphere of radius a . The density is a known value, but the radius will vary from drop to drop. It is, however related to v_f , as is shown by Stokes' Law:

$$a = \sqrt{\frac{9v_f\eta}{2g\rho}}. \quad (5)$$

Here, η is the viscosity of the air and is a linear function of the temperature (see Appendix A). So, it looks like we have a means to calculating everything. Unfortunately Stokes' Law is only valid when v_f is greater than 0.001 m/s, a condition that will not be satisfied in our case. Therefore, we have to add a correction factor to the viscosity, yielding

$$\eta \rightarrow \eta \left(\frac{1}{1 + \frac{b}{pa}} \right), \quad (6)$$

where b is a known constant and p is the barometric pressure. Eq. (5) then becomes

$$a = \sqrt{\frac{9v_f\eta}{2g\rho} \left(\frac{1}{1 + \frac{b}{pa}} \right)}, \quad (7)$$

which can be solved for a by using the quadratic formula to give

$$a = \sqrt{\left(\frac{b}{2p} \right)^2 + \frac{9v_f\eta}{2g\rho} - \frac{b}{2p}}. \quad (8)$$

At this point we have an accurate expression for a , so we plug Eq. (8) into Eq. (4) and insert the resulting expression for m into Eq. (3) to get

$$q = \frac{4}{3}\pi \frac{\rho g (v_f + v_r)}{E v_f} \left(\sqrt{\left(\frac{b}{2p} \right)^2 + \frac{9v_f\eta}{2g\rho} - \frac{b}{2p}} \right)^3. \quad (9)$$

For our purposes, the electric field will be produced by a parallel plate capacitor with plate separation d and voltage difference V . The electric field is then constant and has the value

$$E = \frac{V}{d}. \quad (10)$$

Plugging this into Eq. (9) and rearranging gives our final expression for q :

$$q = \left[\frac{4}{3}\pi d\rho g \right] \left[\sqrt{\left(\frac{b}{2p} \right)^2 + \frac{9v_f\eta}{2g\rho} - \frac{b}{2p}} \right]^3 \left[\frac{v_f + v_r}{V v_f} \right]. \quad (11)$$

The first term in square brackets will be a constant for the setup and need only be calculated once. The second term will be constant for each particular drop, but will have to be calculated again whenever a new drop is observed. (That is true only if the temperature, and thus η remain constant while observing the drop.) The third term will change each time the force from the electric field changes (i.e. when the charge of the drop changes, or the voltage between the plates changes).

Note that this is the total charge on the drop, not the charge of one electron. By observing different drops, and the same drop with many different ionizations, you should be able to deduce the quantization of electric charge.

The symbols in Eq. (11) are defined as:

q : total charge carried by drop (C)

d : separation between plates of capacitor, 7.62×10^{-3} m

ρ : density of oil, 886 kg/m^3

g : local acceleration due to gravity, 9.8 m/s^2 if you're on Earth's surface

b : constant, $8.226 \times 10^{-3} \text{ Pa} \cdot \text{m}$

p : barometric pressure, ask your TA (Pa)

η : viscosity of air, see Appendix A ($\text{N} \cdot \text{s/m}^2$)

v_f : speed of drop in free fall (m/s)

v_r : speed of drop rising under the force of the E field (m/s)

V : potential difference across the plates of the capacitor (V).