World-sheet supersymmetry and supertargets

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Chapel Hill, August 19, 2010

arXiv:1006.5874 Thomas Creutzig, PR

Sigma-models on supergroups/cosets

The Wess-Zumino-Novikov-Witten (WZNW) model of a Lie supergroup is a CFT with additional affine Lie superalgebra symmetry.

Important role in physics

- Statistical systems
- String theory, AdS/CFT

Notoriously hard: Deformations away from WZNW-point

Use and explore the rich structure of dualities and correspondences in 2d field theories

Sigma models on supergroups: AdS/CFT

The group of global symmetries of the gauge theory and also of the dual string theory is a Lie supergroup G.

The dual string theory is described by a two-dimensional sigma model on a superspace.

$AdS_5 \times S^5$	$\dfrac{PSU(2,2\|4)}{SO(4,1) \times SO(5)}$	supercoset
$AdS_2 \times S^2$	$\dfrac{PSU(1,1\|2)}{U(1) \times U(1)}$	supercoset
$AdS_3 \times S^3$	$PSU(1,1\|2)$	supergroup
$AdS_3 \times S^3 \times S^3$	$D(2,1;\alpha)$	supergroup

Obtained using GS or hybrid formalism. What about RNS formalism? And what are the precise relations?

$AdS_3 \times S^3 \times T^4$

D1-D5 brane system on T^4 with near-horizon limit $AdS_3 \times S^3 \times T^4$. After S-duality we have $\mathcal{N} = (1,1)$ WS SUSY WZNW model on $\mathrm{SL}(2) \times \mathrm{SU}(2) \times \mathrm{U}^4$. It has $\mathcal{N} = (2,2)$ superconformal symmetry (as we will see).

Hybrid formalism of string theory on $AdS_3 \times S^3$ gives a sigma model on PSU(1,1|2) .
RR-flux is deformation away from WZ-point.
[Berkovits, Vafa, Witten]

Dual 2-dimensional CFT is deformation of symmetric orbifold $Sym_N(T^4) = (T^4)^N/S_N$. It has $\mathcal{N} = (4,4)$ superconformal symmetry.

Recent progress: Chiral ring

[Kirsch, Gaberdiel][Dabholkar, Pakman]

Computed chiral ring in the NS-formulation.

Results match the boundary computations, but at a different point in moduli space.

Correlators are very simple

$$\langle \mathcal{O}_1(z_1, \bar{z}_1)\mathcal{O}_2(z_2, \bar{z}_2)\mathcal{O}_3(z_3, \bar{z}_3)\rangle = \frac{C_{123}}{\prod |z_i - z_j|^{\Delta_{ij}}}$$

where $C_{123} = 0$ or $C_{123} = 1$.

- Why?

Correspondence

$$S^{\mathcal{N}=(2,2)}_{\mathrm{GL(N)}\times\mathrm{GL(N)}} + \frac{1}{2\pi}\int d\tau d\sigma\ \Phi \quad \xrightarrow{\text{Def. + B-twist}} \quad S_{\mathrm{GL(N|N)}}$$

$$G^{+}(z) \quad \xrightarrow{\text{Def. + B-twist}} \quad J^{F}(z)$$

$$G^{-}(z) \quad \xrightarrow{\text{Def. + B-twist}} \quad \sum J^{F}(z)J^{B}(z) + \partial J^{F}$$

$$U(z) \quad \xrightarrow{\text{Def. + B-twist}} \quad J^{B}(z)\,.$$

Outline

$\mathcal{N} = (1,1)$ world-sheet SUSY WZNW model

[di Vecchia, Knizhnik, Petersen, Rossi]

- Bosonic Lie group H
- Introduce superspace in 2d: θ^1, θ^2
- Map $G : \Sigma_{2|2} \mapsto H$
- Components

$$G(z, \bar{z}, \theta^\alpha) = g + i\bar{\theta}\psi + \frac{1}{2} i\bar{\theta}\theta F$$

- $\mathcal{N} = (1,1)$ WZNW Lagrangian

$$S^{\mathcal{N}=(1,1)}_{\mathrm{WZNW}}[G] = \frac{1}{4\lambda^2} \int d^2z d^2\theta \,\mathrm{tr}(\bar{D}G^{-1}DG) + \frac{k}{16\pi} \int d^2x d^2\theta dt \,\mathrm{tr}(G^{-1}\partial_t G \bar{D}G^{-1}\gamma_5 DG)$$

Decoupling the fermions

- $\mathcal{N} = (1,1)$ for all λ, k
- Enhanced affine symmetry at the WZNW point

$$\lambda^2 = \frac{8\pi}{k}$$

- WZNW model is "rigid": Fermions decouple
- $\{t^a\}$ basis for $\mathfrak{g}$
- Redefine $\psi \mapsto \chi$

$$\chi = \chi_a t^a, \qquad \bar{\chi} = \bar{\chi}_a t^a$$

$$G = \exp(i\theta\chi)\; g\; \exp(-i\bar{\theta}\bar{\chi})$$

$$S_{\mathrm{WZNW}}^{\mathcal{N}=(1,1)}[G] = S'_{WZNW}[g] \;+\; \frac{1}{2\pi}\int d^2z\,(\chi, \bar{\partial}\chi) + (\bar{\chi}, \partial\bar{\chi})$$

Quantum measure

- $(\ ,\ )$ non-degenerate, invariant bi-linear form
- Absorbed k into $(\ ,\ )$
- Quantum measure effect from chiral decoupling of fermions: $S'_{WZNW}[g]$ has bilinear form

$$(\ ,\ ) \mapsto (\ ,\ ) + \frac{1}{2} <\ ,\ >_{\mathrm{Killing}}$$

Current symmetries

- $\hat{G} \times \hat{G}$ affine algebra

$$J^a(z)J^b(w) \sim \frac{(t^a, t^b) + \frac{1}{2} < t^a, t^b >_{\text{Killing}}}{(z-w)^2} + \frac{f^{ab}{}_c J^c(w)}{(z-w)}$$

$$\bar{J}^a(\bar{z})\bar{J}^b(\bar{w}) \sim \frac{(t^a, t^b) + \frac{1}{2} < t^a, t^b >_{\text{Killing}}}{(\bar{z}-\bar{w})^2} + \frac{f^{ab}{}_c \bar{J}^c(\bar{w})}{(\bar{z}-\bar{w})}$$

- Free fermions

$$\chi^a(z)\chi^b(w) \sim \frac{(t^a, t^b)}{(z-w)} \qquad \bar{\chi}^a(\bar{z})\bar{\chi}^b(\bar{w}) \sim \frac{(t^a, t^b)}{(\bar{z}-\bar{w})}$$

- Implies $\mathcal{N} = (1, 1)$ superconformal symmetry (schematically)

$$T(z) = (J(z), J(z)) + (\chi(z), \partial\chi(z))$$

$$G(z) = (J(z), \chi(z)) + ([\chi(z), \chi(z)], \chi(z))$$

$\mathcal{N} = (2, 2)$ superconformal symmetry

- When can we construct the $\mathcal{N} = (2, 2)$ current algebra?

$$
\begin{aligned}
G^+(z)G^-(w) &\sim \frac{c/3}{(z-w)^3} + \frac{U(w)}{(z-w)^2} + \frac{T(w) + \frac{1}{2}\partial U(w)}{(z-w)} \\
G^\pm(z)G^\pm(w) &\sim 0 \\
U(z)G^\pm(w) &\sim \frac{\pm G^\pm(w)}{(z-w)} \\
U(z)U(w) &\sim \frac{c/3}{(z-w)^2}
\end{aligned}
$$

- Fermionic currents $G^\pm$, $\Delta(G^\pm) = 3/2$
- Bosonic $\mathrm{U}(1)$ current U, $\Delta(U) = 1$

Manin decomposition

[Getzler]

- Current construction possible if algebra has Manin decomposition

$$\mathfrak{g} = \mathfrak{a}_+ \oplus \mathfrak{a}_-$$

- $\mathfrak{a}_\pm$ isotropic Lie subalgebras
- $\{x^i\}$ basis $\mathfrak{a}_+$
- $\{x_i\}$ dual basis $\mathfrak{a}_-$

$$\begin{aligned}(x_i, x^j) &= \delta_i^j \\ [x_i, x_j] &= c_{ij}{}^k x_k \\ [x^i, x^j] &= f^{ij}{}_k x^k \\ [x_i, x^j] &= c_{ki}{}^j x^k + f^{jk}{}_i x_k\end{aligned}$$

- Fermions now $(\frac{1}{2},\frac{1}{2})$ ghost systems
  $\chi_j(z)\chi^i(w) \sim \delta^i_j/(z-w)$

$\mathcal{N} = (2,2)$ construction

- Sugawara Virasoro tensor

$$T(z) \;=\; \frac{1}{2}(:J^iJ_i: + :J_iJ^i: + :\partial\chi^i\chi_i: - :\chi^i\partial\chi_i:)$$

- Fermionic currents

$$\begin{aligned} G^+(z) \;&=\; J_i\chi^i - \frac{1}{2}c_{ij}{}^k \;:\chi^i\chi^j\chi_k: \\ G^-(z) \;&=\; J^i\chi_i - \frac{1}{2}f^{ij}{}_k \;:\chi_i\chi_j\chi^k: \end{aligned}$$

- $\mathrm{U}(1)$ current

$$U(z) \;= :\chi^i\chi_i: + \rho^k J_k + \rho_k J^k + c_{mn}{}^i f^{mn}{}_j :\chi^j\chi_i: \;.$$

where

$$\rho = -[x^i, x_i]$$

Deformations

- Wide range of deformations with background charges
- Define $\mathfrak{a}_0$
  $\mathfrak{a}_0 = \{x \in \mathfrak{g} \,|\, (x,y) = 0 \,\forall\, y \in [\mathfrak{a}_+, \mathfrak{a}_+] \oplus [\mathfrak{a}_-, \mathfrak{a}_-]\}$
- $a = p^i x_i + q_i x^i \in \mathfrak{a}_0$

$$
\begin{aligned}
G_a^+ &= G^+ + q_i\,\partial\chi^i \\
G_a^- &= G^- + p^i\,\partial\chi_i
\end{aligned}
$$

- Implies

$$
U_a(z) = U(z) + p^i\, I_i(z) - q_i\, I^i(z)
$$

$$
T_a(z) = T(z) + \frac{1}{2}(p^i\,\partial I_i(z) + q_i\,\partial I^i(z))
$$

- Super-version of affine currents

$$
\begin{aligned}
I_i &= J_i - c_{ij}{}^k :\chi^j\chi_k: - \frac{1}{2} f^{jk}{}_i :\chi_j\chi_k: \\
I^i &= J^i - f^{ij}{}_k :\chi_j\chi^k: - \frac{1}{2} c_{jk}{}^i :\chi^j\chi^k:
\end{aligned}
$$

Deformations

- Chiral ring unchanged as vector space
- Actually, unchanged as ring: Deformation acts as spectral flow on the zero modes
- Does change conformal dimension

$$c_a = c - 6q_i p^i$$

Lie superalgebras and supergroups

- Lie Superalgebra $\mathcal{G} = \mathcal{G}_0 \oplus \mathcal{G}_1$
- $\mathbb{Z}_2$ graded product

$$[\mathcal{G}_0, \mathcal{G}_0] \subset \mathcal{G}_0 \text{ (subalgebra)}$$
$$[\mathcal{G}_0, \mathcal{G}_1] \subset \mathcal{G}_1$$
$$[\mathcal{G}_1, \mathcal{G}_1] \subset \mathcal{G}_0$$

- Graded anti-symmetry

$$[X, Y] = -[Y, X]$$
$$[X, Y] = -[Y, X]$$
$$[X, Y] = [Y, X]$$

- Generalised Jacobi identity

Lie superalgebras and supergroups

- Fundamental supermatrix representation, gl(M|N)

$$M = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$$

- Supertrace (invariant, supersymmetric bilinear form)

$$\mathrm{str} M = \mathrm{tr}(A) - \mathrm{tr}(D)$$

- Grassmann envelope $\eta_a^{(0)} T_a^{(0)} + \eta_b^{(1)} T_b^{(1)}$
  $\eta_a^{(0)}, \eta_b^{(1)}$ Grassmann elements
  $T_a^{(0)}, T_b^{(1)}$ basis for $\mathcal{G} = \mathcal{G}_0 \oplus \mathcal{G}_1$
- Supergroup

$$\exp\left(\eta_a^{(0)} T_a^{(0)} + \eta_b^{(1)} T_b^{(1)}\right)$$

Free fermion resolution

[Quella, Schomerus]

- Introduce generators for super Lie algebra

$$\left(\begin{array}{c|c} K^i & S_1^a \\ \hline S_{2a} & K^i \end{array}\right)$$

- Non-degenerate invariant bi-linear form $\langle A, B\rangle \;=\; k\mathrm{str}(AB)$

$$\langle S_1^a, S_{2b}\rangle = k\delta_b^a$$
$$\langle K^i, K^j\rangle \equiv \kappa^{ij}$$

- Fermions transform in representation of bosonic subgroup

$$[K^i, S_1^a] = -(R^i)^a{}_b S_1^b$$

- Fermionic fields $c, \bar{c}$

$$c = c^a S_{2a}, \quad \bar{c} = \bar{c}_a S_1^a$$

- Parametrization (Type I supergroup)

$$g = e^c g_B e^{\bar{c}}$$

Free fermion resolution

- Free fermion resolution: Introduce auxiliary fields

$$b = b_a S_1^a, \quad \bar{b} = \bar{b}^a S_{2a}$$

- Action

$$S^{\mathrm{WZNW}}[g] = S_0 + S_{\mathrm{pert}}$$

- Screening charges

$$S_{\mathrm{pert}} = -\frac{1}{4\pi k} \int_\Sigma d^2\sigma \, \mathrm{str}(\mathrm{Ad}(g_B)(\bar{b})b)$$

- Bosonic WZNW, background charges, $(1,0)$ bc-ghosts

$$S_0 = S^{\mathrm{WZNW}}_{ren}[g_B] - \frac{1}{4\pi} \int dz^2 \sqrt{h} \mathcal{R}^{(2)} \tfrac{1}{2} \ln \det R(g_B) + \frac{1}{2\pi} \int dz^2 \left(b_a \bar{\partial} c^a - \bar{b}_a \partial \bar{c}^a \right)$$

Free fermion resolution

- Quantum measure (again)

$$\langle K_B^i, K_B^j\rangle_{ren} = \kappa^{ij} - \gamma^{ij}, \quad \gamma^{ij} = \mathrm{tr} R^i R^j,$$

- Free fermion currents
$K^i(z) = K_B^i + b_a(R^i)^a{}_b c^b$
$S_1^a(z) = k\partial c^a + k(R^i)^a{}_b \kappa_{ij} c^b K_B^j - \frac{k}{2}(R^i)^a{}_b \kappa_{ij}(R^j)^c{}_d b_c c^b c^d$
$S_{2a}(z) = -b_a$
- Stress-energy tensor

$$T_G^{\mathrm{FF}} = \frac{1}{2}\left(K_B^i \Omega_{ij} K_B^j + \mathrm{tr}(\Omega R^i)\kappa_{ij}\partial K_B^j\right) - b_a \partial c^a,$$

$$(\Omega^{-1})^{ij} = \kappa^{ij} - \gamma^{ij} + \frac{1}{2} f^{im}{}_n f^{jn}{}_m,$$
$$(\Omega^{-1})^a{}_b = \delta_b^a + (R^i \kappa_{ij} R^j)^a{}_b.$$

Topological conformal field theory

- BRST-currents/charges $Q, \bar{Q}$
- Cohomology by Q or $Q + \bar{Q}$

$$\mathcal{H}_{\text{phys}} = \frac{\text{kernel}(Q)}{\text{image}(Q)}$$

- Conformal tensor exact

$$T(z) = [Q, G(z)] \qquad \bar{T}(\bar{z}) \; = \; [Q, \bar{G}(\bar{z})]$$

- Cohomology ring: Position independent structure constants

$$\phi_i \phi_j \sim a_{ij}{}^k \phi_k$$

TCFT from $\mathcal{N} = (2,2)$ by twisting

- Positive B-twist

$$T^{+}_{\text{twisted}}(z) = T(z) + \frac{1}{2}\partial U(z)\,,$$
$$\bar{T}^{+}_{\text{twisted}}(\bar{z}) = \bar{T}(\bar{z}) + \frac{1}{2}\bar{\partial}\bar{U}(\bar{z})\,.$$

- Gives TCFT with $c = 0$
- $\Delta(G^{+}, G^{-}, U) = (3/2, 3/2, 1) \mapsto (1, 2, 1)$
- $G^{+} \mapsto Q$
- Chiral ring maps to cohomology
- G^{-} cohomology partner to T^{+}_{twisted}

$$T^{+}_{\text{twisted}} = [Q, G^{-}]$$

- U partner to Q

From WS SUSY to target superspaces

- Idea: Consider Lie supergroup G
- Look at free fermion resolution of WZNW model for G
- Construct $\mathcal{N} = (2, 2)$ WS SUSY theory on the bosonic subgroup G_B
- By twisting reach TCFT with $T^{+}_{\text{twisted}} = T^{\mathrm{FF}}_{G}$ and Q one of the fermionic currents
- Immediate constraint $\mathrm{sdim}\mathfrak{g} = 0$
- $\mathrm{PSL}(N|N)$ does not work, and no topological sectors were found
- What about $\mathrm{GL}(N|N)$?

$\mathrm{GL}(N|N)$

- Generators

$$\left(\begin{array}{c|c} E_{+}^{\alpha\beta} & F_{+}^{\alpha\beta} \\ \hline F_{-}^{\alpha\beta} & E_{-}^{\alpha\beta} \end{array}\right)$$

- Invariant form $k\mathrm{str}$ (positive/negative on bosonic part)

$$k\mathrm{str}(E_{\epsilon}^{\alpha\beta} E_{\epsilon'}^{\gamma\delta}) = \kappa^{\binom{\alpha\beta}{\epsilon}\binom{\gamma\delta}{\epsilon'}} = k\epsilon\delta_{\epsilon\epsilon'}\delta^{\alpha\delta}\delta^{\beta\gamma}$$

- Can calculate free fermion currents and stress-energy tensor
- Consider $\mathrm{GL}(N) \times \mathrm{GL}(N)$ bosonic base generated by $E_{\pm}^{\alpha\beta}$
- Choose initial metric on bosonic base

$$\kappa_{\text{start}} = \kappa^{ij} - \gamma^{ij} + \frac{1}{2} f^{im}{}_{n} f^{jn}{}_{m}$$

- Extra terms only changes metric on $U(1)$ factors

$$\kappa_{\text{start}} = (E_{\epsilon}^{\alpha\beta}, E_{\epsilon'}^{\gamma\delta}) = k\epsilon\delta_{\epsilon\epsilon'}\delta^{\alpha\delta}\delta^{\beta\gamma} - \epsilon\epsilon'\delta^{\alpha\beta}\delta^{\gamma\delta}$$

- Simply corresponds to field redefinition of $U(1)$ factors

Choose Manin decomposition

- The right choice turns out to be

$$gl(N) \oplus gl(N) = \mathfrak{a}_+ \oplus \mathfrak{a}_-$$

$$\mathfrak{a}_+ = \text{span}\{E_+^{\alpha\beta} + E_-^{\alpha\beta}\}$$

$$\mathfrak{a}_- = \text{span}\{\frac{E_+^{\alpha\alpha} - E_-^{\alpha\alpha}}{2k} + \frac{1}{2k^2}\mathrm{Id}, \frac{E_+^{\alpha\beta}}{k}, \frac{E_-^{\beta\alpha}}{-k} \,|\, \alpha > \beta\}$$

- Can then calculate SUSY currents $G^{\pm}$, U, $T^{\mathcal{N}=(2,2)}$

Fix deformations

- Deformation parameter

$$\mathfrak{a}_0 = \text{span}\{x_{\alpha\alpha}, \sum_\alpha x^{\alpha\alpha}\}$$

$$a = p^{\alpha\alpha} x_{\alpha\alpha} + q \sum_\alpha x^{\alpha\alpha}$$

- Twist

$$T^+_{\text{twisted}} = T_a^{\mathcal{N}=(2,2)} + \frac{1}{2}\partial U_a$$

- $p^{\alpha\alpha}$ fixed by

$$T^+_{\text{twisted}} = T_{\mathrm{GL}(N|N)}$$

- The single remaining parameter q fixed by

$$G_a^+ \text{ is fermionic current}$$

- $c = 3N^2 \mapsto c_a = 0$ (before twisting)
- Whole deformation can be seen as background charge

WS SUSY currents as super Lie currents

- After the twist χ^i, χ_i ghost system $(1/2, 1/2) \mapsto (1, 0)$
- Fix dictionary-automorphism of ghosts

$$c^{\alpha\beta} = -\chi^{\beta\alpha}, \quad b_{\alpha\beta} = -\chi_{\beta\alpha}$$

- We got what we aimed for

$$G_a^+ = \sum_\alpha F_+^{\alpha\alpha}$$

- ... and a surprise

$$U_a = \frac{1}{2k}\sum_\alpha (k+N+1-2\alpha)\tilde{E}_+^{\alpha\alpha} + \frac{1}{2k}\sum_\alpha(-k+N+1-2\alpha)\tilde{E}_-^{\alpha\alpha}$$
$$G_a^- = \frac{1}{2k}\sum_\alpha F_-^{\alpha\alpha}\Big(\tilde{E}_+^{\alpha\alpha} - \tilde{E}_-^{\alpha\alpha} + \frac{1}{k}\sum_\beta(\tilde{E}_+^{\beta\beta} + \tilde{E}_-^{\beta\beta})\Big) -$$
$$\frac{1}{k}\sum_{\alpha>\beta}\tilde{F}_-^{\alpha\beta}\tilde{E}_-^{\beta\alpha} + \frac{1}{k}\sum_{\alpha<\beta}F_-^{\alpha\beta}\tilde{E}_+^{\beta\alpha} - \frac{1}{2k}\sum_\alpha(2N-2\alpha+1)\partial F_-^{\alpha\alpha}$$

Deformations with chiral fields

- $\mathcal{N} = (2,2)$ WS SUSY: chiral and twisted chiral supermultiplets (and anti-)
- Classical SUSY-preserving deformation by F- and D-terms
- Chiral multiplet generated by field in (++)-ring

$$[G^+_{-1/2}, \phi_{++}] = [\bar{G}^+_{-1/2}, \phi_{++}] = 0$$

- Remaining components

$$\psi_{++} = -[G^-_{-1/2}, \phi_{++}], \quad \bar{\psi}_{++} = -[\bar{G}^-_{-1/2}, \phi_{++}]$$

$$F_{++} = -\{G^-_{-1/2}, [\bar{G}^-_{-1/2}, \phi_{++}]\}$$

- F-term perturbation non-exact in twisted theory
- Anti-chiral and twisted (anti-)chiral F-term perturbations are exact in $G^+ + \bar{G}^+$ cohomology

Screening charges in supergroup model

- We achieved $T^{+}_{\text{twisted}} = T_{\mathrm{GL}(N|N)} \Rightarrow$
  $S^{\mathrm{GL}(N)\times\mathrm{GL}(N)}_{\mathcal{N}=(1,1)} + \text{deformations} + \text{twist} = S_0^{\mathrm{GL}(N|N)}$
- Free fermion resolution boson-fermion interaction terms

$$S_{\text{pert}} = -\frac{1}{4\pi k}\int_\Sigma d^2\sigma\, \mathrm{str}(\mathrm{Ad}(g_B)(\bar{b})b)$$

- Are screening charges:

$$J(z)\mathcal{L}_{\text{pert}} \sim \text{total derivatives}$$

- Conclusion: Is (classical) SUSY deformation
- Is in fact chiral F-term perturbation: Take $g_B = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$
- $\phi = \mathrm{tr}(A^{-1}B)$ in $(+,+)$-ring, has dimension $(1/2, 1/2)$

$$F_{++} = -\{G^{-}_{-1/2}, [\bar{G}^{-}_{-1/2}, \phi_{++}]\} = \frac{4\pi}{k}\mathcal{L}_{\text{pert}}$$

Bosonic screening charges

- Free fermion resolution gave us

$$S_0 = S^{\mathrm{WZNW}}_{ren}[g_B] - \frac{1}{4\pi}\int dz^2 \sqrt{h}\mathcal{R}^{(2)}\tfrac{1}{2}\ln\det R(g_B)$$
$$+\frac{1}{2\pi}\int dz^2 \left(b_a\bar\partial c^a - \bar b_a \partial \bar c^a\right)$$

- GL(2) Gauss decomposition

$$\exp\gamma\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\exp\left(\phi_0\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\phi_z\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right)\exp\bar\gamma\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$$

- First order formalism: Introduce auxiliary fields β and $\bar\beta$

$$S^{\mathrm{WZNW}}_{ren}[g_B] = S_{\mathrm{kin+b.g.}}[\phi_0,\phi_z,\beta,\gamma] + \int d^2z\,\beta\bar\beta e^{\phi_z/\sqrt{k}}$$

- Screening charges are chiral F-terms.

Exactness

- Free fermion resolution AND bosonic first order formalism

$$S^{\mathrm{GL}(2|2)} = \underbrace{S_{\text{kin+b.g.}}[\phi_0^\pm, \phi_z^\pm, \beta^\pm, \gamma^\pm]}_{G^+ + \bar{G}^+ \text{ exact}} + \underbrace{S_{\text{bos. screening}} + S_{\text{bos.-ferm. interaction}}}_{G^- + \bar{G}^- \text{ exact}}$$

Principal chiral field deformations

- Principal chiral field

$$S_{\text{geom.}} = -\frac{k}{4\pi}\int d^2z\,\langle g^{-1}\partial g, g^{-1}\bar{\partial}g\rangle \equiv \int d^2z\,\Phi_{\text{principal}}$$

- $PSU(1,1|2)$ deformation of coefficient principal chiral field to turning on RR-flux in string theory
- Can show this is D-term
- Is G^+-exact

$$\Phi_{\text{principal}} = -\{G^+_{-1/2}, [\bar{G}^+_{-1/2}, \phi]\}$$

$$\phi = -\frac{1}{4\pi k}\langle \left(\begin{array}{c|c} 0 & \mathrm{Id} \\ \hline \mathrm{Id} & 0 \end{array}\right) g^{-1} \left(\begin{array}{c|c} 0 & \mathrm{Id} \\ \hline \mathrm{Id} & 0 \end{array}\right) Jg\bar{J}\rangle$$

String theory on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$

- The procedure works for string theory on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$
- $\mathrm{AdS}_3 \times \mathrm{S}^3$
  $\mathcal{N} = (2, 2)$ WZNW on $\mathrm{SU}(1, 1) \times \mathrm{SU}(2)$
- T^4
  $\mathcal{N} = (2, 2)$ WZNW on $\mathrm{U}(1)^4$
- String ghosts – can be seen as
  $\mathcal{N} = (2, 2)$ WZNW on $\mathrm{U}(1)^2$
- Divide into

$$\mathrm{U}(1,1) \times \mathrm{U}(2) \mapsto \mathrm{U}(1,1|2)$$

$$\mathrm{U}(1) \times \mathrm{U}(1) \mapsto \mathrm{U}(1|1)$$

$$\mathrm{U}(1) \times \mathrm{U}(1) \mapsto \mathrm{U}(1|1)$$

- String theory has total central charge zero

String theory on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$

- Our method: Before twisting, after deformation

$$c = 0$$

- Comparison to string theory

$$\begin{aligned} G^{-}_{\mathrm{string}} &= G^{-}\,, \\ U_{\mathrm{string}} &= U, \\ G^{+}_{\mathrm{string}} &= G^{+} + \text{extra terms} \end{aligned}$$

- Novel choice of $\mathcal{N} = (2,2)$
- G^{-} chiral ring is the same as in string theory!

Conclusions and Outlook

- Start from bosonic group, add background charges plus twist $\longrightarrow$ Supergroup
- Interaction terms are G^--exact F-terms
- G^- chiral ring is the same as in string theory

- Calculate extremal correlators in string theory on $AdS_3 \times S^3 \times T^4$, higher genus!
- Gives proposal for boundary action via analogy with Landau-Ginzburg model – this should be checked explicitly
- Also works for $\mathrm{SL}(2|1)$
- Other groups? Supergroups?
- $\mathcal{N} = (4,4)$ supersymmetric point
- Is it possible to use equivariant localisation in this context?