Emergent geometry: seeing further from the shoulders of giants.

David Berenstein, UCSB.

Chapel Hill, May 8, 2014

Based mostly on arXiv:1301.3519 + arXiv:1305.2394 w. E. Dzienkowski + work in progress.

Remarks on AdS/CFT

AdS/CFT is a remarkable duality between ordinary (even perturbative) field theories and a theory of quantum gravity (and strings, etc) with specified boundary conditions.

Why emergent geometry?

- Field theory lives in lower dimensions than gravity
- Extra dimensions are encoded “mysteriously” in field theory.
- Not all field theories lead to a reasonable geometric dual.
- If we understand how and when a dual becomes geometric we might understand what geometry is.

Goal

- Do computations in field theory
- Read when we have a reasonable notion of geometry.

When do we have geometry?

We need to think of it in terms of having a lot of light modes: a decoupling between string states and "supergravity"

Need to find one good set of examples.

Some technicalities

Coordinate choice

Global coordinates in bulk correspond to radial quantization in Euclidean field theory, or quantizing on a sphere times time.

In equations

$$ds^2 = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega^2$$

Conformally rescaling to boundary

$$\begin{aligned} ds^2 &\simeq \exp(-2\rho)[-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega^2] \\ &\to_{\rho\to\infty} (-dt^2 + d\Omega^2) \end{aligned}$$

Choosing Euclidean versus Lorentzian time in radial quantization of CFT implements the Operator-State correspondence

$$ds^2 = r^2(dr^2/r^2 + d\Omega_3^2)$$
$$\simeq (d\tau^2 + d\Omega_3^2)$$
$$\simeq (-dt^2 + d\Omega_3^2)$$

$$\mathcal{O}(0) \simeq \mathcal{O}|0\rangle_{R.Q.} \simeq |\mathcal{O}\rangle$$

$$H_{S^3 \times R} \simeq \Delta$$

Hamiltonian is generator of dilatations.

Energy of a state is the dimension (incl. anomalous dimension) of the corresponding operator.

AdS/CFT is a quantum equivalence

Everything that happens in field theory (the boundary) has a counterpart in gravity (the bulk).

Everything that happens in the bulk has a counterpart in the boundary

This implies they have the same Hilbert space of states as representation theory of Conformal group.

For this talk

$AdS_5 \times S^5$ dual to N=4 SYM

(deformations or Orbifolds of) (Deformations or Orbifolds of)

Plan of the (rest of the) talk

- The problem of here and now
- Giant gravitons
- Giant graviton states and collective coordinates
- Strings stretched between giants
- Deformations and geometric limits
- Conclusion/Outlook

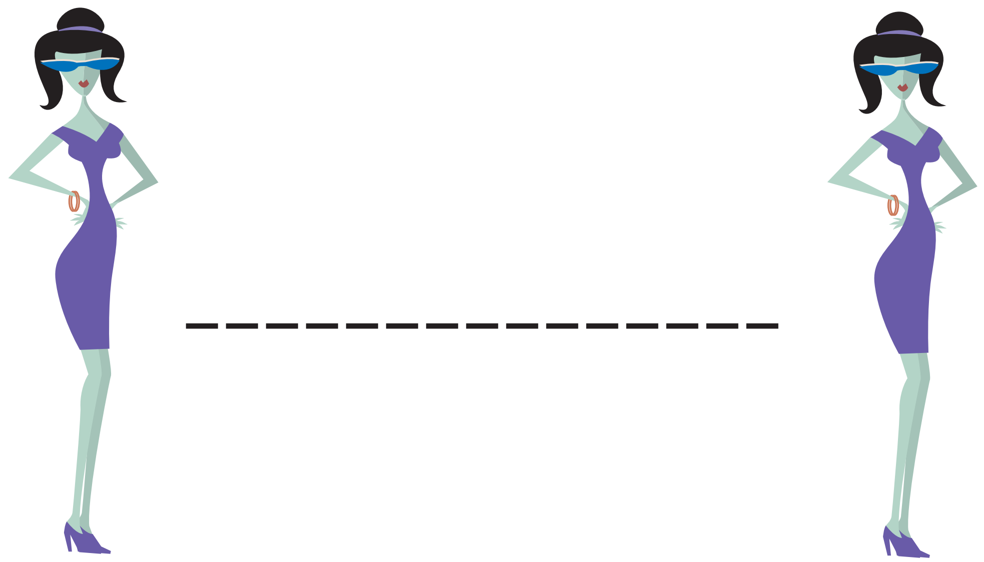
Here and now.

To talk about geometry we need to be able to place an excitation/observer at a given location at a given time.

Then we can talk about the dynamics of such an excitation.

To measure a distance

Two observers and a measure tape between them



Observer: heavy object, so it stays put (classical).
D-branes are natural

Measuring tape: strings suspended between D-branes.

$$E_{string} \simeq T\ell$$

Why giant gravitons, what are giant gravitons?

GIANT GRAVITONS

Gravitons: half BPS states of AdS

Point particles moving on a diameter of sphere and sitting at origin of AdS

Preserve SO(4)x SO(4) symmetry

There are also D-brane (D3-branes) states that respect the same symmetry and leave half the SUSY invariant.

SO(4) x SO(4) invariance implies

Branes wrap a 3-sphere of 5-sphere at origin of AdS (moving in time)

OR

Branes wrap a 3-sphere of AdS, at a point on diameter of 5- sphere

Solution

$$(x^1)^2 + (x^2)^2 + r_{S^3}^2 = 1$$

solving equations of motion gives

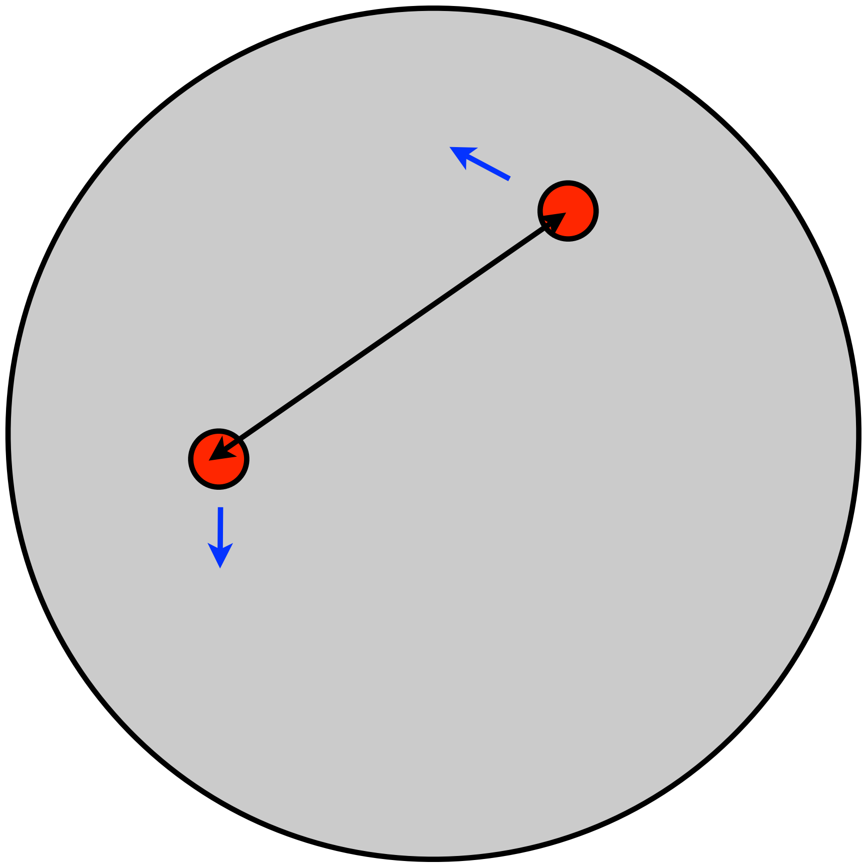
$$x^1 + ix^2 = z = \exp(it)$$

Picture as a point on disk moving with angular velocity one

The one at z=0 has maximum angular momentum

McGreevy, Susskind, Toumbas, hep-th/000307

They are D-branes



Can attach strings

Gauge symmetry
on worldvolume

Gauss' law
Strings in =
Strings out

Mass of strings should be roughly a distance:
depends on geometric position of branes

In gravity, D-branes are localized, but if they have a fixed R-charge in the quantum theory, they are delocalized in the angle variable of z

This is, they correspond to a oscillating wave function on the angle of z (zero mode)

To find masses of strings the branes must also be localized on angles, so they require uncertainty in angular momentum.

Giant graviton states and their collective coordinates.

To preserve SO(4)xSO(4) invariance, gravitons need to look like

$$Tr(Z^n)$$

Where Z is a complex scalar of the N=4 SYM multiplet.

Giant graviton states:

$$\det{}_\ell Z = \frac{1}{N!}\binom{N}{\ell}\epsilon_{i_1,\ldots,i_\ell,i_{\ell+1}\ldots,i_N}\epsilon^{j_1,\ldots,j_\ell,i_{\ell+1}\ldots,i_N} Z^{i_1}_{j_1}\ldots Z^{i_\ell}_{j_\ell}$$

Subdeterminant operators

Balasubramanian, Berkooz, Naqvi, Strassler, hep-th/0107119

Complete basis of all half BPS operators in terms of Young Tableaux,

Corley, Jevicki, Ramgoolam, hep-th/0111222

Interpretation

A giant graviton with fixed R-charge is a quantum state that is delocalized in dual variable to R-charge

To build localized states in dual variable we need to introduce a collective coordinate that localizes on the zero mode: need to introduce uncertainty in R-charge

Introduce collective coordinate for giant gravitons

Consider

$$\det(Z-\lambda)=\sum_{\ell=0}^{N}(-\lambda)^{N-\ell}\det_{\ell}(Z)$$

This is a linear combination of states with different R-charge, depends on a parameter, candidate for localized giant gravitons in angle direction

Can compute norm of state

$$\langle \det(\bar{Z} - \tilde{\lambda}^*) \det(Z - \lambda) \rangle = \sum_{\ell=0}^{N} (\lambda \tilde{\lambda}^*)^{N-\ell} \frac{N!}{(N-\ell)!} = N! \sum_{\ell=0}^{N} (\lambda \tilde{\lambda}^*)^{\ell} \frac{1}{(\ell)!}$$

can be well approximated by

$$\langle \det(\bar{Z} - \tilde{\lambda}^*) \det(Z - \lambda) \rangle \simeq N! \exp(\lambda \tilde{\lambda}^*)$$

For

$$|\lambda| < \sqrt{N}$$

The parameter belongs to a disk

Consider a harmonic oscillator and coherent states

$$|\alpha\rangle = \exp(\alpha a^\dagger)$$

Then

$$\begin{aligned}\langle\beta|\alpha\rangle &= \langle 0|\exp(\beta^* a)\exp(\alpha a^\dagger)|0\rangle \\ &= \exp(\alpha\beta^*)\langle 0|\exp(\alpha a^\dagger)\exp(\beta^* a)|0\rangle \\ &= \exp(\alpha\beta^*)\end{aligned}$$

This means that our parameter can be interpreted as a parameter for a coherent state of a harmonic oscillator.

Can compute an effective action

$$S_{eff} = \int dt \, [\langle \lambda | i\partial_t | \lambda \rangle - \langle \lambda | H | \lambda \rangle]$$

We get an inverted harmonic oscillator in a first order formulation.

$$S_{eff} = \int dt \left[\frac{i}{2}(\lambda^* \dot{\lambda} - \dot{\lambda}^* \lambda) - (N - \lambda\lambda^*) \right]$$

Approximation breaks down exactly when Energy goes to 0

Solution to equations of motion is that the parameter goes around in a circle with angular velocity one.

This is very similar to what happens in gravity

If we rescale the disk to be of radius one,
we get

$$S_{eff} = N \int dt \left[\frac{i}{2}(\xi^* \dot{\xi} - \dot{\xi}^* \xi) - (1 - \xi \xi^*) \right]$$

The factor of N in planar counting suggests that this object can be interpreted as a D-brane

Matches exactly with the fermion droplet picture of half BPS states

D. B. hep-th/0403110

Lin, Lunin, Maldacena, hep-th/0409174

Attaching strings

The relevant operators for maximal giant are

$$\epsilon\epsilon(Z, \ldots Z, W^1, \ldots W^k)$$

Balasubramanian, Huang, Levi and Naqvi, hep-th/0204196

These can be obtained from expanding

$$\det(Z + \sum \xi_i W^i)$$

And taking derivatives with respect to parameters

Main idea: for general giant replace Z by Z-λ in the expansion

$$\det(Z+\sum \xi_i W^i) = \det(Z)\exp(\mathrm{Tr}\log(1+Z^{-a}\sum_i \xi_i W^i Z^{-b}))$$

One loop anomalous dimensions = masses of strings

Want to compute effective Hamiltonian of strings stretched between two giants.

$$\det(Z-\lambda_1)\det(Z-\lambda_2)\mathrm{Tr}((Z-\lambda_1)^{-1}Y(Z-\lambda_2)^{-1}X)$$

I have not done full combinatorics of 2 giants on same group but worked on orbifolds.

$$H_{1-loop} \propto g_{YM}^2 N \mathrm{Tr}[Y, Z][\partial_Z, \partial_Y]$$

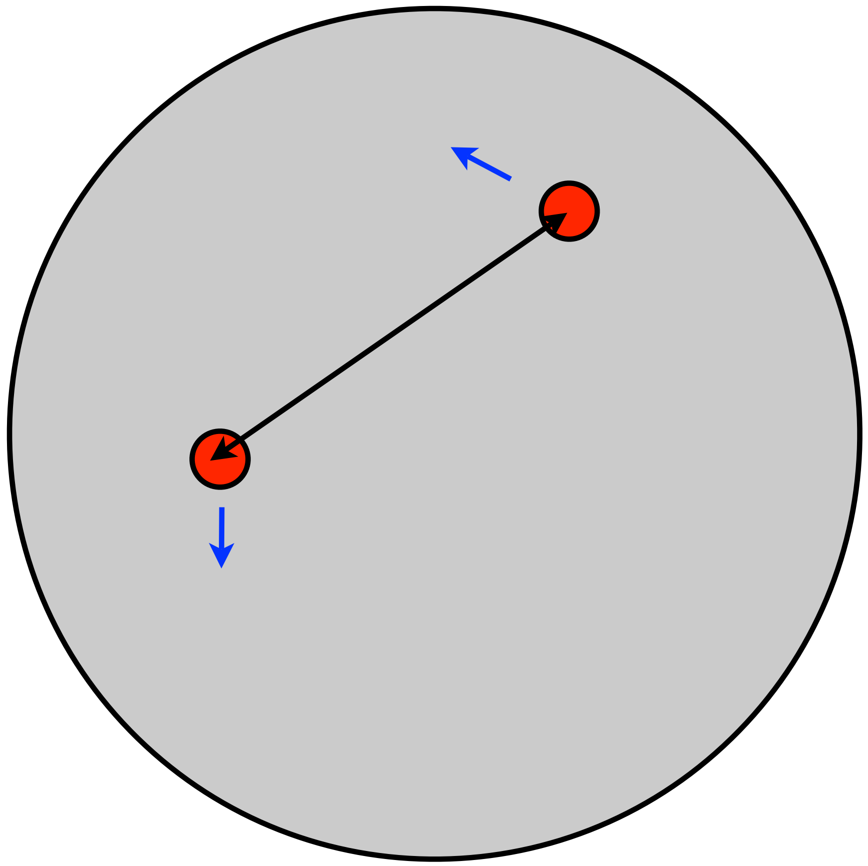
Need following partial results

$$\partial_Z \det(Z-\lambda) = \det(Z-\lambda)\frac{1}{Z-\lambda}$$

$$\partial_Z \mathrm{tr}\left((Z-\lambda)^{-1}W\right) = -(Z-\lambda)^{-1}W(Z-\lambda^{-1})$$

Collect planar contributions.

What we get in pictures



$$m_{od}^2 \simeq g_{YM}^2 |\lambda - \tilde{\lambda}|^2$$

$$E \simeq m_{od}^2 \simeq g_{YM}^2 |\lambda - \tilde{\lambda}|^2$$
$$\simeq g_{YM}^2 N |\xi - \tilde{\xi}|^2$$

Result is local in collective coordinates (terms that could change collective parameters are exponentially suppressed)

Mass proportional to distance is interpreted as Higgs mechanism for emergent gauge theory.

Spin chains

$$Y \to Y^n$$

Need to be careful about planar versus non-planar diagrams.

$$\lambda \simeq N^{1/2}$$

Simplest open chains

$$\det(Z-\lambda)\mathrm{Tr}(\frac{1}{Z-\lambda}YZ^{n_1}Y\ldots Z^{n_k}Y)$$

Just replace the W by n copies of Y: Z can jump in and out at edges. So we need to keep arbitrary Z.

Choose the following labeling for the basis

$$|n_1, n_2, n_3 \ldots\rangle \simeq |\uparrow,\downarrow^{\otimes n_1},\uparrow,\downarrow^{\otimes n_2},\uparrow,\downarrow^{\otimes n_3},\ldots\rangle$$

After some work we can show that the 1-loop anomalous dimension for closed strings is given by

$$H_{eff} = g_{YM}^2 N \sum_i (a_{i+1}^\dagger - a_i^\dagger)(a_{i+1} - a_i)$$

Which clearly shows it is a sum of squares.

Ground states?

We need to try to solve the operator equations

$$a_i = a_{i+1}$$

Acting on some states.

We can try converting them to c-number equations if we introduce generalized coherent states

$$a|z\rangle = z|z\rangle$$

Solving the equations then becomes trivial.

$$|z\rangle = \sum_{k=0}^{\infty} z^k |k\rangle$$

$$\langle z'|z\rangle = \frac{1}{1 - \bar{z}' z}$$

$$\langle z|z\rangle = \frac{1}{1 - |z|^2}$$

Again, we can think of z as a collective coordinate for a site on the chain. $|z|<1$ for convergence.

Same thing for open chains

Use the following basis

$$\det(Z-\lambda)\mathrm{Tr}(\frac{1}{Z-\lambda}YZ^{n_1}Y\ldots Z^{n_k}Y)$$

To simplify combinatorics go to orbifold

$$\det(Z-\lambda)\,\det(\tilde{Z}-\tilde{\lambda})\mathrm{Tr}(\frac{1}{Z-\lambda}Y_{12}\tilde{Z}^{n_1}Y_{21}Z^{n_2}Y\tilde{Z}^{n_3}\ldots Z^{n_k}Y_{12}\frac{1}{\tilde{Z}-\tilde{\lambda}}X_{21})$$

Need to compute

$$\langle \lambda, \tilde{\lambda}; n'_1, \ldots n'_k | H_{eff} | \lambda, \tilde{\lambda}; n_1, \ldots n_k \rangle$$

After some work ....

Again a sum of squares

$$H_{eff} \simeq g_{YM}^2 N \left[\left(\frac{\lambda}{\sqrt{N}} - a_1^\dagger \right) \left(\frac{\lambda^*}{\sqrt{N}} - a_1 \right) + (a_1^\dagger - a_2^\dagger)(a_1 - a_2) + \ldots \right]$$

To find ground state, coherent state ansatz

$$\langle z_1, \ldots z_k | H_{\text{spin chain}} | z_1, \ldots z_k \rangle = g_{YM}^2 N \left[\left| \frac{\lambda^*}{\sqrt{N}} - z_1 \right|^2 + \sum |z_i - z_{i+1}|^2 + \left| \frac{\tilde{\lambda}^*}{\sqrt{N}} - z_k \right|^2 \right]$$

and minimize

$$\frac{\lambda^*}{\sqrt{N}} - z_1 = z_1 - z_2 = \cdots = z_i - z_{i+1} = \cdots = z_k - \frac{\tilde{\lambda}^*}{\sqrt{N}}$$

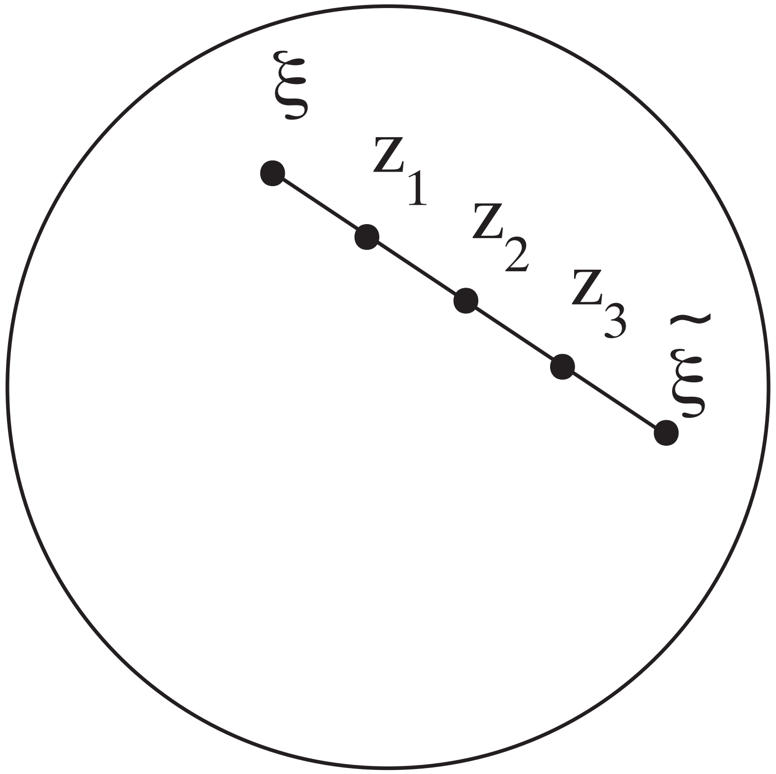
We can add these to solve the linear equations

$$\frac{\lambda^*}{\sqrt{N}} - \frac{\tilde{\lambda}^*}{\sqrt{N}} = (k+1)(z_i - z_{i+1})$$

$$E_0 = \frac{g_{YM}^2 N}{k+1} \left| \frac{\lambda}{\sqrt{N}} - \frac{\tilde{\lambda}}{\sqrt{N}} \right|^2$$

$\xi = \lambda^* N^{-1/2}$

$\tilde{\xi} = \tilde{\lambda}^* N^{-1/2}$



These can be pictured on the "free fermion disk"

The z coordinates also have a geometric interpretation!

$$E(z_0, \dots, z_{k+1}) \simeq g_{YM}^2 N \sum |z_{i+1} - z_i|^2$$

End result:

Full calculation produces a spin chain of Z intertwined in between the Y, and for ground state of spin chain

$$E_n \simeq n + n^{-1} g_{YM}^2 |\lambda - \tilde{\lambda}|^2 \simeq \sqrt{n^2 + g_{YM}^2 |\lambda - \tilde{\lambda}|^2}$$

Starts showing an emergent Lorentz invariance for massive W particles in
the worldsheet fluctuations of giant graviton.

Two loops...

$$= \sum_{l=1}^{L-1} (a_{l+1}^\dagger - a_l^\dagger)^2 (a_{l+1} - a_l)^2 + \left(a_1^\dagger - \frac{\lambda}{\sqrt{N}}\right)^2 \left(a_1 - \frac{\bar{\lambda}}{\sqrt{N}}\right)^2 + \left(a_L^\dagger - \frac{\tilde{\lambda}}{\sqrt{N}}\right)^2 \left(a_L - \frac{\bar{\tilde{\lambda}}}{\sqrt{N}}\right)^2$$

= 0 in ground state

Gives next order in relativistic correction

From the gravity side

Need to modify a calculation in sigma model on a three sphere times time.

Chrysostomos Kalousios, Marcus Spradlin, and Anastasia Volovich, JHEP, 0703:020, 2007

Final answer is

$$\Delta - J = \sqrt{J_2^2 + \frac{\lambda}{4\pi^2}|\xi - \tilde{\xi}|^2}$$

Why? Central charge extension

Acting on a Y

$$Y \to [Z, Y]$$

Beisert hep-th/0511082

in Cuntz basis

$$\sqrt{N}(a_i^\dagger - a_{i+1}^\dagger)$$

OR

$$Y \to [Y, \partial_Z]$$

$$(a_i - a_{i+1})/\sqrt{N}$$

And remember that our ground states are eigenstates of these lowering operators. It gives

$$z_i - z_{i+1}$$

Total central charge

$$\mathfrak{C} = \sum (z_i - z_{i+1}) = z_0 - z_n = \xi - \tilde{\xi}$$

independent of the state, but sourced by D-branes

Small representation of centrally extended PSU(2|2)

$$E = \sqrt{n^2 + g^2 N |\xi - \tilde{\xi}|^2}$$

Exact result to all orders

Now deform N=4 SYM

$$W \simeq \mathrm{Tr}(XYZ - qXZY)$$

Leigh-Strassler

Special case

$$qq^* = 1$$

Preserves integrability

$$q = \exp(2i\beta)$$

$$H_{1-loop} = \sum (a_i^\dagger - q^* q_{i+1}^\dagger)(a_i - q a_{i+1})$$

The q can be removed by twisting (D.B + Cherkis, hep-th/0405215)

This effectively changes

$$\tilde{\xi} \to \tilde{\xi} q^n$$

$$E = \sqrt{n^2 + g^2 N |q^{-n/2}\xi - q^{n/2}\tilde{\xi}|^2}$$

Dispersion relation, which is relativistic + something that looks like a lattice dispersion relation.

Geometric limits: “lots of operators with small anomalous dimensions”

You have a lot of supergravity and field theory modes on branes that do not become stringy, rather, effective field theory on a SUGRA background.

Simplest one

$$q^k = 1 \quad + \quad g^2 N \to \infty$$

$$+ \quad g^2 N |\xi - \tilde{\xi}|^2 \quad \text{fixed or scaled}$$

Only n=km survives at low energies

This indicates a theory on giants of the form

$$S^3/\mathbb{Z}_k$$

We can now consider also “images”

$$\tilde{\xi} = \xi q^s$$

We recover light modes when

$$n = -s \mod k$$

Indicates a relative Wilson line on the quotient sphere.

Another limit, small beta

$$E \simeq \sqrt{n^2 + g^2 N |\xi - \tilde{\xi} - \xi i\beta n + \tilde{\xi}(i\beta)n + \ldots|^2}$$

Now take

$$\xi = \tilde{\xi}$$

$$E \simeq \sqrt{n^2 + g^2 N |\xi|^2 \beta^2 n^2}$$

Is of order n if

$$g^2 N \beta^2 \simeq 1$$

Interpretation

$$E \simeq An$$

Think about this as the spectrum of a relativistic particle on a circle

$$A \simeq \frac{1}{R(\xi)} = \frac{1}{\sqrt{1 + |\xi|^2 g_{YM}^2 N \beta^2}}$$

We start seeing cycles getting squashed

Another limit, small beta

$$E \simeq \sqrt{n^2 + g^2 N|\xi - \tilde{\xi} - \xi i\beta n + \tilde{\xi}(i\beta)n + \ldots|^2}$$

Now take

$$\xi = \tilde{\xi}\exp(-2i\theta)$$

$$E \simeq \sqrt{n^2 + g^2 N|\xi|^2\beta^2(n + \theta/\beta)^2}$$

When we complete the square, we get a "position dependent Wilson line"

This has to be interpreted as the

$$H_{\mu\nu\rho}$$

Field strength in gravity.

Conclusion

- Collective coordinates need to be introduced to resolve a degeneracy problem (geometric zero mode angle)
- Can start obtaining effective actions for giant gravitons with a clean geometric interpretation.
- Attaching strings is no problem, and we start seeing emergent Lorentz symmetry in bulk.
- Can have results to all orders using central charge arguments: truly Lorentzian
- We can play with final answers to understand when we can have geometric limits. Can clarify when SUGRA is valid

Things to do

- Non-integrable deformation |q| different than 1
- Understand higher loop orders, non-renormalization theorem?
- Interacting open strings: can we understand splitting and joining contributions to derive effective interacting field theory on branes?
- Branes at angles?
- Multiple brane combinatorics (reintroduce the technology of Young Tableaux more seriously with collective coordinates takes into account: this is "easy" but requires being careful)