A Naturally Light Dilaton

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- Spontaneous breaking of scale invariance could be very interesting for particle physics phenomenology

- Couplings of Higgs in SM: determined by approximate conformal symmetry of SM

- In absence of Higgs mass parameter SM approximately conformal until QCD scale, and <H>=v breaks conformality spontaneously

- Higgs = dilaton, with f=v, Higgs couplings determined a la Shifman, Vainshtein, Voloshin, Zakharov ’79-’80

- One possibility: Higgs actually dilaton of a broken conformal sector

- Spontaneous breaking of scale invariance could be very interesting for particle physics phenomenology

- Cosmological constant problem

- Only known ways of setting Λ to zero: SUSY or conformal symmetry

- SUSY broken $\rightarrow \Lambda \sim (\text{TeV})^4$ expected

- What is expectation for broken conformal symmetry?

- Aim for this talk

- What does it take to make a dilaton look like the observed Higgs?

- How can we make the dilaton naturally light?

- What are the consequences for a light dilaton for the CC?

Dilaton basics

- Scale transformations $x \to x' = e^{-\alpha} x$
- Operators transform $\mathcal{O}(x) \to \mathcal{O}'(x) = e^{\alpha\Delta}\mathcal{O}(e^{\alpha}x)$
- Δ is full dimension, classical plus quantum corrections
- Change in action:

$$S = \sum_i \int d^4x\, g_i \mathcal{O}_i(x) \longrightarrow S' = \sum_i \int d^4x e^{\alpha(\Delta_i - 4)} g_i \mathcal{O}_i(x)$$

- Assume spontaneous breaking of scale inv. (SBSI)

$$\langle \mathcal{O} \rangle = f^n$$

Dilaton basics

- Dilaton: Goldstone of SBSI, σ, transforms non-linearly under scale transf.:

$$\sigma(x) \to \sigma(e^{\alpha}x) + \alpha f$$

- Restore scale invariance by replacing VEV

$$f \to f\,\chi \equiv f\,e^{\sigma/f}$$

- Effective dilaton Lagrangian is then (using NDA for coeffs)

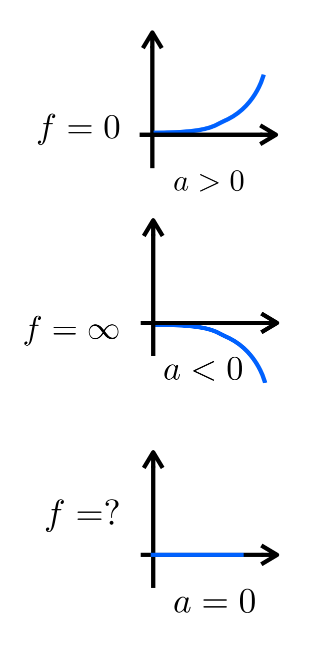
$$\begin{aligned} \mathcal{L}_{eff} &= \sum_{n,m\geqslant 0} \frac{a_{n,m}}{(4\pi)^{2(n-1)}\ f^{2(n-2)}} \frac{\partial^{2n}\chi^m}{\chi^{2n+m-4}} \\ &= -a_{0,0}\,(4\pi)^2 f^4\chi^4 + \frac{f^2}{2}(\partial_\mu\chi)^2 + \frac{a_{2,4}}{(4\pi)^2}\frac{(\partial\chi)^4}{\chi^4} + \ldots \end{aligned}$$

Dilaton dynamics

- Main point of dilaton: effective action can have non-derivative χ^4 term - just the cosmological constant in the composite sector

$$S = \int d^4x \frac{f^2}{2}(\partial\chi)^2 - a f^4 \chi^4 + \text{higher derivatives}$$

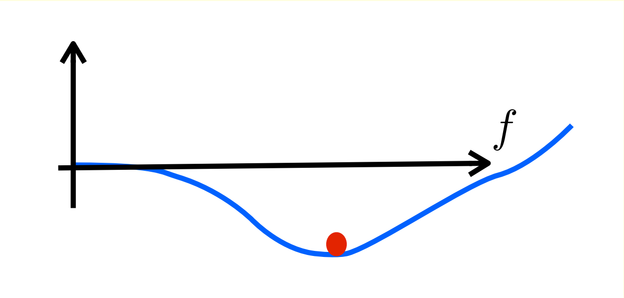
- Generically a≠0. Will make SBSI difficult:
  - a>0: VEV at f=0, no SBSI
  - a<0: runaway vacuum f→∞
  - a=0 arbitrary f



- Need to add additional almost-marginal operator to generate dilaton potential

Dilaton dynamics

•Perturbation: $$\delta S = \int d^4x \lambda(\mu)\mathcal{O}$$



$$af^4 \to f^4 F(\lambda(f))$$

•Dilaton potential: $V(\chi) = f^4 F(\lambda(f))$ vacuum energy in units of f

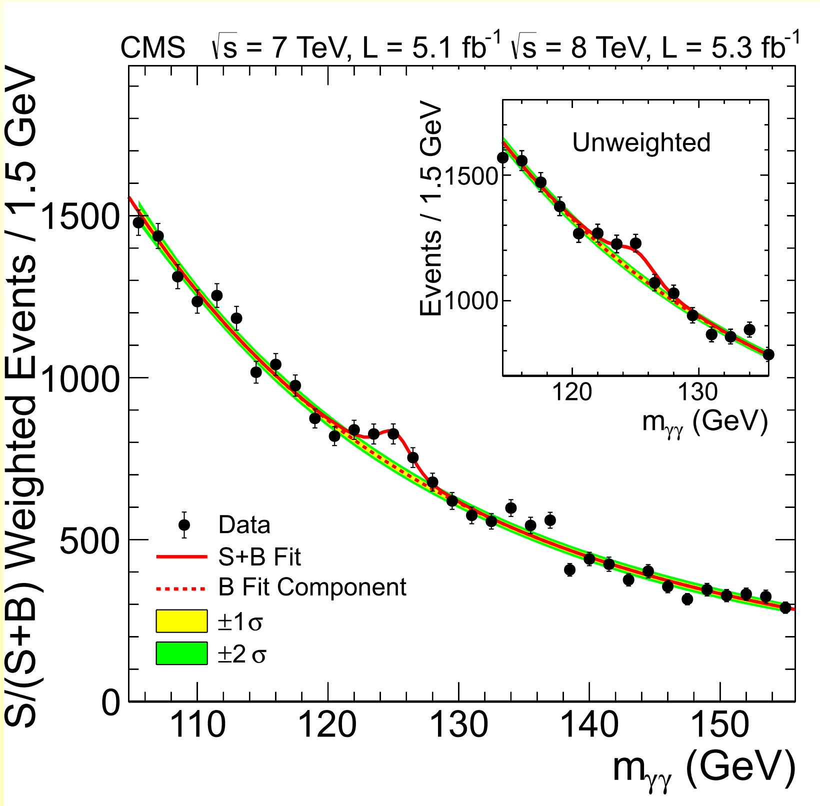
•To have a VEV: $$V' = f^3 \left[4F(\lambda(f)) + \beta F'(\lambda(f))\right] = 0$$

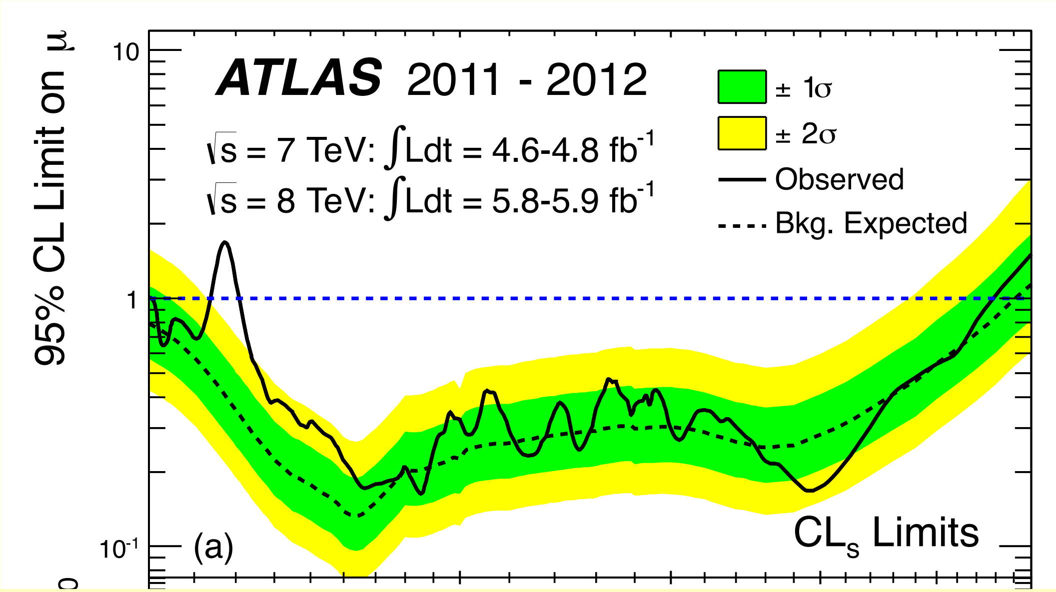
$$\beta = \frac{d\lambda}{d\log\mu}$$

•Dilaton mass:

$$m^2_{dil} = f^2\beta\left[\beta F'' + 4F' + \beta' F'\right] \simeq 4f^2\beta F'(\lambda(f)) = -16f^2 F(\lambda(f))$$

What would it take for the 126 GeV Higgs to be a dilatom





- A new particle at ~ 126 GeV that behaves very similarly to SM Higgs

Dilaton dynamics

- We need $m_{dil} \sim 125\ \mathrm{GeV}$

- With $f \sim v = 246\ \mathrm{GeV}, \Lambda = 4\pi f \sim 3\ \mathrm{TeV}$

- So $m_{dil} \sim f/2 \ll \Lambda$

- But dilaton mass:

$$m_{dil}^2 = f^2\beta\left[\beta F'' + 4F' + \beta' F'\right] \simeq 4f^2\beta F'(\lambda(f)) = -16 f^2 F(\lambda(f))$$

- Naive expectation: one loop vacuum energy

$$F_{NDA} \sim \frac{\Lambda^4}{16\pi^2 f^4} \sim 16\pi^2 \qquad m_{dil} \sim \Lambda$$

Dilaton dynamics

- Generically DO NOT expect a light dilaton, need the dilaton quartic to be suppressed vs. NDA size

- If quartic not suppressed, need large β to stabilize, large explicit breaking a la QCD and TC, no light dilaton

- Need to start with an almost flat direction

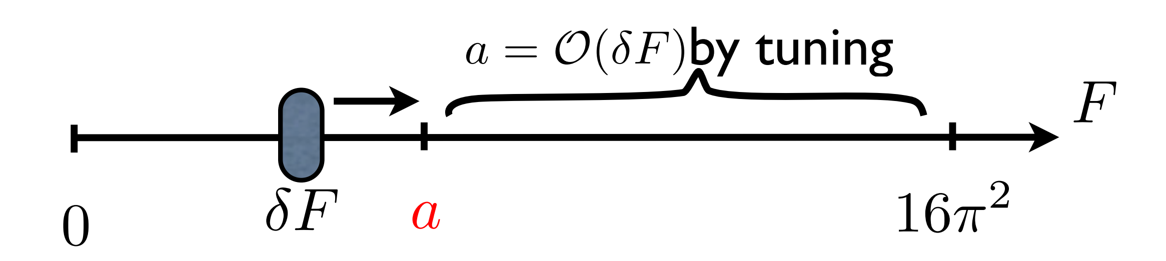
- Dynamics should not generate a large contribution to the vacuum energy...

- Natural in SUSY theories - have flat or almost flat directions

- Not natural in non-SUSY theories

Dilaton dynamics

To find a (non-SUSY) solution we need



- Small vacuum energy (tuning), $a \ll 16\pi^2$

- δF dynamically cancels vs. a

- Perturbation should be close to marginal

Dilaton dynamics

- Detailed examination of the dynamics

- Assume small deviation ε from marginality, and coupling λ:

$$\beta(\lambda) = \frac{d\lambda}{d\ln\mu} = \epsilon\lambda + \frac{b_1}{4\pi}\lambda^2 + O(\lambda^3)$$

- Assume λ perturbative λ<4π, and dilaton quartic very small

$$F(\lambda) = (4\pi)^2\left[c_0 + \sum_n c_n\left(\frac{\lambda}{4\pi}\right)^n\right], \quad c_0 \ll c_n \sim 1\,, \quad a = (4\pi)^2 c_0$$

- Coleman-Weinberg type potential for dilaton

Dilaton dynamics

- For perturbative λ can introduce large hierarchies

$$f \simeq M \left(\frac{-4\pi c_0}{\lambda(M) c_1} \right)^{1/\epsilon}$$

if ε small and negative f<<M (if positive more tuning)

- The dilaton mass:

$$\frac{m_{dil}^2}{\Lambda^2} \sim \frac{\beta}{\pi} \simeq \epsilon \frac{\lambda}{\pi}$$

- Could make it very small by taking ε→0?

Dilaton dynamics

- When ε very small, λ^2 term in β-function dominates

$$\frac{m_{dil}^2}{\Lambda^2} \sim \frac{\beta}{\pi} \sim \frac{\lambda^2}{4\pi^2}$$

- Shows need perturbative coupling for light dilaton

- QCD and (walking)-TC will not have a light dilaton, since there λ=g~4π

- Fine-tuning in weakly coupled models: min. condition gives $\lambda(f) \sim 4\pi c_0/c_1 \equiv 4\pi/\Delta$ where Δ is FT

$$\Delta \gtrsim 2\Lambda/m_{dil} \simeq 50 \left(\frac{f}{246\mathrm{GeV}}\right)\left(\frac{125\mathrm{GeV}}{m_{dil}}\right)$$

A SUSY example for a light dilaton

- Look at 3-2 model

	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_R$
Q	$\square$	$\square$	$1/3$	1
L	$\mathbf{1}$	$\square$	-1	-3
$\overline{U}$	$\overline{\square}$	$\mathbf{1}$	$-4/3$	-8
$\overline{D}$	$\overline{\square}$	$\mathbf{1}$	$2/3$	4

- Classical flat directions $Q\bar{D}L,\ Q\bar{U}L$ and $\det(\bar{Q}Q)$
- Lifted by superpotential $W = \lambda Q\bar{D}L$
- Dynamical ADS superpotential $W_{\rm dyn} = \frac{\Lambda_3^7}{\det(\overline{Q}Q)}$
- Will push fields to large VEVs $\gg\Lambda_3$ as long as $\lambda \ll 1$
- Spontaneous conformality breaking, expect light dilaton

A SUSY example for a light dilaton

- The potential $V \approx \frac{\Lambda_3^{14}}{f^{10}} + \lambda \frac{\Lambda_3^7}{f^3} + \lambda^2 f^4$

- VEVs: $f \approx \frac{\Lambda_3}{\lambda^{1/7}}\ , \quad V \approx \lambda^{10/7} \Lambda_3^4$

- Dilaton mass: $m_{dil} \approx \lambda f \approx \lambda^{\frac{6}{7}} \Lambda_3$

- Of course here SUSY is playing the essential role of keeping the dilaton light, unlike in the non-SUSY examples we are interested in

The radion in RS/GW

- The effective potential w/o stabilization

$$V_{eff} = V_0 + V_1 \left(\frac{R}{R'}\right)^4 + \Lambda_{(5)} R \left(1 - \left(\frac{R}{R'}\right)^4\right)$$

- With f=1/R' get a characteristic SBSI potential with quartic

$$V_{eff}(\chi) = \underbrace{V_0 + \Lambda_{(5)} R}_{\text{CC, FT1}} + f^4 \left(\underbrace{V_1 R^4 - \Lambda_{(5)} R^5}_{\text{quartic, FT2}}\right)$$

- Natural size of quartic: NDA in 5D like in 4D EFT

$$\delta a_{(bulk)} \sim \Lambda_{(5)} R^5 \sim \frac{12^{\frac{5}{2}}}{24\pi^3} \sim \mathcal{O}(1)$$

$$\delta a_{(IR)} = -V_1 R^4 = -V_1 \left(\frac{R}{R'}\right)^4 R'^4 = \frac{\widetilde{V}_1}{\left(\frac{\Lambda}{4\pi}\right)^4} \sim 16\pi^2$$

The radion in RS/GW

- Assumption for GW: quartic is set to zero/very small, then bulk scalar added with non-trivial profile and small bulk mass

- Potential:

$$V = f^4 \left\{ (4+2\epsilon)\left[v_1 - v_0 (fR)^\epsilon\right]^2 - \epsilon v_1^2 + \delta a + O(\epsilon^2) \right\} = f^4 F(f)$$

- ε is bulk mass, $v_{1,0}$ IR/UV VEVs in units of AdS curvature, δa the remaining quartic

- VEV: $$f = \frac{1}{R}\left(\frac{v_1 + \sqrt{-\delta a/4}}{v_0} + O(\epsilon)\right)^{1/\epsilon}$$

- Tuning determined by $\sqrt{-\delta a/4} \lesssim v_1$

- Amount: $\Delta = \frac{a}{|\delta a|} \gtrsim \frac{4\pi^2}{v_1^2}$ ~ 4000 for v_1 ~ 0.1.

Radion as Higgs?

- Radion kinetic term normalization gives

$$f^{(RS)} = \frac{1}{R'}\sqrt{12(M_* R)^3}$$

- For calculability need $N = \sqrt{12(M_* R)^3} \gg 1$, so

- For higgsless: $$\frac{v}{f^{(RS)}} = \frac{2}{g}\frac{1}{N\sqrt{\log\frac{R'}{R}}}$$

- For models with very heavy higgs: $$\frac{v}{f^{(RS)}} = \frac{vR'}{N}$$

- Both cases couplings very suppressed, but mass light

$$m_{dil} \sim M_{KK}\frac{2v_1\sqrt{\epsilon}}{\sqrt{12(M_* R)^3}}$$

Dilaton couplings

- Assumption: composite sector + elementary sector
- Composite sector close to conformal, breaks scale inv. spontaneously
- Elementary sector is external to composite, but weak couplings
- Dilaton coupling in composite sector: assume in UV

$$\mathcal{L}_{CFT}^{UV} = \sum_i g_i \mathcal{O}_i^{UV}$$

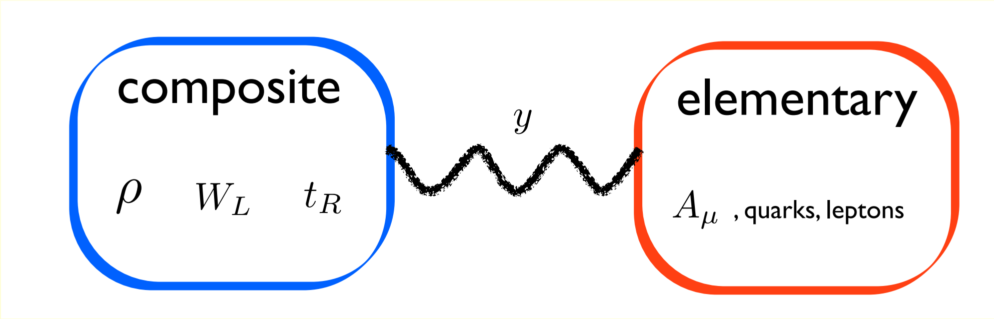
- All operators dim 4 or small explicit breaking $\quad [g_i] = 4 - \Delta_i^{UV}$
- Generic IR Lagrangian $\quad \mathcal{L}_{CFT}^{IR} = \sum_i c_j \left(\Pi g_i^{n_i}\right) \mathcal{O}_j^{IR} \chi^{m_j}$

Dilaton couplings I. Composites

- Power of χ fixed $\mathcal{L}^{IR}_{CFT} = \sum_i c_j \left(\Pi g_i^{n_i}\right) \mathcal{O}_j^{IR} \chi^{m_j}$
- $m_j = 4 - \Delta_j^{IR} - \sum_i n_i (4 - \Delta_i^{UV})$
- Single coupling: $\mathcal{L}^{IR}_{breaking} = \sum_j c_j \, g_i \, \left(\Delta_i^{UV} - \Delta_j^{IR}\right) \mathcal{O}_j^{IR} \frac{\sigma}{f}$
- If no explicit breaking $\mathcal{L}^{IR}_{symmetric} = \sum_j c_j \, \left(4 - \Delta_j^{IR}\right) \mathcal{O}_j^{IR} \frac{\sigma}{f}$
- Coupling to Tr of energy-momentum tensor: $\mathcal{L}_{eff} = -\frac{\sigma}{f} \mathcal{T}^\mu_\mu$
- Trace anomaly included, for $\mathcal{O}_j^{IR} = -(F_{\mu\nu})^2/(4g^2)$

$$4 - \Delta_j^{IR} = 2\gamma(g) = \frac{2\beta(g)}{g}$$

Dilaton couplings II. Partially composite



- Mixing between composite and elementary sectors

$$\mathcal{L}^{UV} = \mathcal{L}^{UV}_{CFT} + \mathcal{L}_{elem} + \sum_i y_i \, O_{elem,i} \, \mathcal{O}^{UV}_{CFT,i}$$

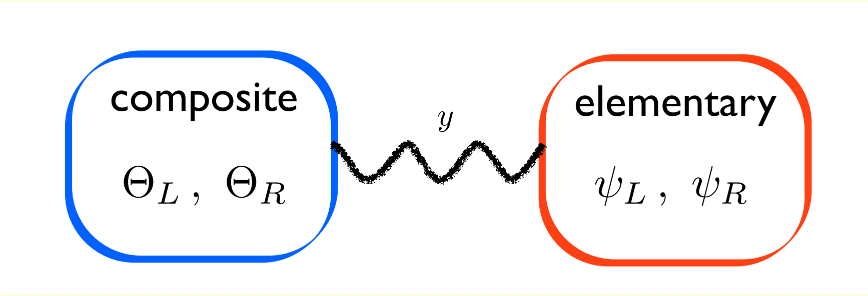
- Treat y as spurion with dimension $[y_i] = 4 - \Delta^{UV}_{elem,i} - \Delta^{UV}_{CFT,i}$

- Effective Lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}^{IR}_{CFT} + \mathcal{L}_{elem} + \sum_j c_j \, y_i \, O_{elem,i} \, \mathcal{O}^{IR}_{CFT,j} \, \chi^{m_j} + \mathcal{O}(y^2)$$

- Power of χ: $\Delta^{UV}_{elem,i} - \Delta^{IR}_{elem,i} + \Delta^{UV}_{CFT,i} - \Delta^{IR}_{CFT,j}$

Example I: Partially comp. fermions



- Mixing between elementary and composite fermions:

$$\mathcal{L}_{int} = y_L \psi_L \Theta_R + y_R \psi_R \Theta_L + h.c.$$

- Spurion dimensions: $[y_L] = 4 - \Delta^{UV}_{\psi_L} - \Delta^{UV}_{\Theta_R}\,, \quad [y_R] = 4 - \Delta^{UV}_{\psi_R} - \Delta^{UV}_{\Theta_L}$

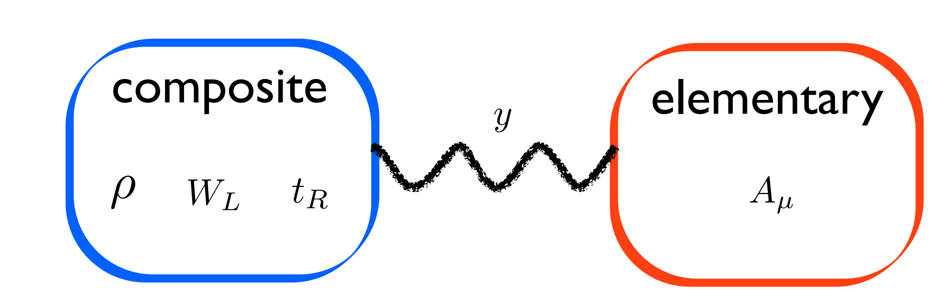
- The effective fermion mass: $\mathcal{L}_{eff} = -M\, y_L\, y_R\, \psi_L \psi_R \chi^m + h.c.$

$$\Delta^{UV}_{\psi_L} - \Delta^{IR}_{\psi_L} + \Delta^{UV}_{\psi_R} - \Delta^{IR}_{\psi_R} + \Delta^{UV}_{\Theta_L} + \Delta^{UV}_{\Theta_R} - 4$$

- Coupling to dilaton: $\Delta^{UV}_{\Theta_L} = 2 + c_L\,, \quad \Delta^{UV}_{\Theta_R} = 2 - c_R\,,$

- In RS language: $\mathcal{L}_{eff} \;=\; -M\, y_L\, y_R\, \psi_L \psi_R \chi^{c_L - c_R}$

Example II: Partially comp. gauge field



- Mixing between gauge field and composite current:

$$\mathcal{L} = -\frac{1}{4g_{UV}^2} F_{\mu\nu}F^{\mu\nu} + A_\mu \mathcal{J}^\mu$$

- Spurion dimension: $[g_{UV}] = \Delta_A^{UV} - 1$

- Low energy coupling: $\mathcal{L}_{eff} = -\frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu}\chi^m$

- Coupling: $m = 4 - 2[1 + \Delta_A^{IR}] + 2[g] = 2(\frac{\beta_{IR}}{g} - \frac{\beta_{UV}}{g})$

Example II: Partially comp. gauge field

- Can also find this from matching of coupling

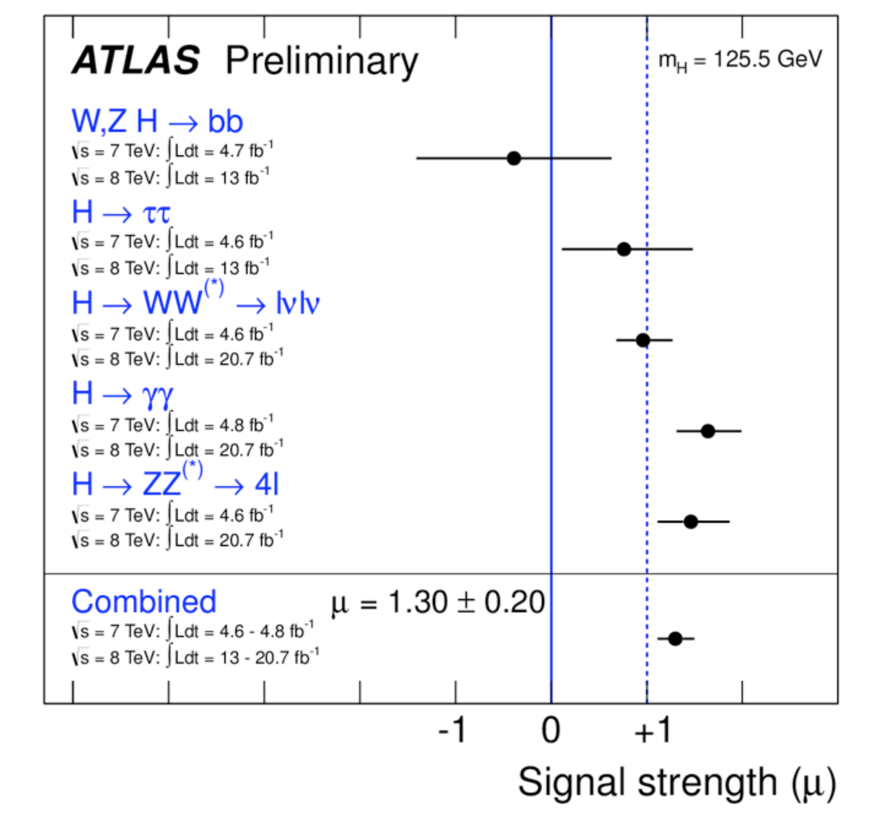
$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\mu_0)} - \frac{b_{UV}}{8\pi^2} \ln \frac{\mu_0}{f} - \frac{b_{IR}}{8\pi^2} \ln \frac{f}{\mu}$$

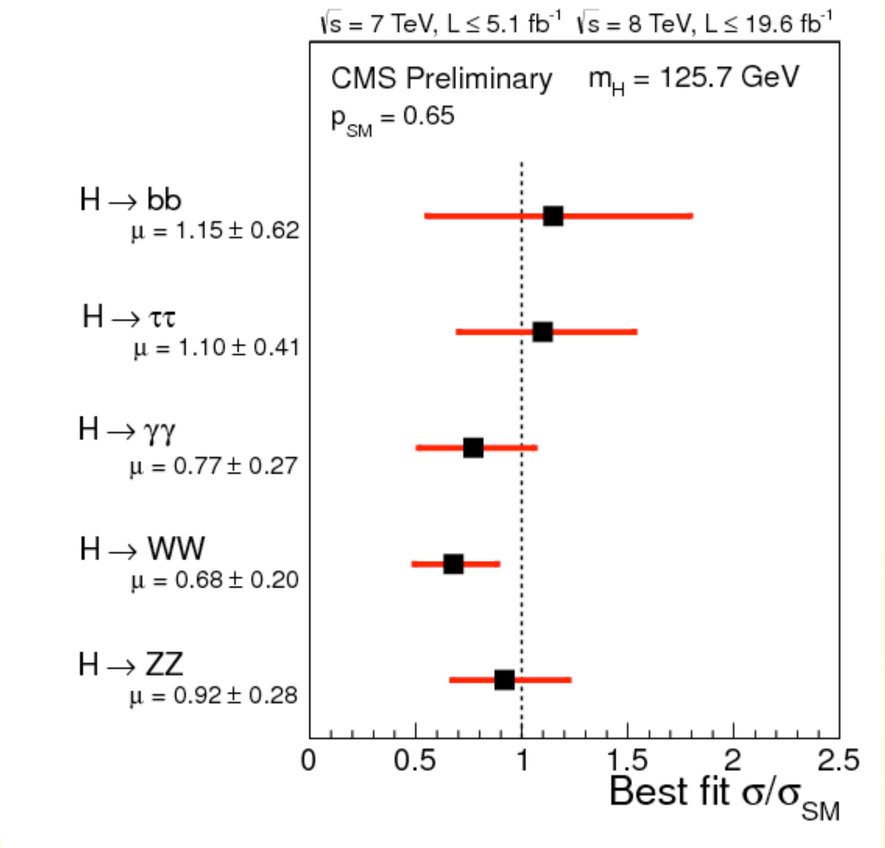
- With replacement $f \to f e^{\frac{\sigma}{f}}$

- Coupling again

$$\frac{g^2}{32\pi^2} \left(b_{IR} - b_{UV}\right) F^{\mu\nu} F_{\mu\nu} \frac{\sigma}{f}$$

Could this be the 126 GeV particle?





- Couplings compatible with SM values, but at this point some could also be somewhat off.

Dilaton coupling to SM

- Couplings to massive fields:

$$\delta\mathcal{L}_{mass} = \left(2m_W^2 W_\mu^+ W^{-\mu} + m_Z^2 Z_\mu^2\right)\frac{\sigma}{f} - Y_\psi \frac{v}{\sqrt{2}}\psi_L\psi_R(1+\gamma_L+\gamma_R)\frac{\sigma}{f} + h.c.$$

- Anomalous dimensions $\gamma_{L,R}$ might be flavor dependent. Assume flavor symmetry to tame dilaton mediated FCNCs

- Coupling to massless gauge bosons:

$$\delta\mathcal{L}_{kin} = \frac{g_A^2}{32\pi^2}\left(b_{IR}^{(A)} - b_{UV}^{(A)}\right)\left(F_{\mu\nu}^{(A)}\right)^2\frac{\sigma}{f}$$

- Assuming photon, gluon partially composite

$$-\left(b_{UV}^{(3)} + b_{t_L}^{(3)}\right)\frac{\alpha_s}{8\pi}G_{\mu\nu}^2\frac{\chi}{f} \quad - \left(b_{UV}^{(EM)} + b_{W_T^\pm}^{(EM)} + N_c\, b_{t_L}^{(EM)}\right)\frac{\alpha}{8\pi}A_{\mu\nu}^2\frac{\chi}{f}$$

Dilaton coupling to SM

- In terms of generic parametrization

$$\mathcal{L}_{eff} = c_V \left(\frac{2m_W^2}{v} W_\mu^+ W^{-\mu} + \frac{m_Z^2}{v} Z_\mu^2 \right) h$$
$$- c_t \frac{m_t}{v} \bar{t}t\, h - c_b \frac{m_b}{v} \bar{b}b\, h - c_\tau \frac{m_\tau}{v} \bar{\tau}\tau\, h$$
$$+ c_g \frac{\alpha_s}{8\pi v} G_{\mu\nu}^2 h + c_\gamma \frac{\alpha}{8\pi v} A_{\mu\nu}^2 \,,$$

- For massive fields

$$c_{t,\chi} = \frac{v}{f}(1+\gamma_t)\,, \quad c_{b,\chi} = \frac{v}{f}(1+\gamma_b)\,, \quad c_{\tau,\chi} = \frac{v}{f}(1+\gamma_\tau)\,,$$

- For massless GBs including top and W loops:

$$\hat{c}_{g,\chi} \simeq \frac{v}{f}\left(b_{IR}^{(3)} - b_{UV}^{(3)} + \frac{1}{2} F_{1/2}(x_t) \right) \equiv \frac{v}{f} b_{eff}^{(3)}\,,$$
$$\hat{c}_{\gamma,\chi} \simeq \frac{v}{f}\left(b_{IR}^{(EM)} - b_{UV}^{(EM)} + \frac{4}{3} F_{1/2}(x_t) - F_1(x_W) \right) \equiv \frac{v}{f} b_{eff}^{(EM)}$$

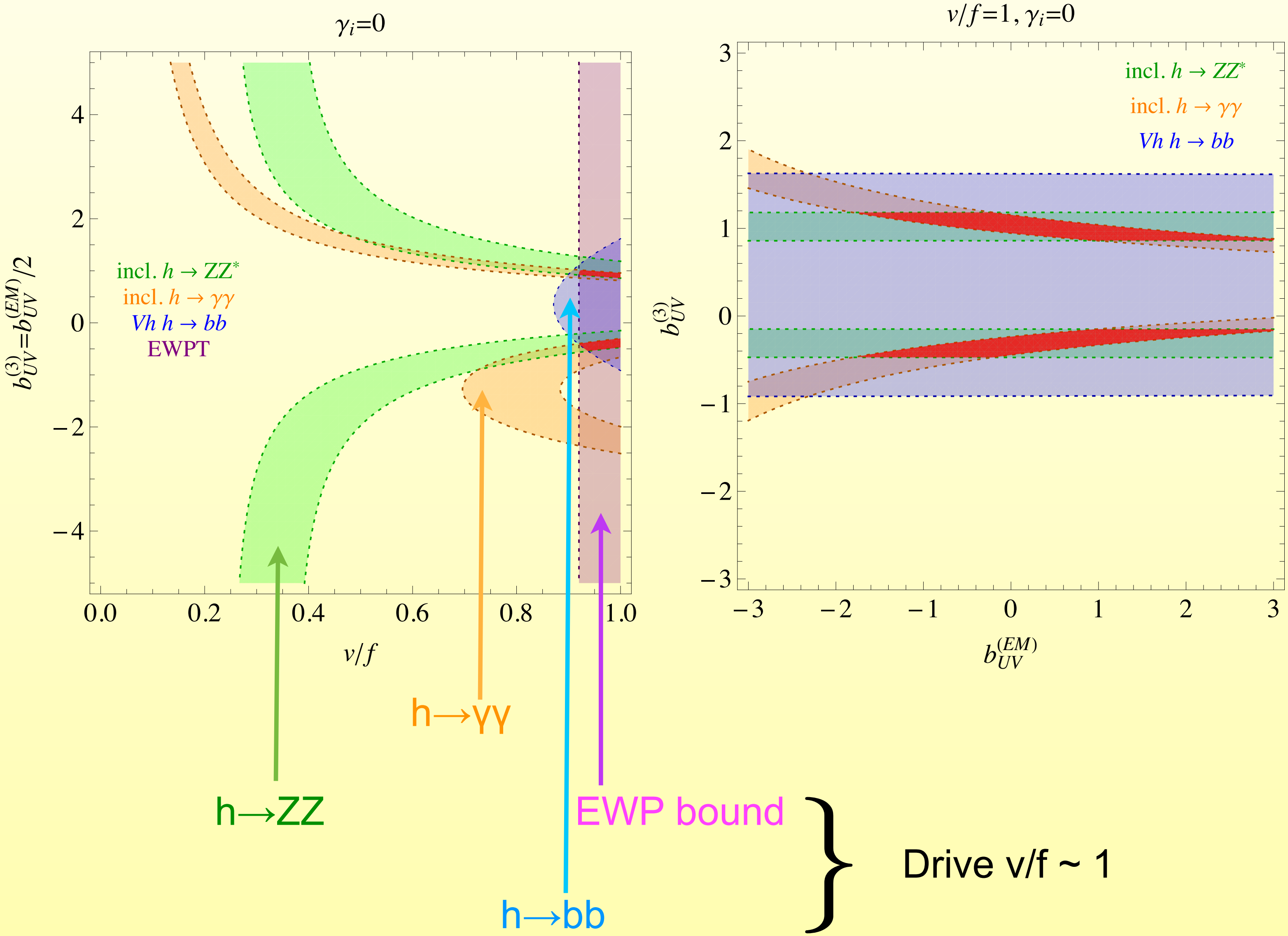
Dilaton rates and production

- Decay rates: $$\frac{\Gamma_{WW}}{\Gamma_{WW,SM}} = \frac{\Gamma_{ZZ}}{\Gamma_{ZZ,SM}} \simeq |c_V|^2\,, \quad \frac{\Gamma_{bb}}{\Gamma_{bb,SM}} \simeq |c_b|^2\,, \quad \frac{\Gamma_{\tau\tau}}{\Gamma_{\tau\tau,SM}} \simeq |c_\tau|^2$$
$$\frac{\Gamma_{gg}}{\Gamma_{gg,SM}} \simeq \frac{|\hat{c}_g|^2}{|\hat{c}_{g,SM}|^2}\,, \quad \frac{\Gamma_{\gamma\gamma}}{\Gamma_{\gamma\gamma,SM}} \simeq \frac{|\hat{c}_\gamma|^2}{|\hat{c}_{\gamma,SM}|^2}$$
- Production rates: $$\frac{\sigma_{GF}}{\sigma_{GF,SM}} \simeq \frac{|\hat{c}_g|^2}{|\hat{c}_{g,SM}|^2}\,, \quad \frac{\sigma_{VBF}}{\sigma_{VBF,SM}} \simeq |c_V|^2\,, \quad \frac{\sigma_{Vh}}{\sigma_{Vh,SM}} \simeq |c_V|^2$$
- Rates for individual channels: $$R \simeq (\sigma\Gamma)/(\sigma\Gamma)_{SM} \times |C_{tot}|^{-2}$$

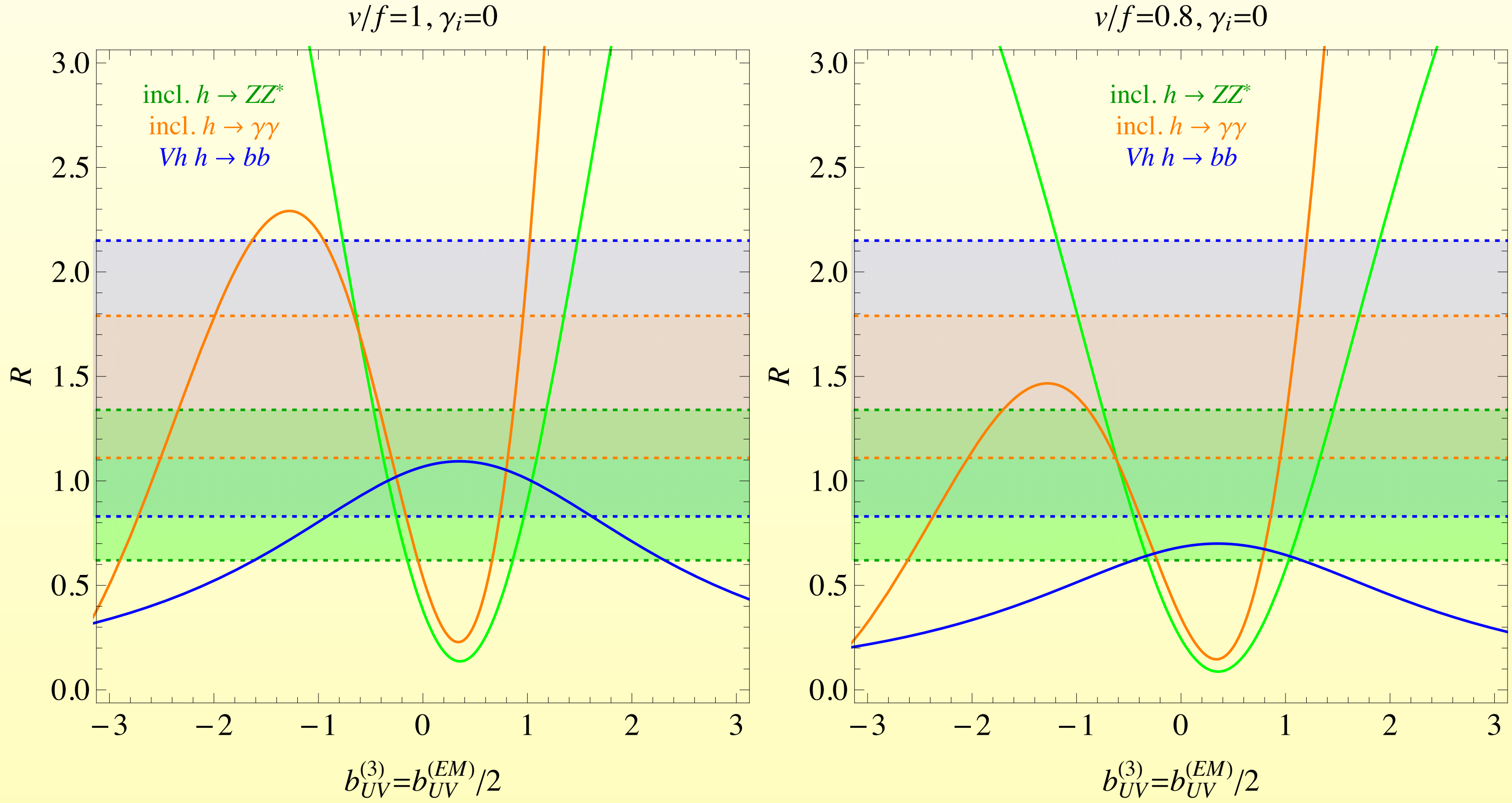
$$R_{GF,(WW,ZZ)} \simeq \frac{v^2}{f^2}\frac{1}{C^2}\left(\frac{b^{(3)}_{eff}}{b^{(3)}_t}\right)^2\,, \quad R_{GF,\gamma\gamma} \simeq \frac{v^2}{f^2}\frac{1}{C^2}\left(\frac{b^{(3)}_{eff}\, b^{(EM)}_{eff}}{b^{(3)}_t\, b^{(EM)}_{t+W}}\right)^2\,,$$
$$R_{GF,\tau\tau} \simeq \frac{v^2}{f^2}\frac{1}{C^2}\left(\frac{b^{(3)}_{eff}\,(1+\gamma_\tau)}{b^{(3)}_t}\right)^2\,, \quad R_{VBF,\gamma\gamma} \simeq \frac{v^2}{f^2}\frac{1}{C^2}\left(\frac{b^{(EM)}_{eff}}{b^{(EM)}_{t+W}}\right)^2\,,$$
$$R_{VBF,(WW,ZZ)} \simeq \frac{v^2}{f^2}\frac{1}{C^2}\,, \quad R_{VBF,\tau\tau} \simeq \frac{v^2}{f^2}\frac{1}{C^2}(1+\gamma_\tau)^2\,, \quad R_{Vh,bb} \simeq \frac{v^2}{f^2}\frac{1}{C^2}(1+\gamma_b)^2$$

- where $$C = \left[\mathrm{BR}_{WW,SM} + \mathrm{BR}_{ZZ,SM} + (1+\gamma_b)\mathrm{BR}_{bb,SM} + \frac{(b^{(3)}_{eff})^2}{(b^{(3)}_t)^2}\mathrm{BR}_{gg,SM}\right]$$

LHC and EWPT constraints



Enhancement in h→γγ



Rates for

h→γγ
h→ZZ
h→bb

} Can be easily enhanced for largish b's

A naturally light dilaton

- We have seen, hard to get light dilaton

- Large quartic expected for dilaton in non-SUSY models

- To remove quartic w/o tuning, Contino, Pomarol, Rattazzi suggested

- Start with exactly conformal theory

- Add close to marginal perturbation with dimension 4-ε

- Make sure β function remains small even when coupling is large - very non-trivial requirement!

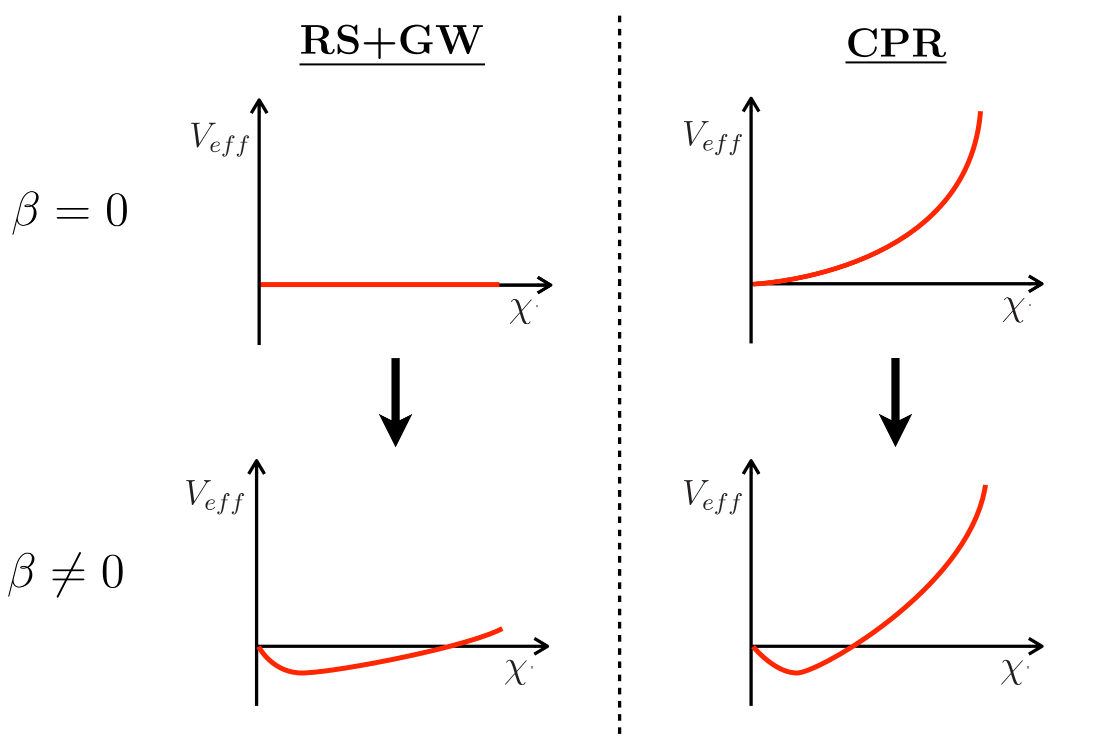
- Quartic will relax to close to zero, dilaton light cc small

A naturally light dilaton

- By adding small explicit breaking quartic will be slowly running
- Model will slowly scan through space of quartics
- SBSI happens when quartic is small
- If β function small dilaton will remain light
- Minimum expected at small CC also!
- Similar construction by Weinberg (no-go thm)
- Zero CC requires exact scale invariance, but then dilaton can not be fixed

A naturally light dilaton

- RS-GW vs. CPR approaches



- RS-GW starts with a tuned setup (IR brane tension)

- CPR approach allows arbitrary IR tension, but quartic will slowly relax, that is where IR brane stabilized

A naturally light dilaton

- Expression for effective potential

$$V_{eff} = F\chi^4 \rightarrow \quad V_{eff} = \chi^4 F(\lambda(\chi))$$

- Due to running coupling:

$$\frac{d\lambda}{d\log\mu} = \beta(\mu) \equiv \epsilon\, b(\lambda) \ll 1$$

- After long running $\delta F \sim (\Lambda_{UV}/\mu)^{\epsilon}$
- At some scale $F(\lambda(\mu_{IR})) \sim 0.$
- Can check explicitly in a warped 5D setup!

5D picture of naturally light dilaton

- General warped metric with scalar action

$$S = \int d^5x \sqrt{g} \left(-\frac{1}{2\kappa^2}\mathcal{R} + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - V(\phi) \right) - \int d^4x \sqrt{g_0} V_0(\phi) - \int d^4x \sqrt{g_1} V_1(\phi)$$

- Metric $ds^2 = e^{-2A(y)} dx^2 - dy^2$

- Identification of scale $\mu = k e^{-A(y)}$

- And dilaton: location of IR brane $\chi = e^{\frac{\sigma}{f}} = e^{-A(y_1)}$

The effective potential

- It is a pure boundary term

$$V_{UV/IR} = e^{-4A(y_{0,1})}\left[V_{0,1}\left(\phi(y_{0,1})\right) \mp \frac{6}{\kappa^2}A'(y_{0,1})\right]$$

- Dilaton potential will be

$$V_{IR} = \chi^4\left[V_1\left(\phi\left(A^{-1}(-\log\chi)\right)\right) + \frac{6}{\kappa^2}A'\left(A^{-1}(-\log\chi)\right)\right]$$

- In accordance with expectation $V_{eff}(\chi) = \chi^4 F(\lambda(\chi))$

- With $F = V_1 + \frac{6}{\kappa^2}A'$

Toy case: constant bulk potential
Flat dilaton via tuning two condensates

- Provides of a dimension 4 condensate - a soft-wall version of RS (=spontaneous breaking of SI with dim 4 rather than ∞ dimensional)

- Will be the IR region of the full problem with bulk mass for scalar

- Bulk equations can be solved explicitly

$$
\begin{aligned}
A(y) &= -\frac{1}{4}\log\left[\frac{\sinh 4k(y_c - y)}{\sinh 4ky_c}\right] \\
\phi(y) &= -\frac{\sqrt{3}}{2\kappa}\log\tanh[2k(y_c - y)] + \phi_0
\end{aligned}
$$

Toy case: constant bulk potential
Flat dilaton via tuning two condensates

- For finite y_c deviates from AdS space. AdS recovered in $y_c \longrightarrow \infty$ limit.

- Location of IR and UV branes:

$$\chi^4 = e^{-4A(y_1)} = \frac{\sinh 4k(y_c - y_1)}{\sinh 4ky_c}, \quad \mu_0^4 = e^{-4A(y_0)} = \frac{\sinh 4k(y_c - y_0)}{\sinh 4ky_c},$$

- Parametrization of deviation from AdS

$$\delta^4 = \frac{1}{\sinh 4ky_c}.$$

Toy case: constant bulk potential
Flat dilaton via tuning two condensates

- The potential will be:

$$V_{IR} = \chi^4 \left[\Lambda_1 + \frac{6k}{\kappa^2}\sqrt{1+\frac{\delta^8}{\chi^8}} + \lambda_1 \left(\phi_0 - v_1 - \frac{\sqrt{3}}{2\kappa} \log \left[\sqrt{1+\frac{\delta^8}{\chi^8}} - \frac{\delta^4}{\chi^4} \right] \right)^2 \right]$$

- The BC for scalar will give (in limit of stiff brane potentials):

$$\begin{aligned} \phi_0 &= v_0 \left(1 + \mathcal{O}(\chi^4/\mu_0^4)\right), \\ \delta^4 &= \chi^4 f_1 \left(v_0(1+\mathcal{O}(\chi^4/\mu_0^4)), \lambda_1, v_1\right) \end{aligned}$$

- Pure quartic up to corrections in UV brane position. Coefficient of quartic:

$$a(v_0) = \Lambda_1 + \frac{6k}{\kappa^2} \cosh\left(\frac{2\kappa}{\sqrt{3}}(v_1 - v_0) \right)$$

- Can TUNE to zero by choosing v_0 properly!

Toy case: constant bulk potential Flat dilaton via tuning two condensates

- A theory that deviates strongly from AdS

- Nevertheless this is a spontaneously broken CFT

- Gravity will be explicit breaking, UV contribution to potential

$$V_{UV} = \mu_0^4 \left[\Lambda_0 - \frac{6k}{\kappa^2}\sqrt{1+\frac{\delta^8}{\mu_0^8}} + \lambda_0 \left(\phi_0 - v_0 - \frac{\sqrt{3}}{2\kappa} \log\left[\sqrt{1+\frac{\delta^8}{\mu_0^8}} - \frac{\delta^4}{\mu_0^4} \right] \right)^2 \right]$$

- μ_0 location of UV brane, in limit $\mu_0 \to \infty$ gravity decoupled

Important comments

- In the limit of no gravity potential is pure quartic (as it should be in a pure CFT)

- Quartic can be tuned to vanish by choosing v_0 (value of Φ on UV brane)

- Different from GW: here we tune UV value of perturbation - if small explicit breaking, this will run $v_0 \to v_0(\chi/\mu_0)^\epsilon$ and will find the position where quartic is vanishing

- Scale invariance of metric non-trivial: $y \to y + a,\ x \to e^\alpha(a)x.$ also requires shift in y_1 and y_c.

The general case: small bulk mass

- Bulk potential $$V(\phi) = -\frac{6k^2}{\kappa^2} - 2\epsilon k^2\phi^2$$

- ε<<1, dimension 4-ε operator.

- Two regions of space:

1. UV region:
   Φ'' can be neglected, slow running of scalar

$$\begin{aligned} A_r'(y) &= k \\ \phi_r(y) &= \phi_0 e^{\epsilon k y} \end{aligned}$$

Space remains AdS, RGE running of scalar

The general case: small bulk mass

2. IR region (``condensate region"):
Scalar dominated by Φ'',Φ', mass term can be neglected: just like the solution without mass

$$
\begin{aligned}
A_c'(y) &= -k \coth\left(4k(y-y_c)\right) \\
\phi_c(y) &= \phi_m - \frac{\sqrt{3}}{2\kappa}\log\left(-\tanh\left(2k(y-y_c)\right)\right)
\end{aligned}
$$

- Need to match up these two solutions

- Asymptotic matching for boundary layer theory

- Full solution:

$$
\begin{aligned}
A_{full}'(z) &= \left(-1+\frac{2z^8}{z^8+\chi^8\tanh^2\left(\frac{\kappa}{\sqrt{3}}(v_1-v_0(\mu_0/\chi)^\epsilon)\right)}\right)^{-1}, \\
\phi_{full}(z) &= v_0\left(\frac{\mu_0}{z}\right)^\epsilon - \frac{\sqrt{3}}{2\kappa}\log\left[-1+\frac{2z^4}{z^4+\chi^4\tanh\left(\frac{\kappa}{\sqrt{3}}(v_1-v_0(\mu_0/\chi)^\epsilon)\right)}\right]
\end{aligned}
$$

The matched solutions

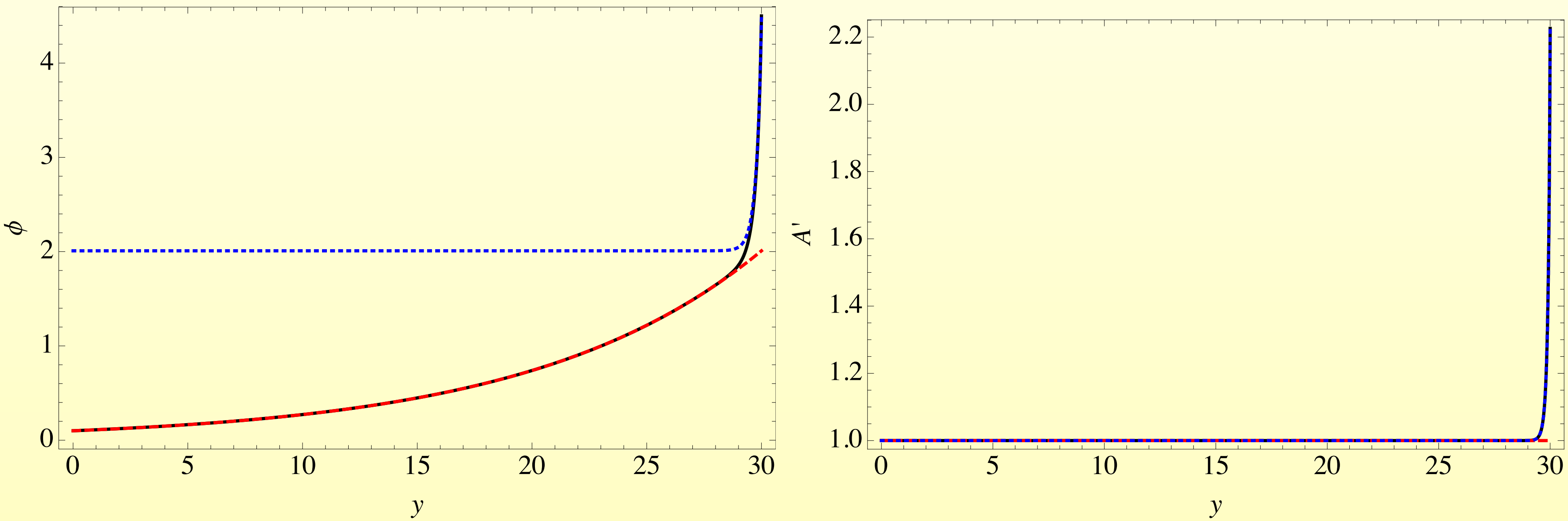
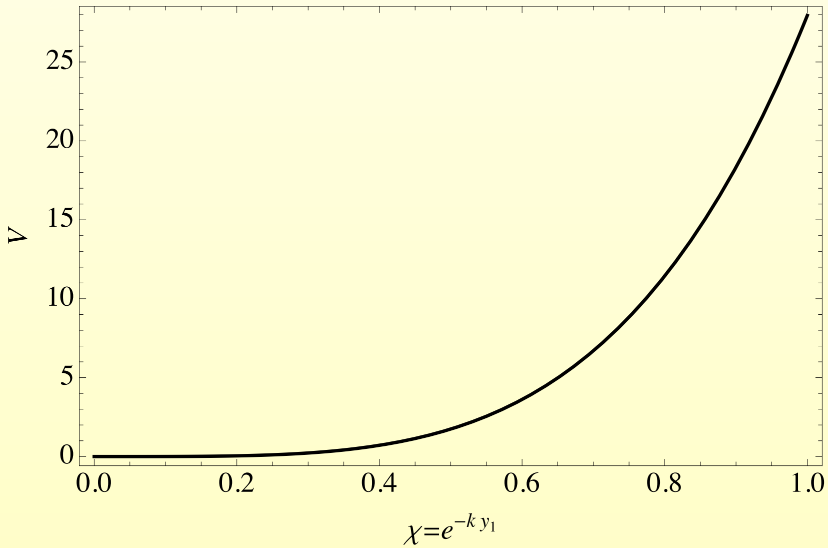
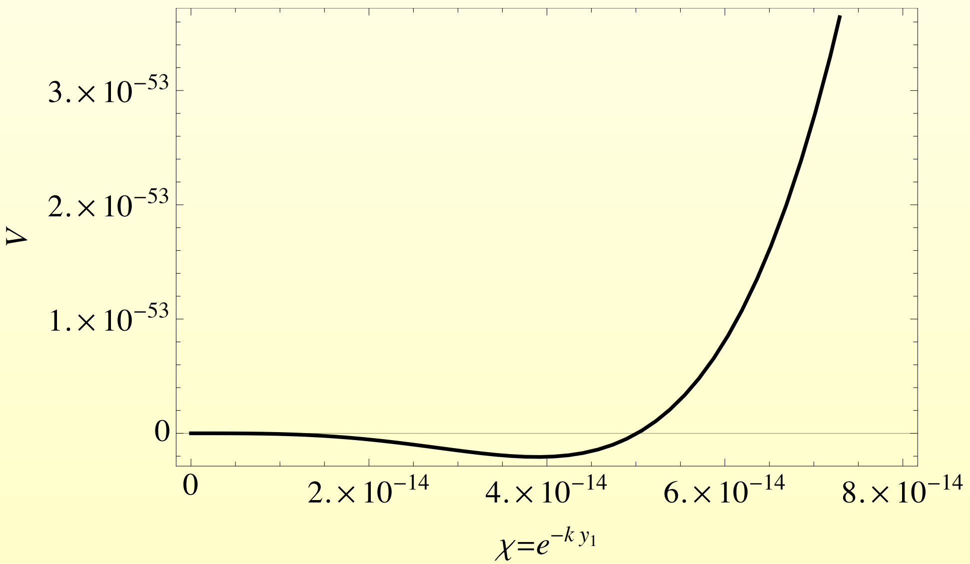


Figure 2: Left, bulk scalar profile: ϕ_{full} (solid black), ϕ_r (dashed red), and ϕ_b (dotted blue). Right, effective AdS curvature, $A'(y)$: same color code.

The effective dilaton potential





- Dilaton VEV hierarchical:

$$\frac{\langle\chi\rangle}{\mu_0} = \left(\frac{v_0}{v_1 - \mathrm{sign}(\epsilon)\frac{\sqrt{3}}{2\kappa}\,\mathrm{arcsech}(-6k/\kappa^2\Lambda_1)}\right)^{1/\epsilon} + O(\epsilon)$$

Dilaton mass and CC

- Dilaton mass:

$$m_\chi^2 \sim \epsilon \frac{32\sqrt{3}kv_0}{\kappa} \tanh\left(\frac{\kappa}{\sqrt{3}}(v_1 - v_0(\mu_0/\chi)^\epsilon)\right) \langle\chi\rangle^2 (\mu_0/\chi)^\epsilon + O(\epsilon^2)$$

$$m_\chi^2 \sim \epsilon \langle\chi\rangle^2$$

- Vacuum energy:

$$V_{IR}^{min} = -\epsilon \frac{2\sqrt{3}kv_0}{\kappa} \tanh\left(\frac{\kappa}{\sqrt{3}}(v_1 - v_0(\mu_0/\chi)^\epsilon)\right) \langle\chi\rangle^4 (\mu_0/\chi)^\epsilon \sim -m_\chi^2 \frac{\langle\chi\rangle^2}{16}$$

$$\Lambda \sim \epsilon \langle\chi\rangle^4$$

Dilaton mass and CC

- Dilaton naturally light, no tuning here (except UV CC)

$$m_\chi^2 \sim \epsilon \langle \chi \rangle^2$$

- Vacuum energy: $$\Lambda \sim \epsilon \langle \chi \rangle^4$$

- Suppressed compared to SUSY, but non-zero.

- Need conformal symmetry to set CC to zero. To stabilize scales need to break it - reintroduces CC, but small breaking can do it.

- Here ε also sets hierarchy - can not be too small.

Conclusions

- Spontaneous breaking of scale invariance could be interesting for phenomenology

- Dilaton could be Higgs-like particle, motivated

- Large quartic expected for dilaton in non-SUSY models

- Hard to get light dilaton, but can fit LHC data

- To obtain light dilaton need small explicit breaking that remains small even at large coupling

- Explicit 5D construction possible

- $m_\chi^2 \sim \epsilon \langle \chi \rangle^2, \Lambda \sim \epsilon \langle \chi \rangle^4$