Dualities

in the Wilderness of Orientifolds

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JHEP 1310 (2013) 006 (1307.1701)
& forthcoming work

Motivation

Why study dualities?

String theory
or other quantum gravity

AdS/CFT
(large N_c)

Strongly coupled gauge theory

S-duality
(finite N_c)

Weakly coupled gauge theory

* Can use dualities to better understand strong coupling in gauge theories
* Gauge theory dualities can help us learn more about string theory

Symbiotic relationship between gauge theory and string theory

Motivation II

Why strong coupling?

* QCD is strongly coupled at low energies
* Anthropics: If we live in landscape of vacua then other vacua may have different strongly coupled / confining gauge groups
* Move away from the lamppost...

Why string theory?

* Consistent theory of quantum gravity (only one?)
* Playground of dualities
* Symbiosis w/ gauge theories via AdS/CFT

(Why supersymmetry?)

* Needed for computability

Example #1

Source-free Maxwell's eqns:

$$\boxed{\begin{array}{ll} \vec{\nabla}\cdot\vec{E} = 0 & \vec{\nabla}\times\vec{B} = \frac{1}{c}\frac{\partial\vec{E}}{\partial t} \\ \vec{\nabla}\cdot\vec{B} = 0 & \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \end{array}}$$

* Invariant under $\vec{E}\to\vec{B}$, $\vec{B}\to-\vec{E}$

  ($F_{\mu\nu} \to \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$)

* "Electromagnetic duality" also holds in <u>full</u> <u>quantum</u> theory

  ↳ but this theory is free

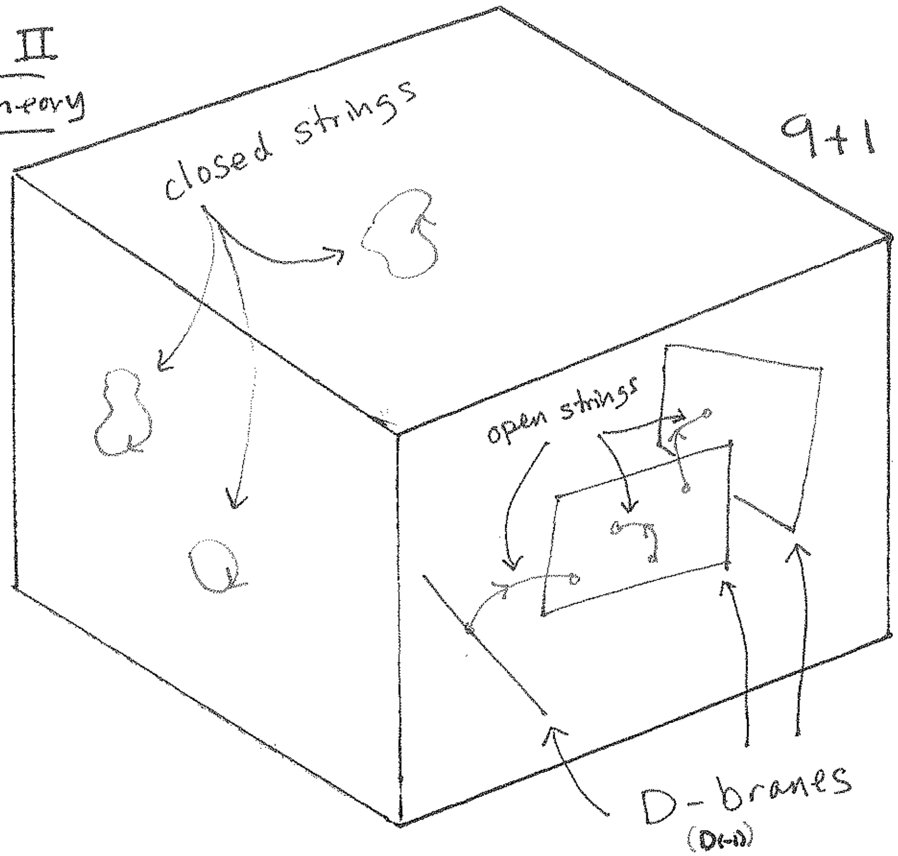
* Adding sources (charged particles) usually spoils the duality

  ... except in some special cases

Example #2

Type II String theory

9+1 dimensions



Dp: p+1 dimensional

IIA: D0, D2, D4, D6, D8

C_1: elec. → D0, mag. → D6

C_3: elec. → D2, mag. → D4

IIB: D1, D3, D5, D7, (D9)

C_2: elec. → D1, mag. → D5

C_4: elec. / mag. → D3

C_0: mag. → D7

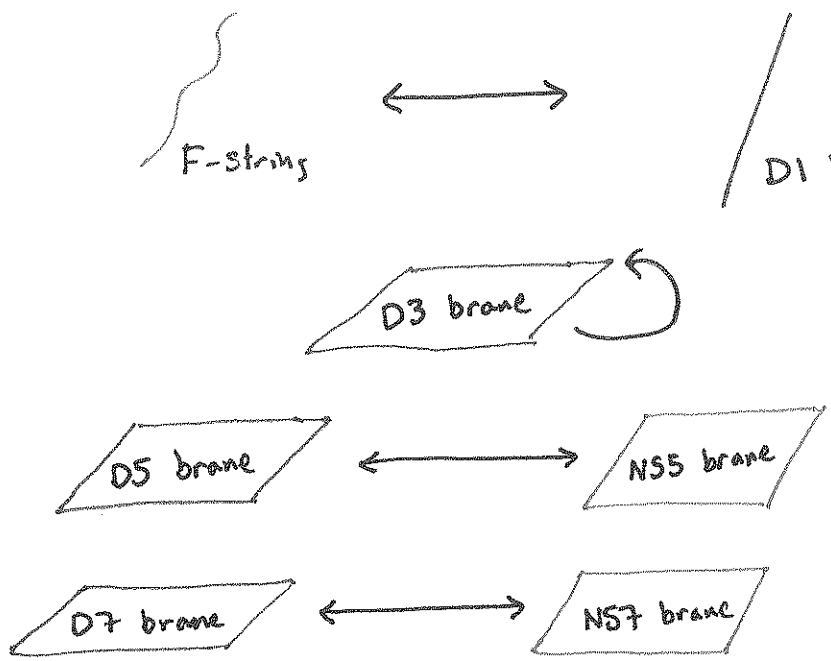
F1 NS5

elec. → F1, mag. → NS5

+ O-planes ...

Example #2

Type IIB string theory is self-dual



$C_4 \to C_4$

$B_2 \to C_2$
$C_2 \to -B_2$

$\boxed{g_s \to 1/g_s}$ for $C_0 = 0$

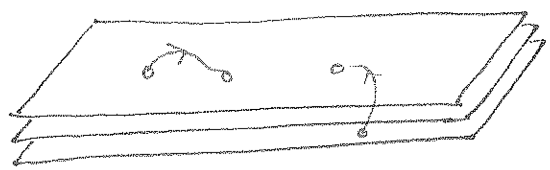
in <u>general</u> $\tau \to -1/\tau$

where $\tau \equiv C_0 + i/g_s$

<u>S-duality!</u>

D3 branes & S-duality

D3 has 4d worldvolume gauge theory:



Gauge bosons are F-string excitations

But $f \to f$ under S-duality

... gauge field transforms: $F_{\mu\nu} \to \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$

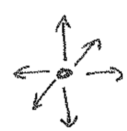
<u>Electromagnetic duality</u>!

What is this theory?

This is $\underline{\mathcal{N}=4}$ vector multiplet.

For N D3's, get

<u>(S)U(N) $\mathcal{N}=4$ SYM</u>

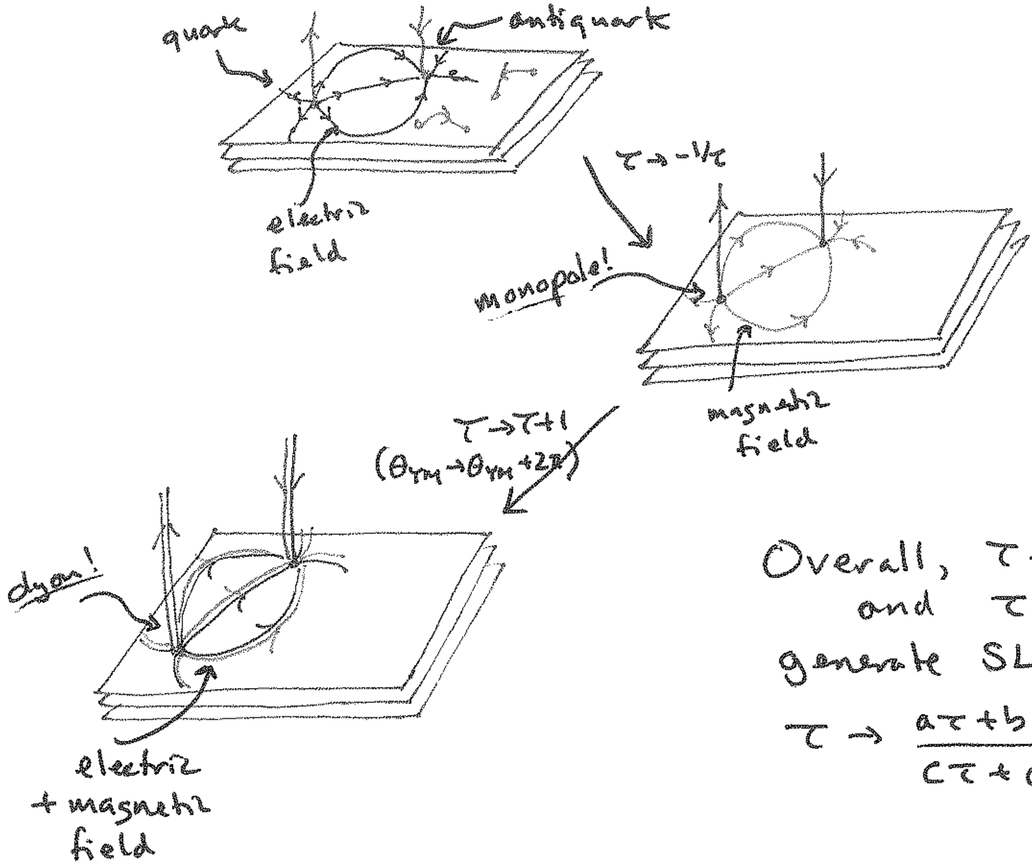


* 6 real scalars control position in transverse space — <u>vector</u> of SO(6)
* 4 Weyl fermions — <u>spinor</u> of SO(6)
* 1 gauge boson

$$\tau = C_0 + i/g_s = \frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g_{YM}^2}$$

Monopoles & Dyons

Consider attaching long strings:

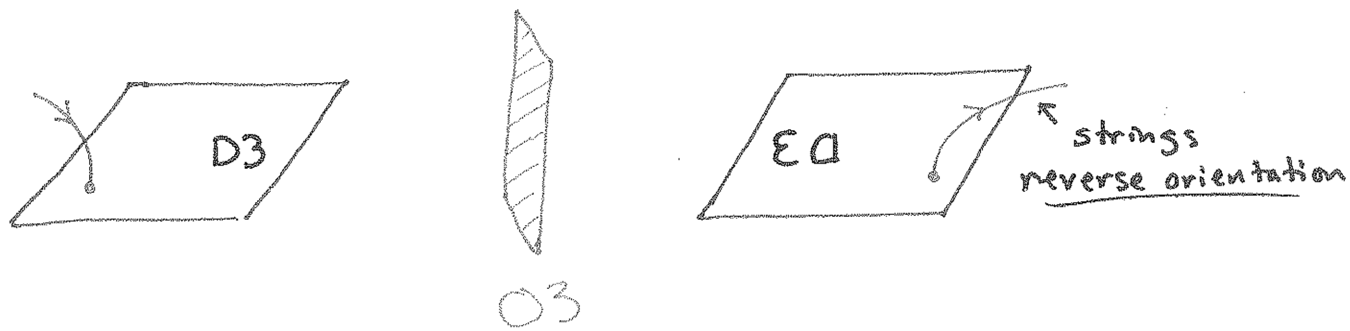


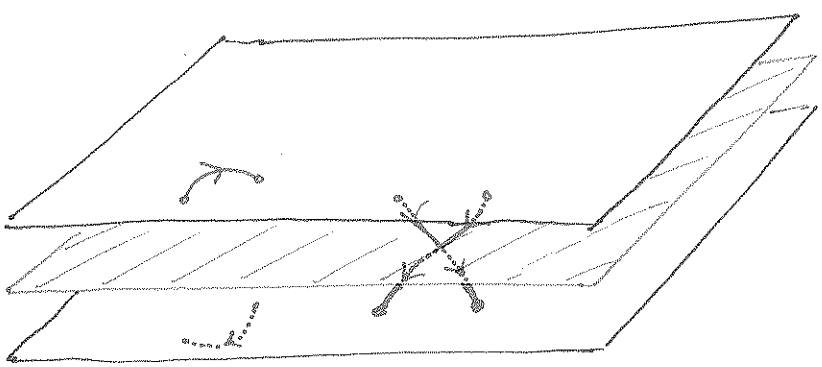
Overall, $\tau \to -1/\tau$
and $\tau \to \tau + 1$
generate $SL(2,\mathbb{Z})$

$$\tau \to \frac{a\tau + b}{c\tau + d} \quad (ad - bc = 1)$$

Orientifolds

Now we introduce orientifold planes:





D3 brane can interact w/ its own image $\Rightarrow$ extra light states, enhanced gauge group on top of O-plane,

e.g. $U(1) \longrightarrow Sp(2)$ $(O3^+)$

$\underline{Sp(2n) = USp(2n)}$

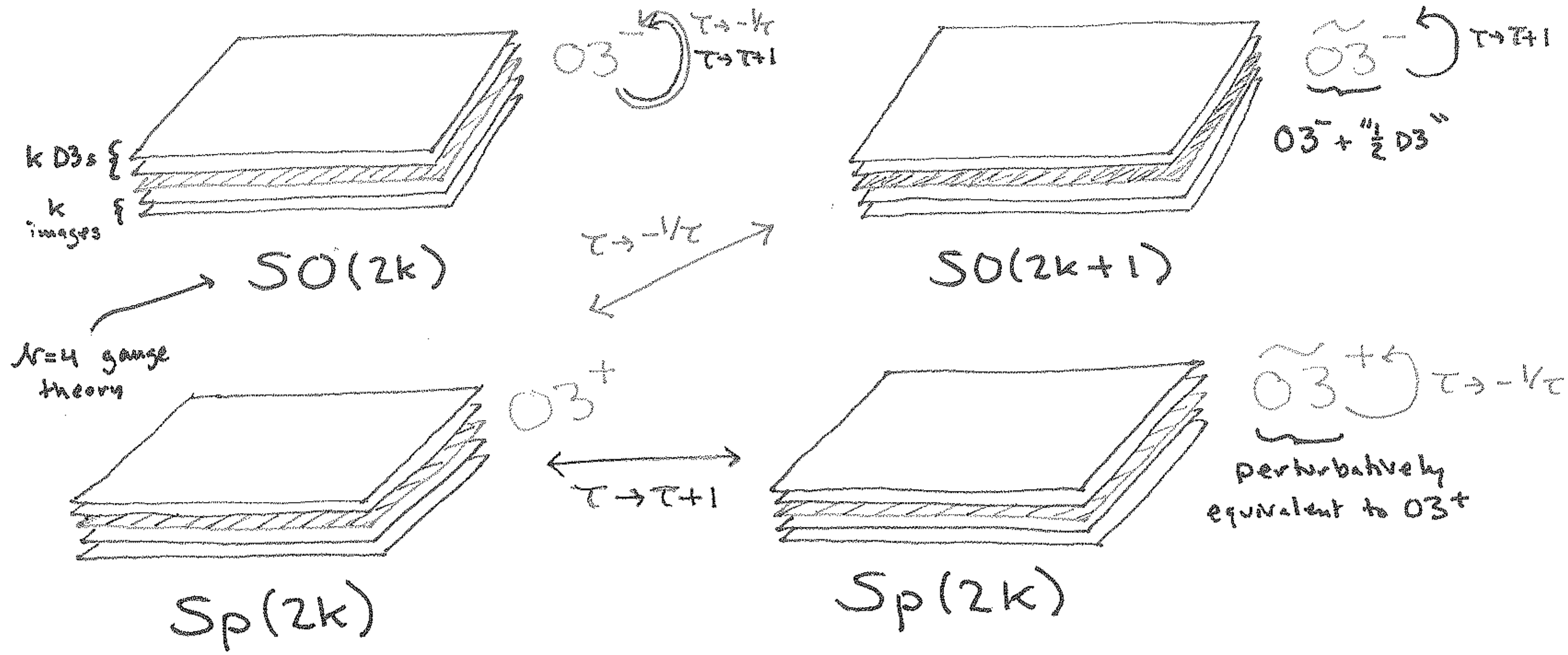
$2n \times 2n$ unitary matrices s.t.

$$M \Omega M^T = \Omega$$

for $\Omega = -\Omega^T$ symplectic form

$$\Omega = \begin{pmatrix} 0 & 1 & & & \\ -1 & 0 & & & \\ & & 0 & 1 & \\ & & -1 & 0 & \\ & & & & \ddots \end{pmatrix}$$

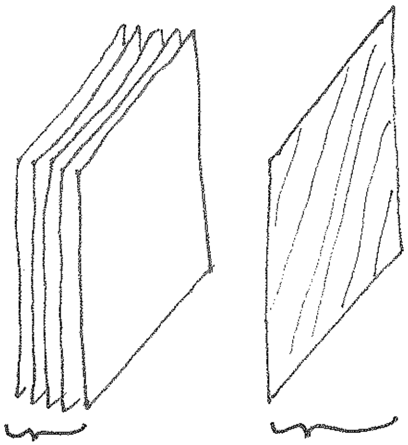
O3-planes & SL(2, $\mathbb{Z}$)



Electromagnetic duality can change the gauge group b/c O-plane is not always SL(2, $\mathbb{Z}$) invariant.

The closed-string picture ($g_s N \gg 1$)

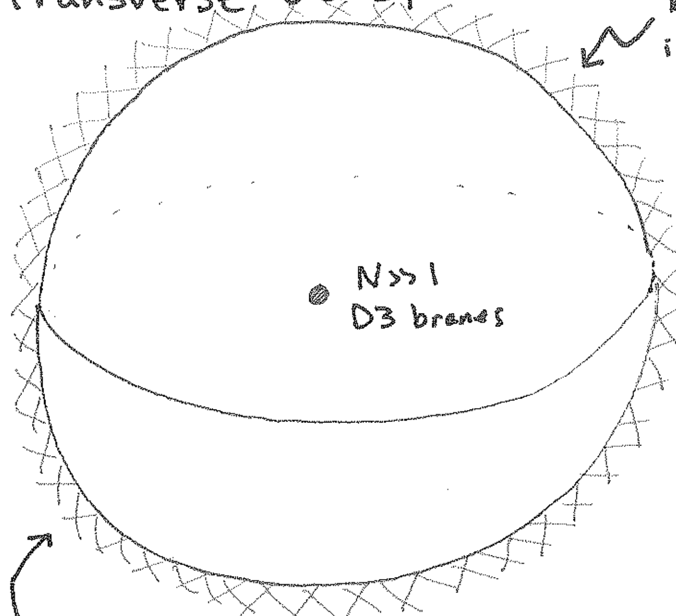
Closed strings see things differently:



$N \gg 1$
D3 branes

event
horizon

Transverse 6d space:



Near-horizon region is $\underbrace{AdS_5}_{\text{5d space w/ negative C.C.}} \times S^5$

<u>Maldacena '97</u>: Closed strings in the near-horizon region are <u>dual</u> to the 4d worldvolume gauge theory on the branes

"AdS/CFT"

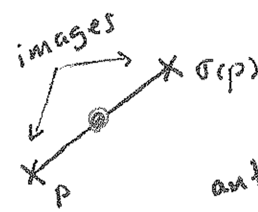
★ <u>Very general</u>

Orientifolds & AdS/CFT

Different orientifold planes must give different near-horizon geometries.

* Orientifold implies a projection

$$p \sim \sigma(p)$$



$(\sigma(\vec{x}) = -\vec{x},\quad \vec{x} = 6d \text{ vector})$

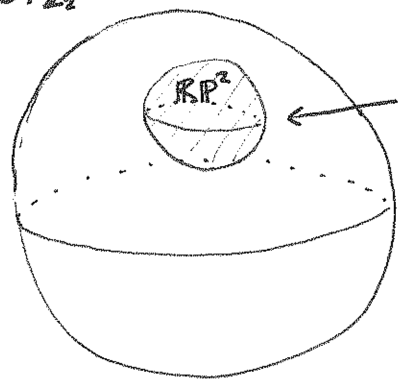
$\hookrightarrow$ near horizon geometry is $AdS_5 \times \overbrace{S^5/\mathbb{Z}_2}^{\text{antipodal pts identified}}$

Witten '98: $O3^{\pm}, \widetilde{O3}^{\pm}$ distinguished non-trivial two form connection:

$$B_2, C_2 \in H^3(S^5/\mathbb{Z}_2, \widetilde{\mathbb{Z}}) \cong \mathbb{Z}_2$$

"twisted" b/c $\sigma^* B_2 = -B_2$

$S^5/\mathbb{Z}_2 = \mathbb{RP}^5$



wrap D5 brane

$\hookrightarrow$ can't contract to a point b/c

$H_2(S^5/\mathbb{Z}_2, \widetilde{\mathbb{Z}}) \cong \mathbb{Z}_2$ nontrivial

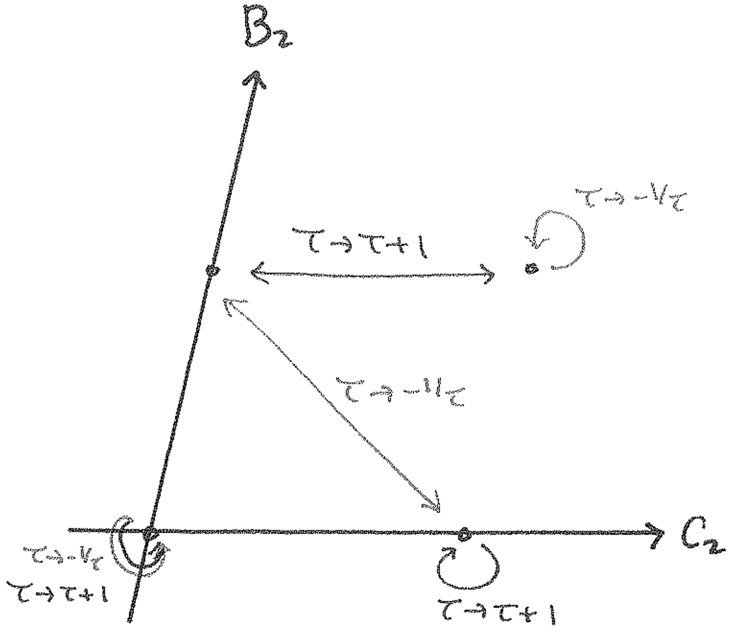
same as above by Poincare duality

$\sim$ $\boxed{\text{Dropping D5 through horizon induces nontrivial } C_2 \text{ torsion}}$

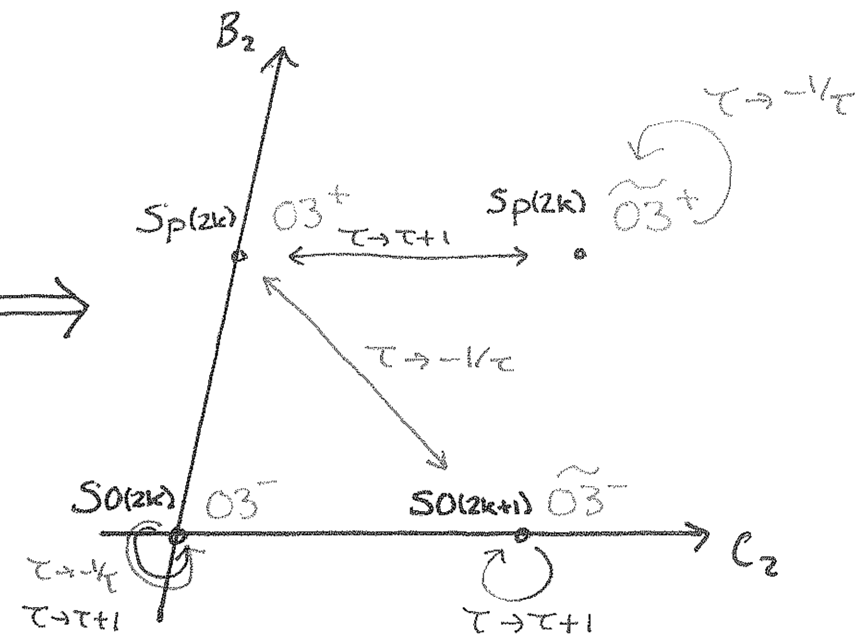
Orientifolds & AdS/CFT II

$\tau \to -1/\tau$ takes $B_2 \to C_2$, $C_2 \to -B_2$

$\tau \to \tau+1$ takes $B_2 \to B_2$, $C_2 \to C_2 + B_2$



$\Longrightarrow$



So:

Closed strings		Open strings
$SL(2,\mathbb{Z})$ action on B_2, C_2 torsion	$\longleftrightarrow$	$SL(2,\mathbb{Z})$ action on O3 planes

Into the woods...

EM duality of $\mathcal{N}=4$ gauge theories a beautiful subject

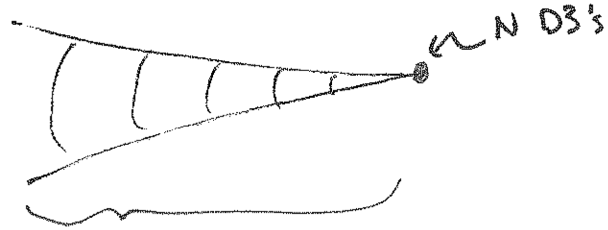
But... Very limited in scope

* Maximal SUSY
* No chirality
* Only adjoint matter

$\Longleftrightarrow$ Special class of S.T. backgrounds: D3's in $\mathbb{R}^6$ (w/o flux)

Look a little further from the lamppost:

D3 branes @ singularities



preserves $\mathcal{N}=1$ if geometry is Calabi-Yau

Why singularities?

* Branes in smooth background only see their immediate vecinity at low energies
* Any smooth space looks flat after zooming in enough

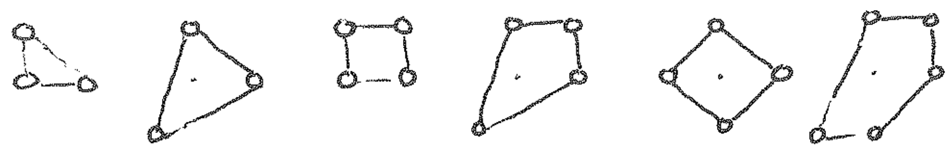
$\Rightarrow$ Only way to change the IR is to put branes @ singularity (or w/ background flux)

Toric Singularities

* We'll focus on a class of singularities:

  Toric singularities ($\approx U(1)^3$ or larger isometry)

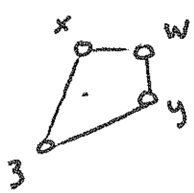
* These are classified by convex rational polytopes:



$\mathbb{C}^3$ $\quad$ $\mathbb{C}^3/\mathbb{Z}_3$ $\quad$ $\mathcal{C}$ $\quad$ dP_1 $\quad$ $\mathbb{F}_0$ $\quad$ dP_2 $\quad$ ...

(up to rotations, reflections, <u>shears</u>, translations)

* Geometry can be read off:



Identify under: $x \to \mu_x x$, $y \to \mu_y y$, $\vdots$

w/ $\mu_x \mu_y \mu_z \mu_w = 1 \quad (\prod_i \mu_i = 1)$

$\mu_x \mu_w \mu_z^{-1} = 1 \quad (\prod_i \mu_i^{p_i} = 1)$

$\underbrace{\mu_y \mu_w \mu_z^{-1} = 1}_{\text{solution is } \mu_x = \mu_y = \mu_z^{-2},\ \mu_w = \mu_z^3} \quad (\prod_i \mu_i^{q_i} = 1)$

So "dP_1" is the space:

$$\mathbb{C}^* \ \begin{array}{cccc} x & y & z & w \\ \hline 2 & 2 & -1 & -3 \end{array}$$

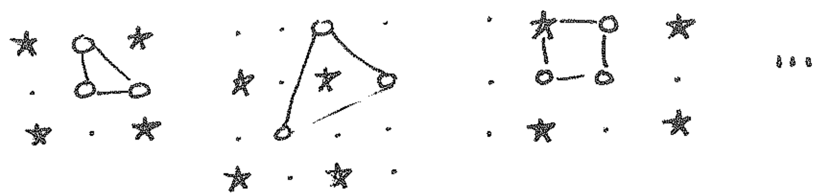
Toric Orientifolds

Generalize this to orientifolds...

* need to specify <u>involution</u>, σ ($\sigma(p)$ is "mirror image" of p)

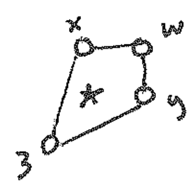
* Focus on <u>special class</u> such that $U(1)^3$ <u>unbroken</u>:

Choice of σ is choice of even sublattice:



... <u>4 choices</u> for each singularity

* How to read of involution:



σ:
$x \to \tilde{\mu}_x x$
$y \to \tilde{\mu}_y y$
$\vdots$

w/ $\tilde{\mu}_x \tilde{\mu}_y \tilde{\mu}_z \tilde{\mu}_w = -1$ $\quad (\prod_i \tilde{\mu}_i = -1)$

$$\underbrace{\begin{aligned} \tilde{\mu}_x \tilde{\mu}_w \tilde{\mu}_z^{-1} &= 1 \\ \tilde{\mu}_y \tilde{\mu}_w \tilde{\mu}_z^{-1} &= 1 \end{aligned}}_{\text{Pick any one solution, e.g.}} \qquad \underbrace{\begin{aligned} (\textstyle\prod_i \tilde{\mu}_i^{p_i} &= 1) \\ (\textstyle\prod_i \tilde{\mu}_i^{q_i} &= 1) \end{aligned}}_{\text{where } p_i = q_i = 0 \text{ lies on chosen sublattice}}$$

Pick any one solution, e.g.
$\cancel{\tilde{\mu}_z = \tilde{\mu}_w}$ $\tilde{\mu}_w = 1$
$\tilde{\mu}_x = \tilde{\mu}_y = \tilde{\mu}_z = -1$

where $p_i = q_i = 0$ lies on chosen sublattice

<u>dP_1 orientifold</u>

	x	y	z	w
$\mathbb{C}^*$	2	2	-1	-3
$\sigma: \mathbb{Z}_2$	-	-	-	+

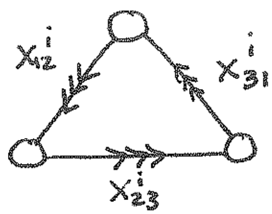
Toric gauge theories

What kind of gauge theory lives on the D3 branes?

Quiver gauge theory:

(Moose diagram)

$\mathbb{C}^3/\mathbb{Z}_3$:



same gauge theory

$$W = \epsilon_{ijk} X_{12}^i X_{23}^j X_{31}^k$$

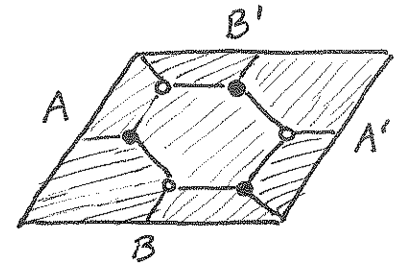
Here: $\otimes$ = (S)U(N) + adjoint vector mult. (gauge boson + gaugino)

$① \longrightarrow ②$ = $X^{n_1}{}_{n_2}$ fund. under 1, antifund. under 2

X = chiral mult = Weyl spinor + c'plex scalar

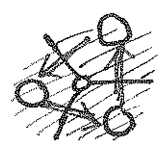
Superpotential built from loops in the quiver

Brane tiling



Bipartite graph on torus (glue A ⟷ A' and B ⟷ B')

* Face ⟷ (S)U(N) gauge group (+ vector mult)
* Edge ⟷ bifund. chiral mult.
* Vertex ⟷ superpotential term



(coefficient +1 (−1) for white (black))

Quivers v. Tilings

* Quivers are more general
* Superpotential must be specified in addition

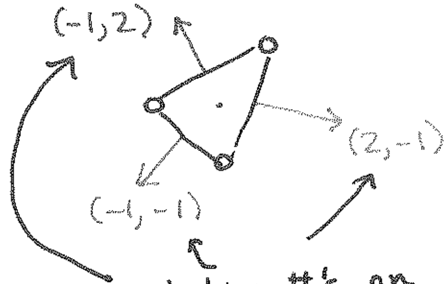
* Brane tilings possible for specific class of $\mathcal{N}=1$ gauge theories
* Superpotential is <u>built in</u>

This is exactly the class we are interested in (gauge theories of D3's @ toric singularities)

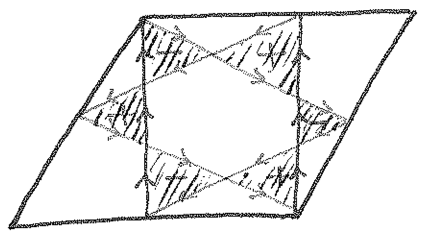
... quivers are still useful & more intuitive

The inverse algorithm

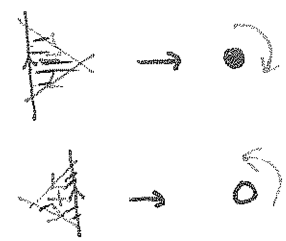
How to find the right brane tiling from toric diagram?

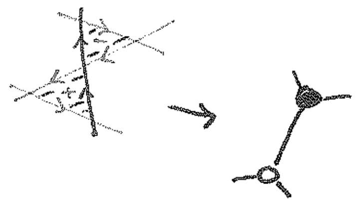


winding #'s on torus

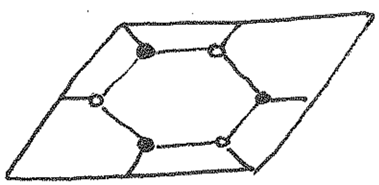


← shade <u>oriented</u> faces





We get:



(same as before)

<u>BUT</u> "algorithm" is ambiguous...
could have <u>multiple</u> <u>solutions</u>.

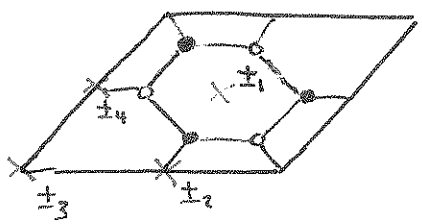
*In this case, find that the different "phases" are <u>Seiberg dual</u> to each other
⇒ IR physics is the same
"Toric duality"

Orientifold gauge theories

(Franco, Hanany, Krefl, Park, Uranga, Vegh '07)

Orientifold gauge theories can be found by "orientifolding" the tiling:

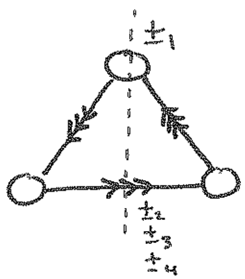
e.g.



For the $U(1)^3$-inv. case where $\pm_1 \pm_2 \pm_3 \pm_4 = -1$ $\left(= (-1)^{N_W} \quad N_W = \#\text{ of white nodes} \right)$

$\underbrace{\circ \quad \times \quad \bullet}$ orientifold exchanges white $\leftrightarrow$ black

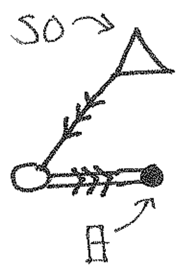
OR:

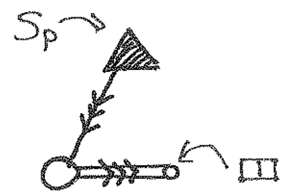


where $+$ $(-)$ fixed face [node] $\rightarrow$ SO (Sp) gauge grp.

$+$ $(-)$ fixed edge $\rightarrow$ symmetric (antisymmetric) tensor matter ($\square\square$ or $\begin{smallmatrix}\square\\\square\end{smallmatrix}$)

Notation: "quiverfold"





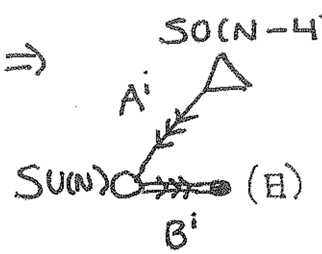
New $\mathcal{N}=1$ dualities

(García-Etxebarria, BJH, Wrase, '12)

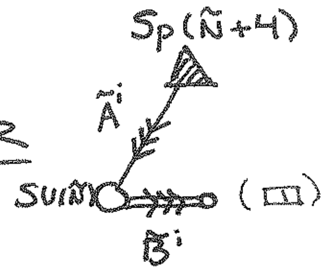
Look at these examples in detail:

★ Only anomaly-free cases are for $\pm_2 = \pm_3 = \pm_4$

$\Rightarrow$



<u>OR</u>



$$W = \frac{1}{2}\epsilon_{ijk} A^i A^j B^k \qquad\qquad W = \frac{1}{2}\epsilon_{ijk} \tilde{A}^i \tilde{A}^j \tilde{B}^k$$

<u>Observation:</u> These two theories are <u>dual</u> for odd N, $\boxed{N = \tilde{N}+3}$

<u>Checks:</u> ★ Anomaly matching

★ Moduli space matching

★ Matching of superconformal index

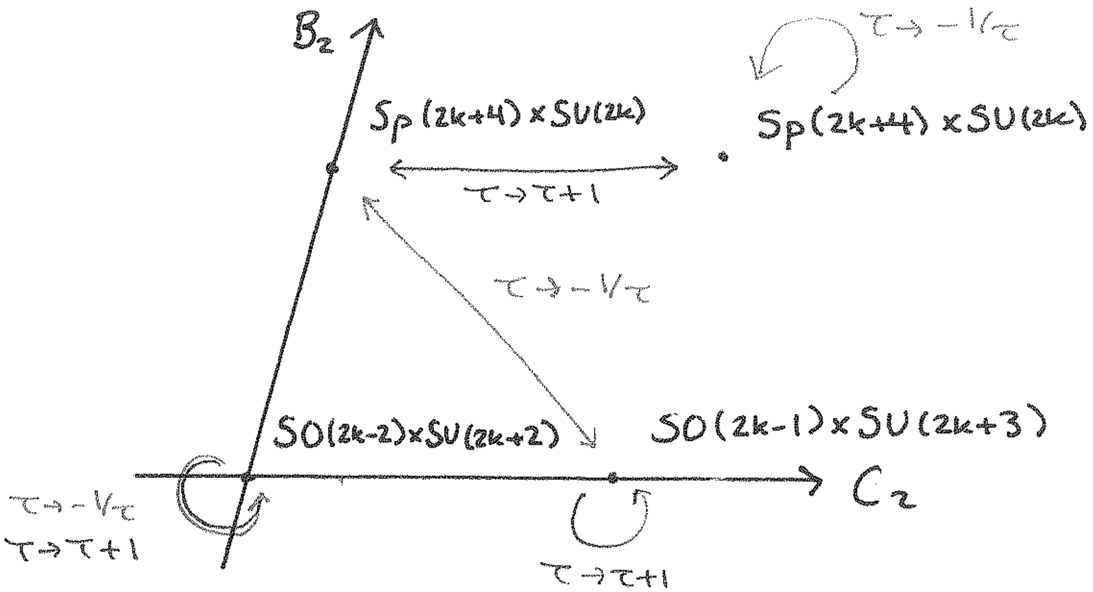
★ Low N checks

The AdS dual

What distinguishes the gravity duals?

* As in $\mathcal{N}=4$ cases, $H_2(S^5/\mathbb{Z}_6, \tilde{\mathbb{Z}}) = \mathbb{Z}_2$

Can argue that right torsion assignments are:
(I. G-E, B. H., T. W.) '13)



So the observed duality comes from $SL(2,\mathbb{Z})$ in IIB
<u>just like</u> EM duality!

Orbifolds in general

(I. G-E, B. H., T. W. '13; Bianchi et al, '13)

* Let's generalize to other orbifolds (3-sided T.D.'s) subject to two restrictions:

I. Isolated singularity:

NOT e.g.

II. Compact O-planes

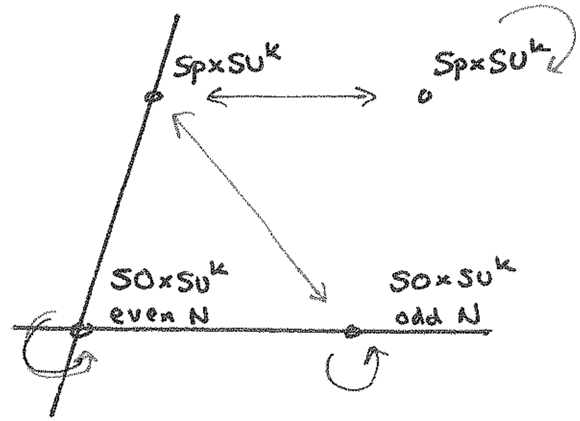
NOT e.g.

Then in general can show $G = SO \times SU \times SU \times \ldots$ ∋ even or odd N

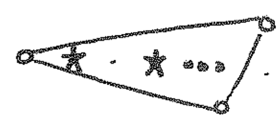
OR $G = Sp \times SU \times SU \times \ldots$ ∋ even $\tilde{N}$

$H_2(S^5/\mathbb{Z}_{2k+1}, \tilde{\mathbb{Z}}) = \mathbb{Z}_2$

$\Rightarrow$ guess



☆ Anomaly matching Checked explicitly for:



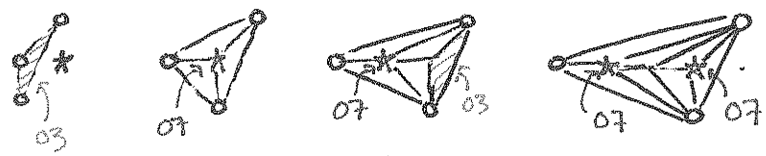
$\mathbb{C}^3/\mathbb{Z}_{2k+1}$

$\underline{n = 2k+1}$: $z_{1,2} \to e^{2\pi i/n} z_{1,2}$

$z_3 \to e^{-4\pi i/n} z_3$

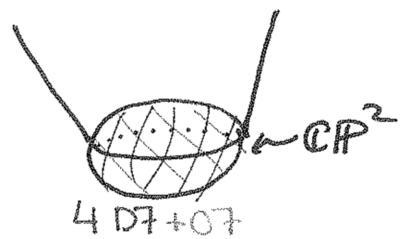
Brane interpretation

Resolve the singularity:



$\mathbb{C}^3$ $\mathbb{C}^3/\mathbb{Z}_3$ $\mathbb{C}^3/\mathbb{Z}_5$ $\mathbb{C}^3/\mathbb{Z}_7$ ... get's complicated fast...

e.g. $\mathbb{C}^3/\mathbb{Z}_3$:



★ Action of $SL(2,\mathbb{Z})$ on the branes hard to track

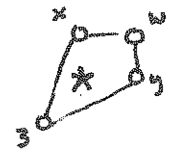
... done using F-theory / fractional branes for $\mathbb{C}^3/\mathbb{Z}_3$

(I. G-E, BJH, T.W., '13)

Beyond orbifolds

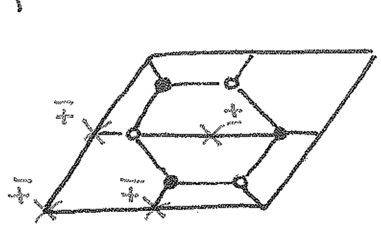
(I. G-E, BJH, T.W., '12 & Forthcoming)

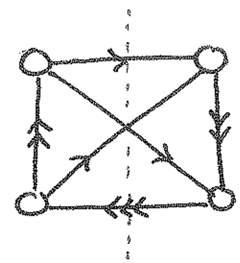
Consider dP_1 orientifold:



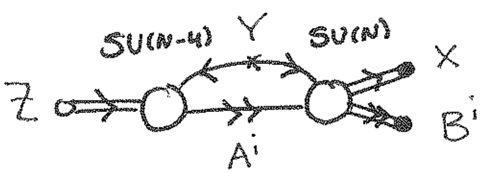
	x	y	3	w
$\mathbb{C}^*$	2	2	-1	-3
$\mathbb{Z}_2$	−	−	−	+

* Only one phase
* Tiling is



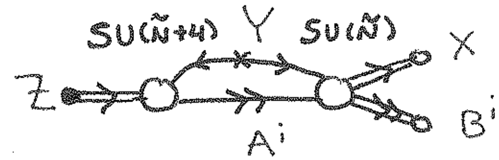


I_A:



$$W = \varepsilon_{ij} B^i A^j Y + \tfrac{1}{2}\varepsilon_{ij} X A^i Z A^j$$

I_B:



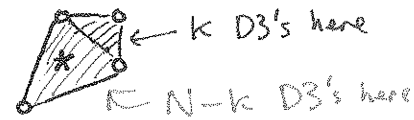
$$W = \varepsilon_{ij} B^i A^j Y + \tfrac{1}{2}\varepsilon_{ij} X A^i Z A^j$$

These theories are dual for odd N, $\boxed{N = \tilde{N} + 2}$

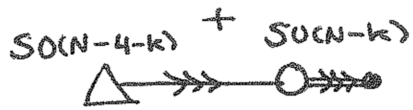
(not dual for even N)

A puzzle

What happens for even N, $\tilde{N}$?

Look at partial resolution: 

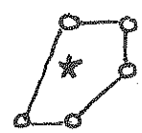
Get (from I_A): 

+ 

$\Rightarrow$ <u>k even</u>

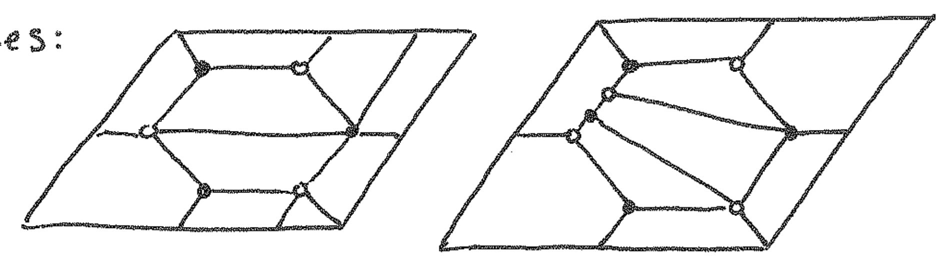
... but ~~neith~~ Sp(k) component (w/ $\mathcal{N}=4$) <u>not</u> self-dual,
so I_A, even N <u>not</u> self-dual. (Similarly for I_B, even $\tilde{N}$)

<u>So where are their duals?</u>

<u>A bigger puzzle</u>: Consider dP$_2$ orientifold: 

Two phases:

<u>Neither admits an involution</u>



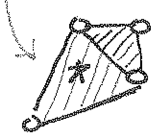
Insight from the gravity dual

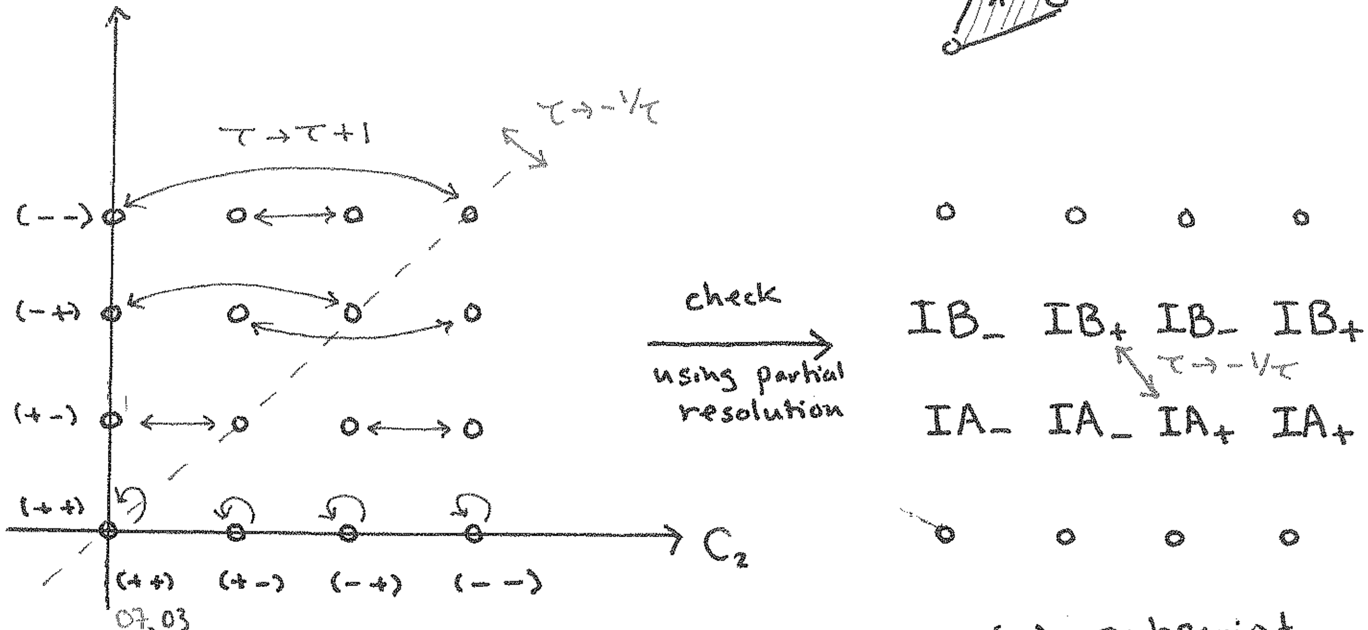
We find $H_2\left(\frac{S^2\times S^3}{\mathbb{Z}_2}, \widetilde{\mathbb{Z}}\right) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$

torsion for O7

torsion for O3

Under partial resolution:

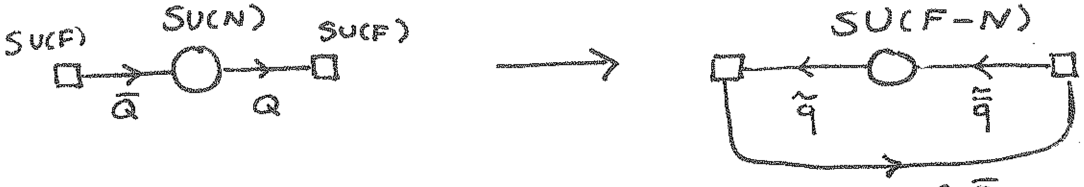




where + (−) subscript denotes the case where $S \equiv N-1 = \tilde{N}+1$ is even (odd)

★ Explains $IA_+ \leftrightarrow IB_+$ duality, but first/last row still missing...

Seiberg duality & tilings

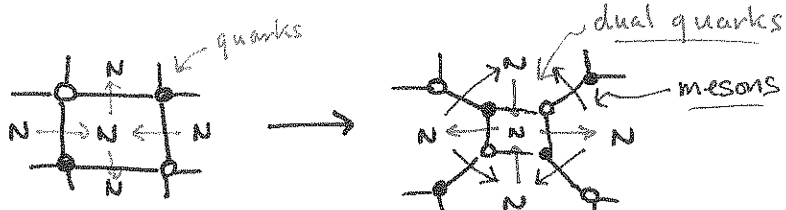


$$W = \frac{1}{\mu} M \tilde{q} \tilde{\bar{q}}$$

$$M = Q\bar{Q}$$

"Electric theory" "Magnetic theory"

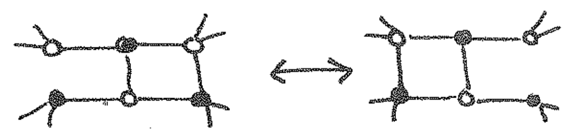
S.D. in brane tilings:



$F = 2N$

$\Rightarrow F - N = N$

Ex:

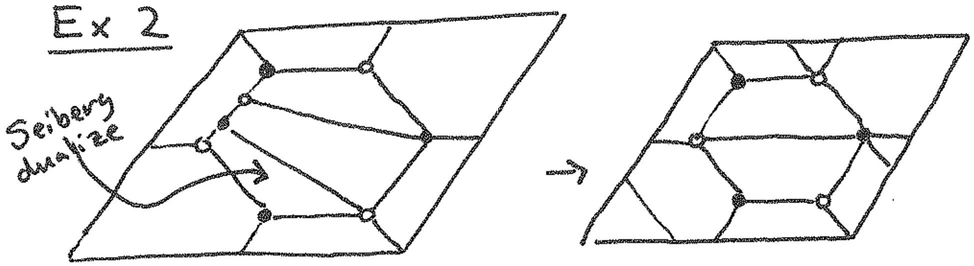


where 

$\rightarrow$ 

upon integrating out massive fields

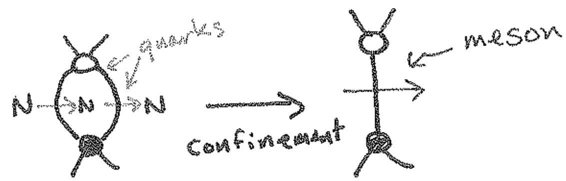
Ex 2



Relates phases of dP_2 as expected.

(De)confinement

Another example of S.D. in brane tilings:



$F = N$

$\Rightarrow F - N = 0$

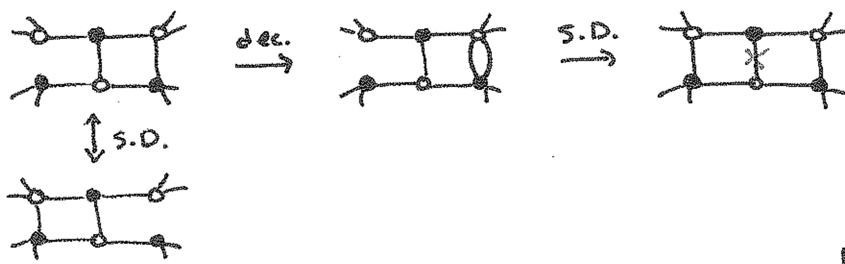
$\Rightarrow$ no magnetic gauge group!

(Actually, this is confinement w/ chiral symmetry breaking

$\Rightarrow$ have to be a bit careful

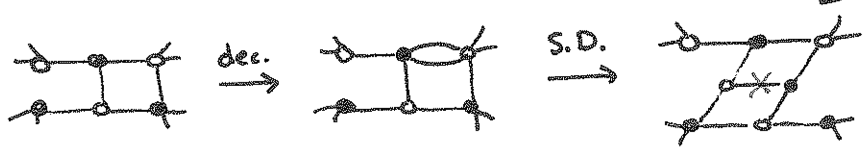
... but ignore this subtlety for now)

Ex 1:



admits involution!

Ex 2:

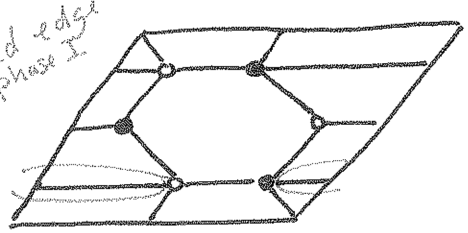


So deconfinement allows new involutions...

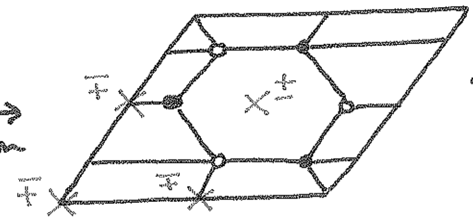
dP_1 phase II

Use method of Ex. 1 to construct new phase for dP_1:

added edge rel. to phase I

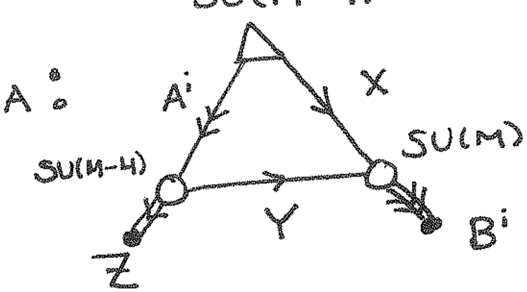


$\xrightarrow[\text{involution}]{\text{admits}}$



"Phase II"

$\Rightarrow$ II_A:



$$W = \tfrac{1}{2}\varepsilon_{ij} A^i A^j Z + \varepsilon_{ij} A^i Y B^j X$$

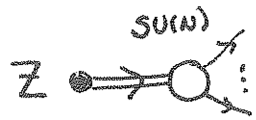
Can this fill in any of the gaps ... ?

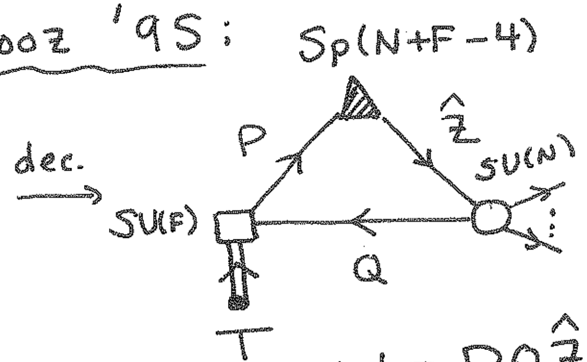
Not quite! The global symmetries don't match!

Deconfinement & Flavors

We were too cavalier about deconfinement.

c.f. e.g. Berkooz '95:





$$Z = \hat{Z}^2$$

$$W = PQ\hat{Z} + TP^2$$

$F = N \bmod 2$

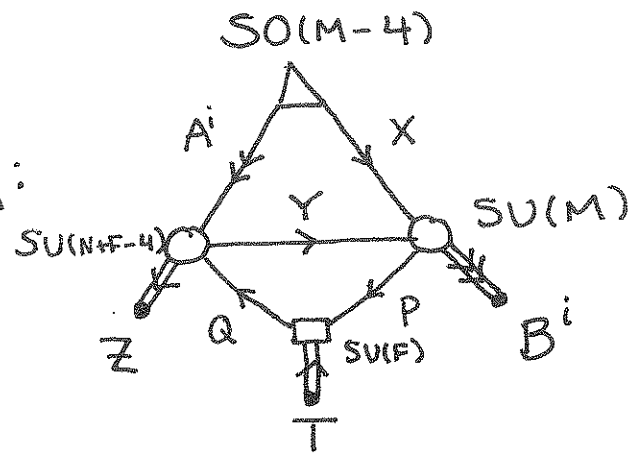
★ Extra matter fields
T, P, Q + $W(T, P, Q, \hat{Z})$
are engineered to remove constraints
on the confined meson (e.g. being rank-deficient)

★ Also needed to match global symmetries

Phase II "with flavors"

With more care, we find:

II_A:



$$W = \varepsilon_{ij} A^i Y B^j X + \tfrac{1}{2} \varepsilon_{ij} A^i A^j Z + YPQ + TQ^2 Z$$

formally ~~same as~~ similar to adding flavor D7 branes

* Can show using S.D., deconfinement, that all values of F lead to same IR physics, up to parity.
* Anomalies match w/ phase I for $M = N+1 = S+2$

$\Rightarrow$ 4 theories: $\mathrm{II}A^+_+$, $\mathrm{II}A^-_+$, $\mathrm{II}A^+_-$, $\mathrm{II}A^-_-$ ← flavor parity $(-1)^F$ ← color parity $(-1)^S$

Right # to fill the bottom row...

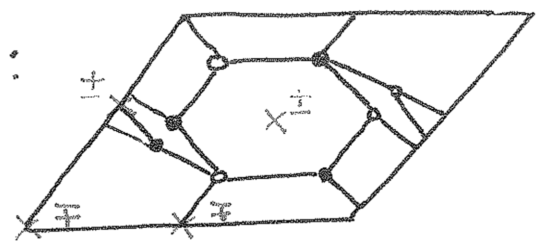
Phase III

★ II_B (II_A w/ $SO \to Sp$, $\begin{smallmatrix}\square\\\square\end{smallmatrix} \to \square\square$) doesn't seem to work

... look elsewhere for theories to fill top row.

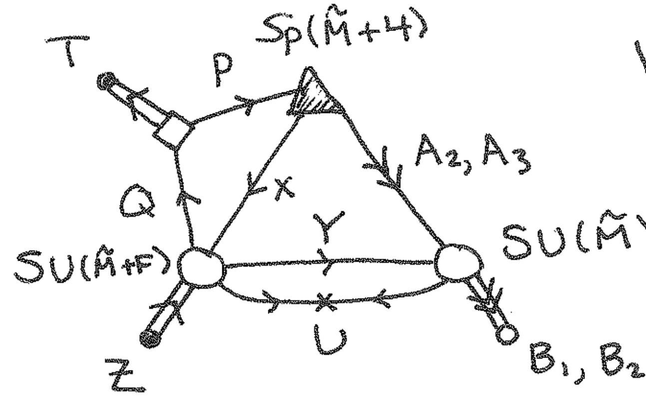
★ Apply deconfinement method of Ex. 2 to get another phase:

Phase III:



SU(2) is hidden (only $U(1) \subset SU(2)$ manifest) but otherwise symmetries match

$\Rightarrow$ III_B:



$$W = A_2A_3B_1 + XUA_2 + ZYU + XYB_2A_3 + XPQ + TQ^2Z$$

Anomaly matching:

$\boxed{\tilde{M} = S-2}$

$\Rightarrow$ two theories, III B^+, III B^- ← flavor parity

Right # for top row

Torsion & dualities

Now we can fill in the square using partial resolution:

$$\begin{tikzcd}
\mathrm{IIB}^+ \arrow[rrr, bend left, leftrightarrow, "\tau \to \tau+1"] & \mathrm{IIB}^- \arrow[r, leftrightarrow] & \mathrm{IIB}^- & \mathrm{IIB}^+ \\
\mathrm{IB}_- \arrow[rr, bend left, leftrightarrow] & \mathrm{IB}_+ \arrow[rr, bend right, leftrightarrow] & \mathrm{IB}_- & \mathrm{IB}_+ \\
\mathrm{IA}_- \arrow[r, leftrightarrow] & \mathrm{IA}_- & \mathrm{IA}_+ \arrow[r, leftrightarrow] & \mathrm{IA}_+ \\
\mathrm{IIA}^+_+ \arrow[loop below] & \mathrm{IIA}^-_- \arrow[loop below] & \mathrm{IIA}^+_- \arrow[loop below] & \mathrm{IIA}^-_+ \arrow[loop below]
\end{tikzcd}$$

$\tau \to -1/\tau$

(Placement ambiguity from partial resolution... resolved by duality considerations)

Predicts the dualities:

$\mathrm{IA}_- \leftrightarrow \mathrm{IIA}^-_-$

$\mathrm{IB}_- \leftrightarrow \mathrm{IIA}^+_-$

$\mathrm{IIB}^+ \leftrightarrow \mathrm{IIA}^-_+$

$\mathrm{IA}_+ \leftrightarrow \mathrm{IB}_+ \leftrightarrow \mathrm{IIB}^-$

... checked by computing the superconformal index to high order. ✓

Conclusions

* $SL(2,\mathbb{Z})$ invariance of type IIB string theory gives a broad (largely unexplored) class of $\mathcal{N}=1$ gauge theory dualities ... generalizing EM duality.

* These dualities reveal our <u>incomplete</u> understanding of D3 branes @ orientifold singularities.

* Dualities provide a highly non-trivial check on any attempt to solve the problem (as w/ dP_1 above)

* dP_1 example now seems understood ... other examples under study ($\mathbb{F}_0$, dP_2, dP_3, ...)

* In general, $\underbrace{H_2(Y/\mathbb{Z}_2, \tilde{\mathbb{Z}}) = \mathbb{Z}_2^k}_{\text{grows rapidly!}}$ where $k+2 = \#$ of sides of toric dia.

... this is the "wilderness" of orientifolds.