Split attractor flows and D-brane stability

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Outline

1. Setup, context and motivation

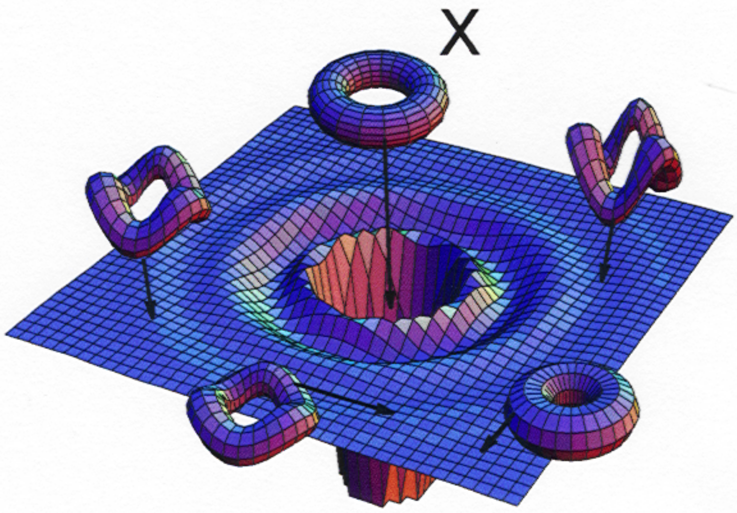
2. BPS configurations in $d = 4$, $N = 2$ supergravity and their associated (split) attractor flows [or "string webs" on moduli space].

3. Applications to the quintic spectrum.

4. Open questions.

1. Setup, context and motivation

Type IIA string theory on Calabi-Yau X, or IIB on its mirror Y:



Our main example: $X =$ quintic, with mirror Y given by:

$$Y : \sum_{i=1}^{5} x_i^5 - 5\,\psi\, x_1 x_2 x_3 x_4 x_5 = 0$$

in CP^4. One complex structure modulus: ψ.

Low energy effective theory: d=4, N=2 supergravity coupled to n_v massless $U(1)$ vector multiplets [and n_h massless hyper multiplets].

In IIB: $n_v = \#$ complex structure moduli of Y.
In IIA: $n_v = \#$ Kähler moduli of X.

Bosonic fields:

- metric $G_{\mu\nu}$
- $n_v + 1$ $U(1)$ gauge fields A^I_μ
- n_v scalars z^a, parametrizing moduli space $\mathcal{M}_v$

Electric and magnetic sources for gauge fields: Dp-branes wrapped around nontrivial cycles of the CY; $p = 3$ in IIB, $p = 0, 2, 4, 6$ in IIA.

Central and highly nontrivial problem: in a given vacuum, parametrized by $(z^a)_{r=\infty}$, what is the spectrum of BPS states of the theory?

State with electric and magnetic charges (q_I, p^I) is BPS iff

$$M = |Z|_\infty \equiv |q_I X^I - p^I F_I|_\infty \,.$$

Here the $X^I(z,\bar{z})$, $F_I(z,\bar{z})$ are the (normalized) periods, specific model-dependent functions on $\mathcal{M}_v$ (more precisely: on its covering space). For quintic: $\sim$ Meijer G-functions.

Not all charges support BPS states:

1. From microscopic geometric D-brane perspective, one needs existence of supersymmetric cycle in given homology class: special lagrangian 3-cycle in IIB, holomorphic cycle (+holomorphic bundle) in IIA.

2. From microscopic CFT perspective, one needs existence of BPS boundary state.

3. From 4d low energy perspective, one needs existence of a BPS solution to the equations of motion; black holes *or more general configurations*

Furthermore, spectrum can depend on moduli: there can be lines of marginal stability where BPS states decay.

At first sight, the 4d low energy picture did not seem sufficiently rich in structure to tell much about the rather complex properties of such N=2 BPS spectra.

However, some surprises were met in the past few years: existence of solutions, even spherically symmetric ones, turned out to be very subtle, and a rich class of multicentered "bound state"-solutions emerged, exhibiting decay at marginal stability and other remarkable features.

$\Longrightarrow$ Conjecture:

Existence of sugra BPS solution

$\Leftrightarrow$

Existence of BPS state in the full theory

2. BPS solutions and (split) attractor flows

A. Spherically symmetric solutions

Metric: $ds^2 = -e^{2U}dt^2 + e^{-2U}d\mathbf{x}^2$.

Moduli fields z^a and metric function U only dependent on inverse radial coordinate $\tau = 1/|\mathbf{x}|$.

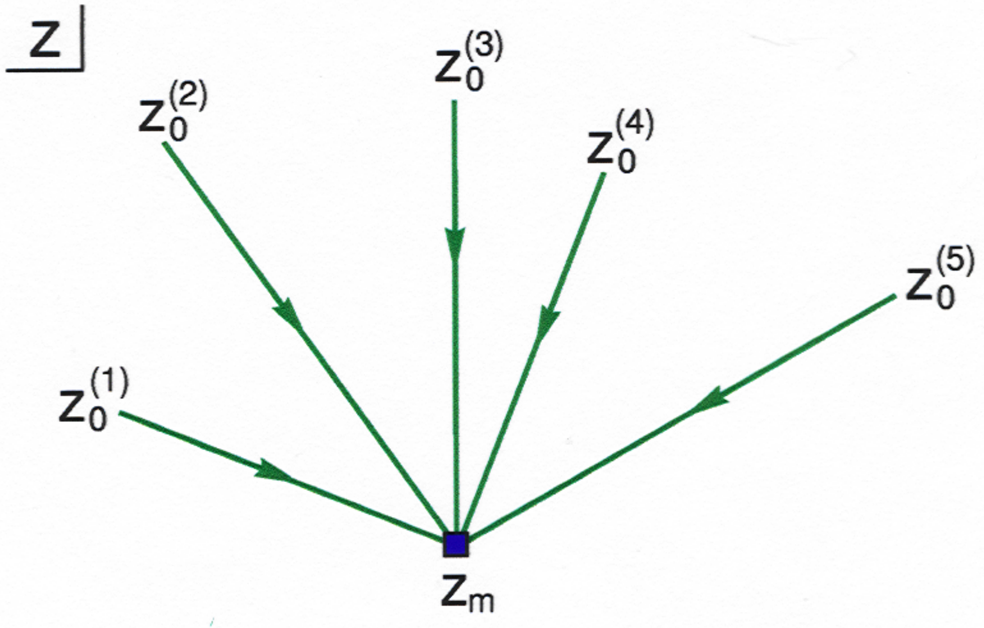
BPS eqs. of motion have gradient flow form:

$$\begin{aligned} \partial_\tau U &= -e^U |Z| \\ \partial_\tau z^a &= -2e^U g^{a\bar{b}}\, \bar{\partial}_{\bar{b}} |Z| \end{aligned}$$

Which, with $e^{i\alpha} \equiv Z/|Z|$, can be integrated to:

$$\begin{aligned} e^{-U}\,\mathrm{Im}[e^{-i\alpha}X^I] &= p^I\,\tau + \mathrm{Im}[e^{-i\alpha}X_I]_0 \\ e^{-U}\,\mathrm{Im}[e^{-i\alpha}F_I] &= q_I\,\tau + \mathrm{Im}[e^{-i\alpha}X_I]_0 \end{aligned}$$

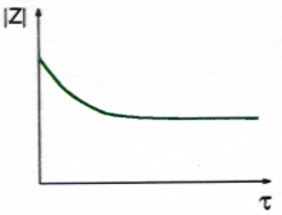
Crucial: BPS e.o.m. define flow in moduli space with decreasing $|Z|$:



$\Rightarrow$ For given charge, all flows run to fixed point z_m, minimum of $|Z|$, $\equiv$ attractor point.

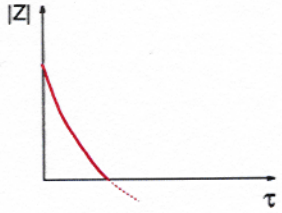
3 cases (for z_m at finite distance in $\mathcal{M}$):

1. $|Z|_m > 0$:



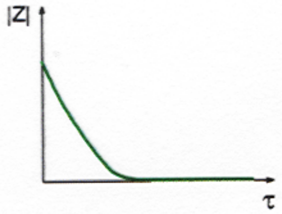
$\rightarrow$ regular BPS black hole, $A = 4\pi |Z|_m{}^2$.

2. reg. zero: $|Z|_m = 0$ and $g^{a\bar{b}}\bar{\partial}_{\bar{b}}|Z|_m \neq 0$:



$\rightarrow$ **no** solution.

3. zero at (conif.) sing.: $|Z|_m = 0$ and $g^{a\bar{b}}\bar{\partial}_{\bar{b}}|Z|_m = 0$:



$\rightarrow$ empty hole / fuzzy Q-ball / QFT monopole.

(4. LCS runaway ; e.g. D2-D0)

The attractor flows in moduli space can also be interpreted as geodesic strings in moduli space, stretched between vacuum and attractor point.

Geodesic with respect to action

$$S = \int \sqrt{V}\, ds$$

where $V = 4g^{a\bar{b}}\partial_a|Z|\bar{\partial}_{\bar{b}}|Z|$, and ds is the line element on moduli space: $ds^2 = g_{a\bar{b}}dz^a d\bar{z}^{\bar{b}}$.

In suitable rigid (QFT) limit, this reduces to usual F-theory picture of QFT BPS states as stretched strings.

B. General multicentered stationary solutions

Metric:

$$ds^2 = -e^{2U}\left(dt + \omega_i\, dx^i\right)^2 + e^{-2U} d\mathbf{x}^2 .$$

Integrated BPS equations:

$$\begin{aligned} 2\, e^{-U}\, \mathrm{Im}[e^{-i\alpha} X^I] &= H^I \\ 2\, e^{-U}\, \mathrm{Im}[e^{-i\alpha} F_I] &= H_I \\ \epsilon^{ijk}\partial_j \omega_k &= H_I \partial_i H^I - H^I \partial_i H_I \end{aligned}$$

with the H_I, H^I harmonic functions, with source centers at positions $\mathbf{x}_r$, $r = 1, \ldots, N$:

$$H = \sum_{r=1}^{N} \frac{Q_r}{\mathbf{x} - \mathbf{x}_r} + \text{const.}$$

where $Q_r = (q_{rI}, p_r^I)$.

Third eq. implies constraint on positions:

$$\sum_{r=1}^{N} \frac{\langle Q_r, Q_s \rangle}{|\mathbf{x}_r - \mathbf{x}_s|} = 2\, \mathrm{Im}[e^{-i\alpha} Z(Q_s)]_{r=\infty},$$

Here $\langle \cdot, \cdot \rangle$ denotes the intersection product:

$$\langle Q, \tilde{Q} \rangle = q_I \tilde{p}^I - p^I \tilde{q}_I$$

(=Dirac-Schwinger-Zwanziger product, measures mutual nonlocality)

For just two charges, this determines equilibrium distance:

$$R = \frac{1}{2} \langle Q_1, Q_2 \rangle \left. \frac{|Z_1 + Z_2|}{\mathrm{Im}(Z_1 \bar{Z}_2)} \right|_{r=\infty}$$

Note:

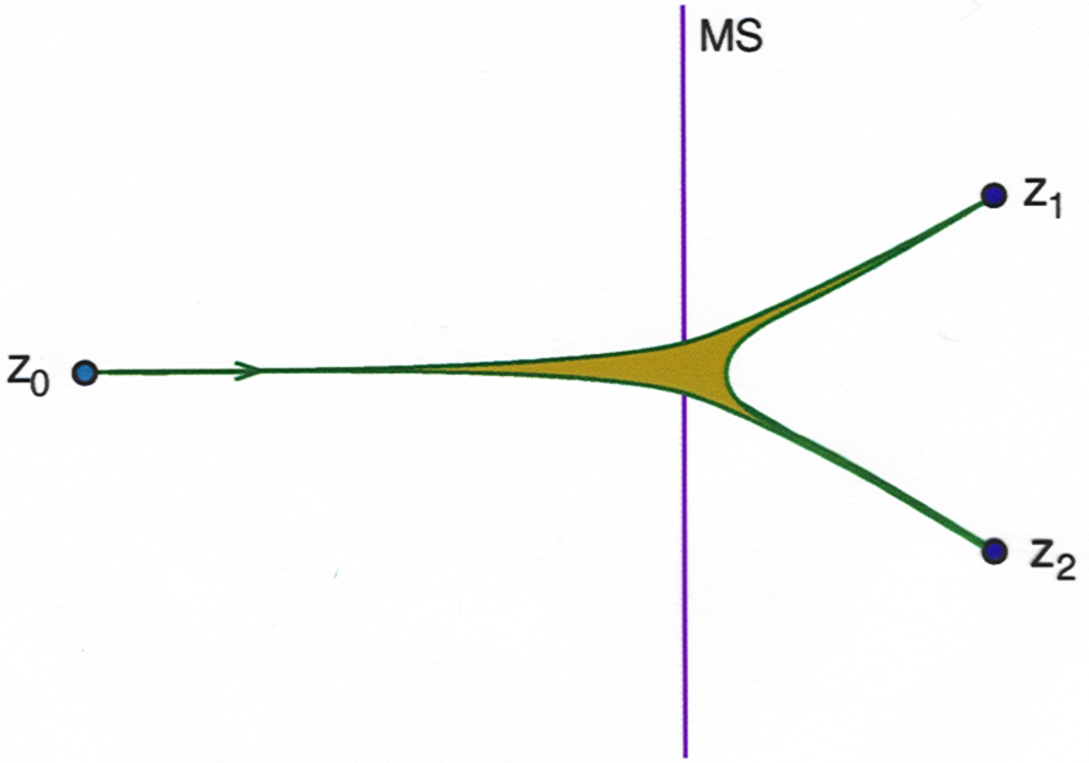
- $R \to \infty$ when approaching MS-line: decay.
- condition $R > 0 \sim$ Pi-stability criterion arising in microscopic D-brane picture.

These mutually nonlocal multicenter configurations have generically a nonzero intrinsic angular momentum:

$$\mathbf{J} = \frac{1}{2} \sum_{r<s} \langle Q_r, Q_s \rangle \, \mathbf{e}_{rs}$$

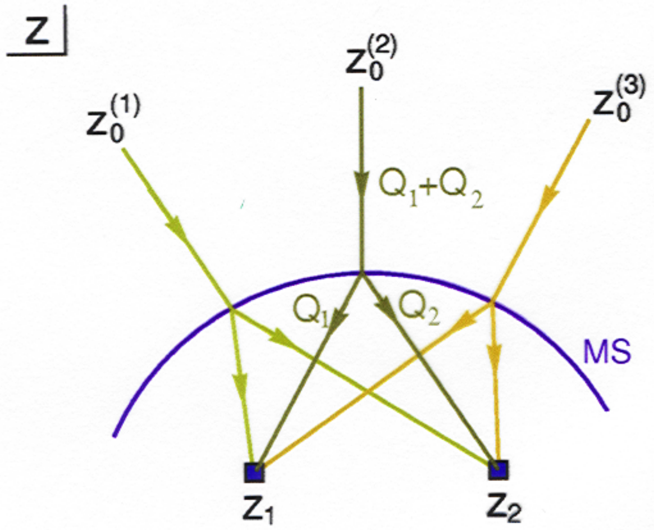
with $\mathbf{e}_{rs} =$ unit vector from $\mathbf{x}_r$ to $\mathbf{x}_s$.

Generically, image $z^a(R^3)$ in moduli space looks like "fattened split attractor flow". For configurations with two charge types:



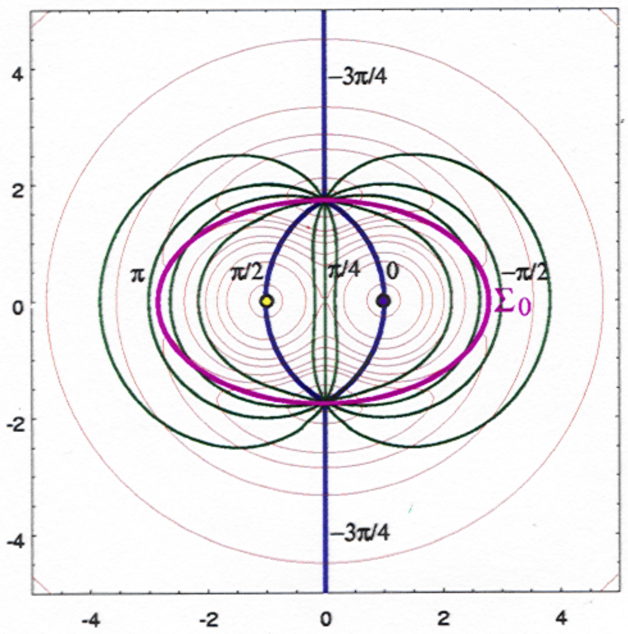
In "spherical cloud limit" (charge NQ_1 in center and N charges Q_2 distributed over spherical shell with radius R, $N \to \infty$),

this approaches exact split flow:

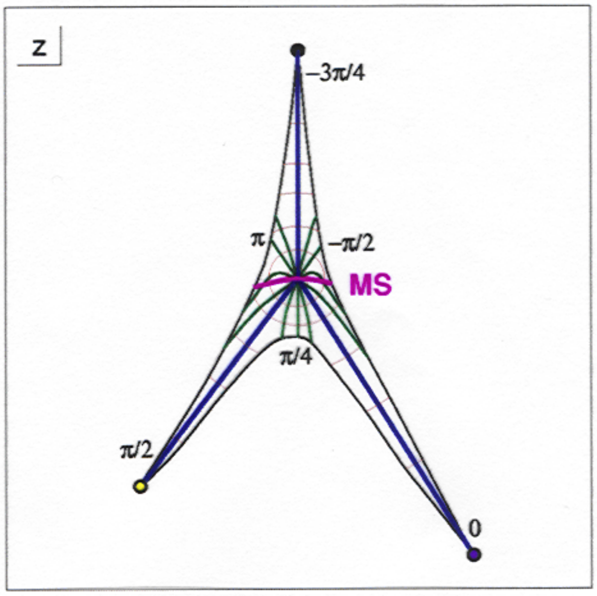


Split point will always be on line of marginal stability.

For more general configurations, e.g. 2 centers, split flow will still be "skeleton" of solution:



(a)



(b)

Conclusion: general BPS solutions correspond to split attractor flows, or *string webs on moduli space* (geodesic w.r.t. same action as given earlier).

Note: For centers of empty hole type (vanishing $|Z|_m$), situation is more subtle (...)

(~ S-RULE PROBLEM)

3. The quintic

→ Predictions about BPS spectrum of IIA string theory compactified on quintic Calabi-Yau.

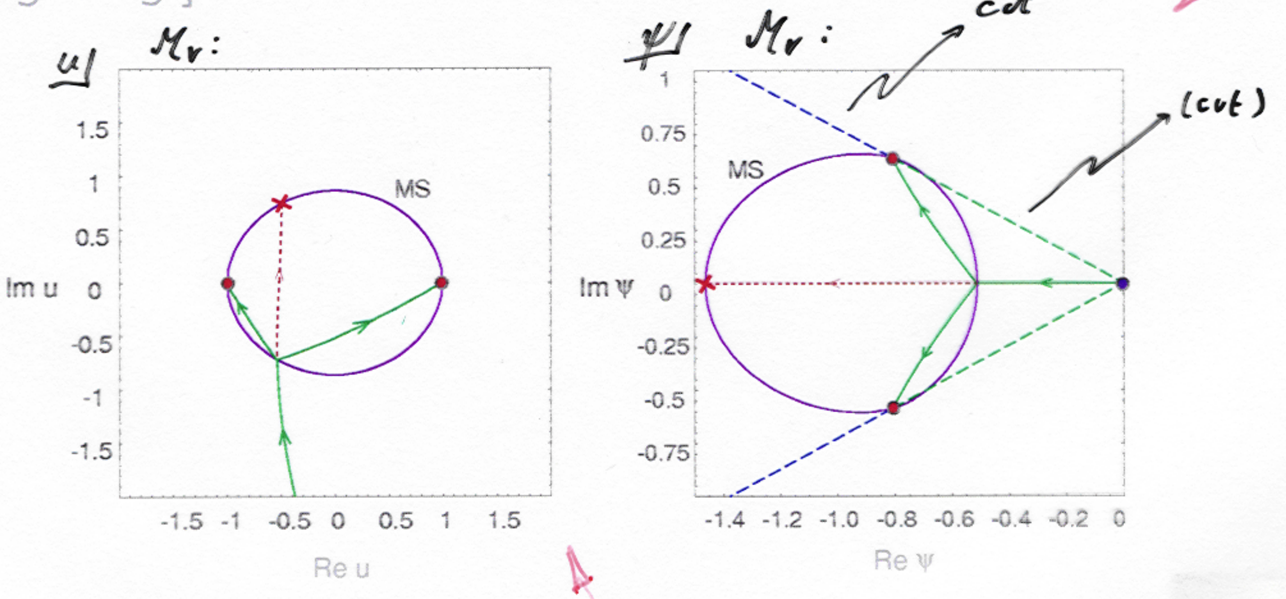
Note: verification is not easy at this point, since not many concrete results known in microscopic D-brane picture.

For known cases [BDLR], supergravity results are correct (with surprising predictive power. e.g. decay products).

Comparison with [BDLR] results at Gepner point $\psi = 0$:

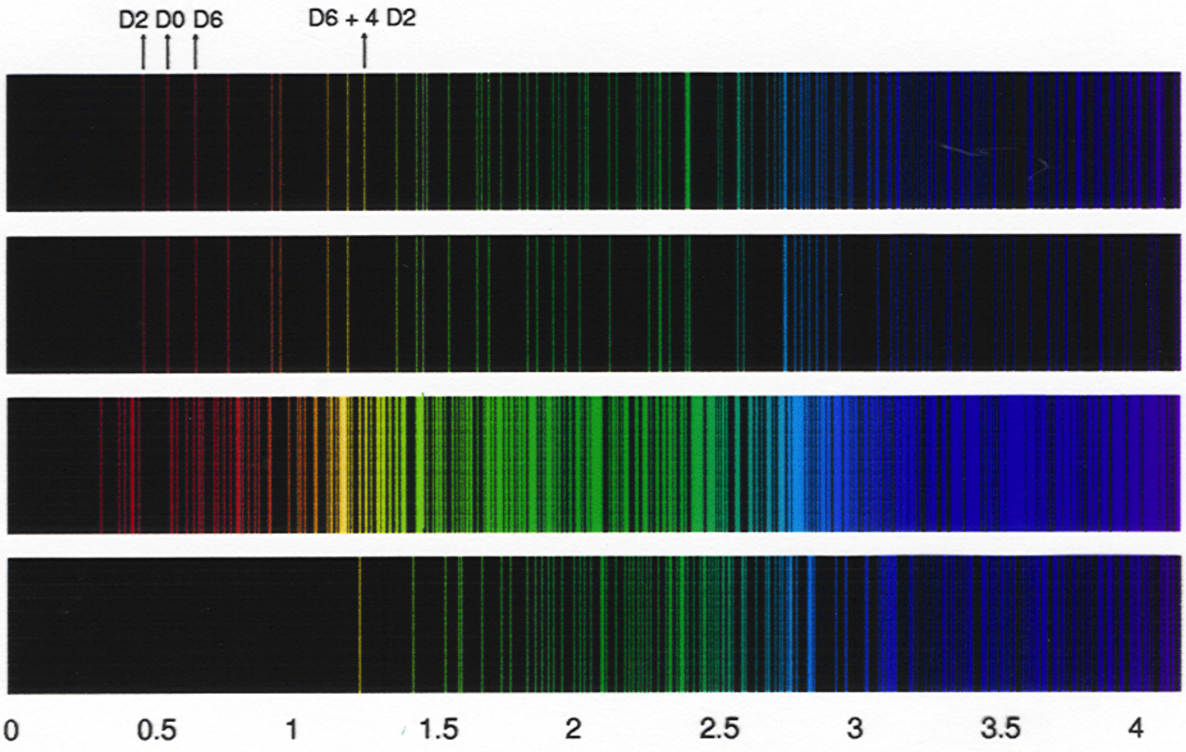
CFT label	$Q(D6, D4, D2, D0)$	$\lvert Z\rvert_m$	ψ_m
00000	$(-1, 0, 0, 0)$	0	1
10000	$(2, 0, 5, 0)$	0	-1.46
11000	$(1, 0, 5, 0)$	1.61	0.85
11100	$(3, 0, 10, 0)$	2.78	-0.51
11110	$(4, 0, 15, 0)$	4.58	-0.15
11111	$(7, 0, 25, 0)$	7.43	-0.07

So state $|10000\rangle$ does *not* have spherically symmetric BPS solution. Instead, split flow (unique in this case) with decay products $(-4, -3, -14, 10)$ and $(6, 3, 19, -10)$ [right fig.]:



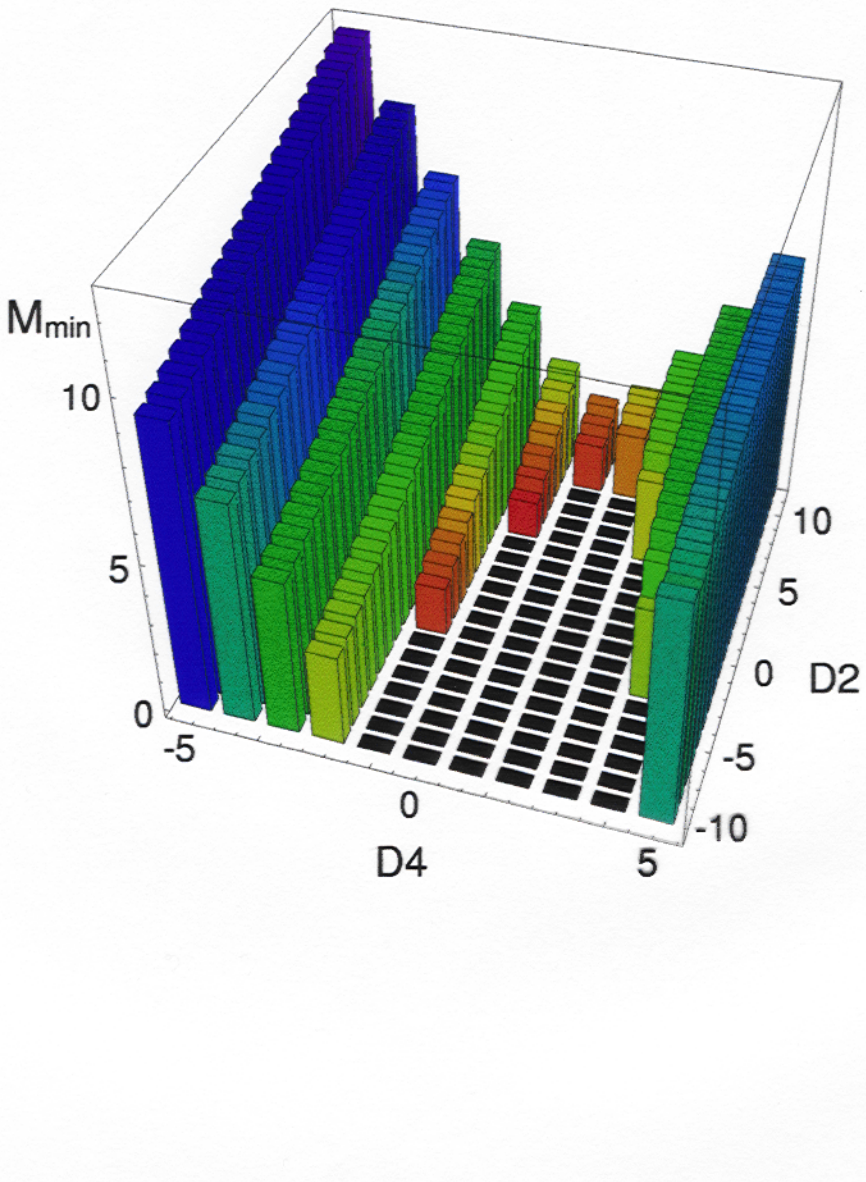
Other example [left fig.]: W-boson in SW theory.

First step in analysis BPS spectrum from supergravity perspective is computing single flow spectra [charge not.: $(D6, D4, D2, D0)$].



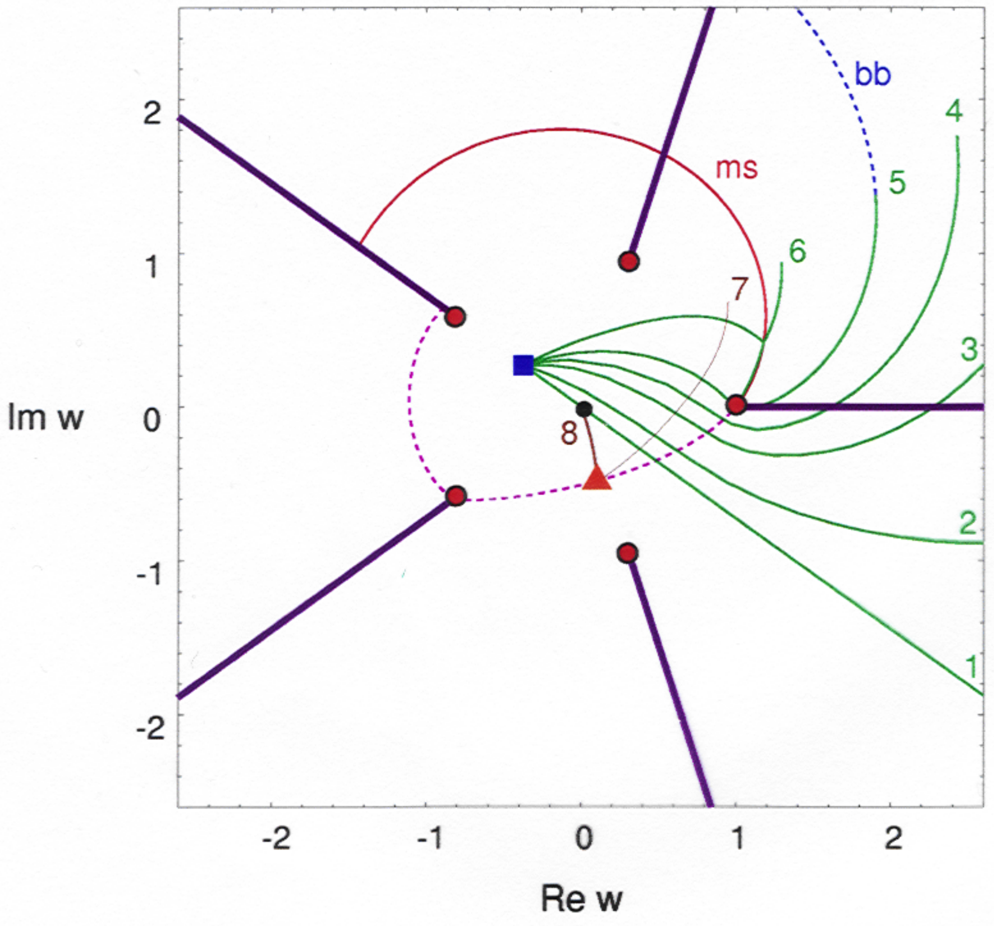
here resp. at Gepner point $\psi = 0$, at $\psi = 0$ without reg. black holes, at $\psi = 0.0851 - 0.3997\,i$, and reg. BH at their resp. min. mass points. Lightest is $(1, 0, 4, 0)$, $M_{min} \approx 1.250947$. Note: discrete.

M_{min} for set of charges $(1, q_4, q_2, 0)$, with $\psi_\infty = 0$. Black bar of zero height means flow does not exist:

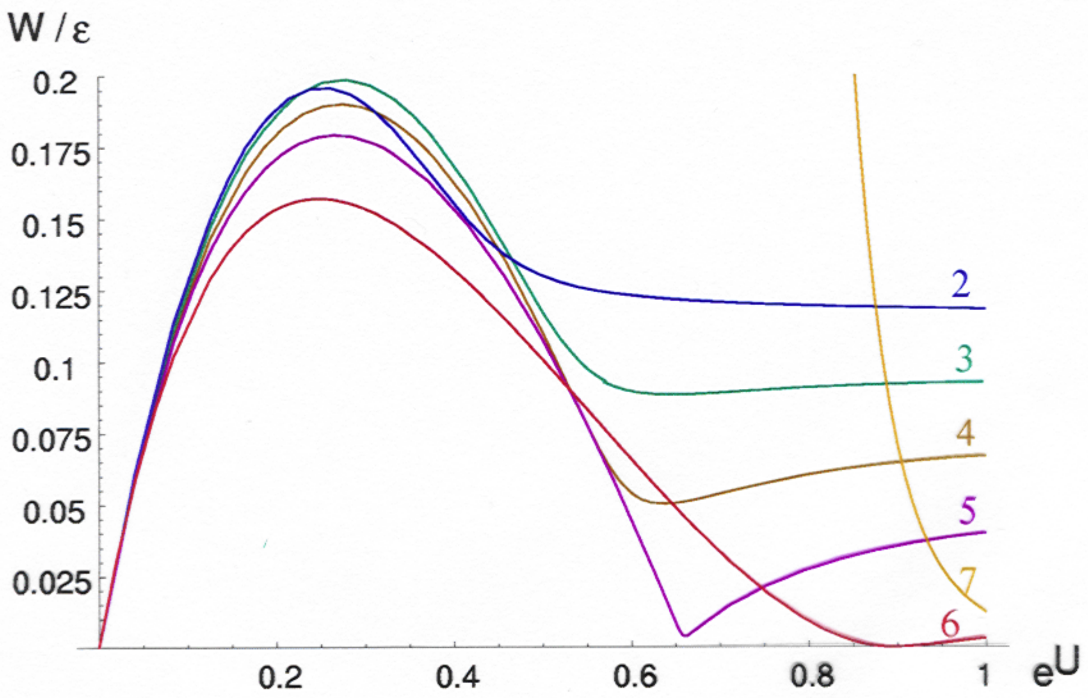


Example of prediction of a decay of a [SPH. SYMM. B.H.] BPS state when $\psi : \infty \to 0$. $Q = (2, -1, -2, 2) \to 2(1, 0, 0, 0) + (0, -1, -2, 2)$. Decay happens via conifold branch creation mechanism:

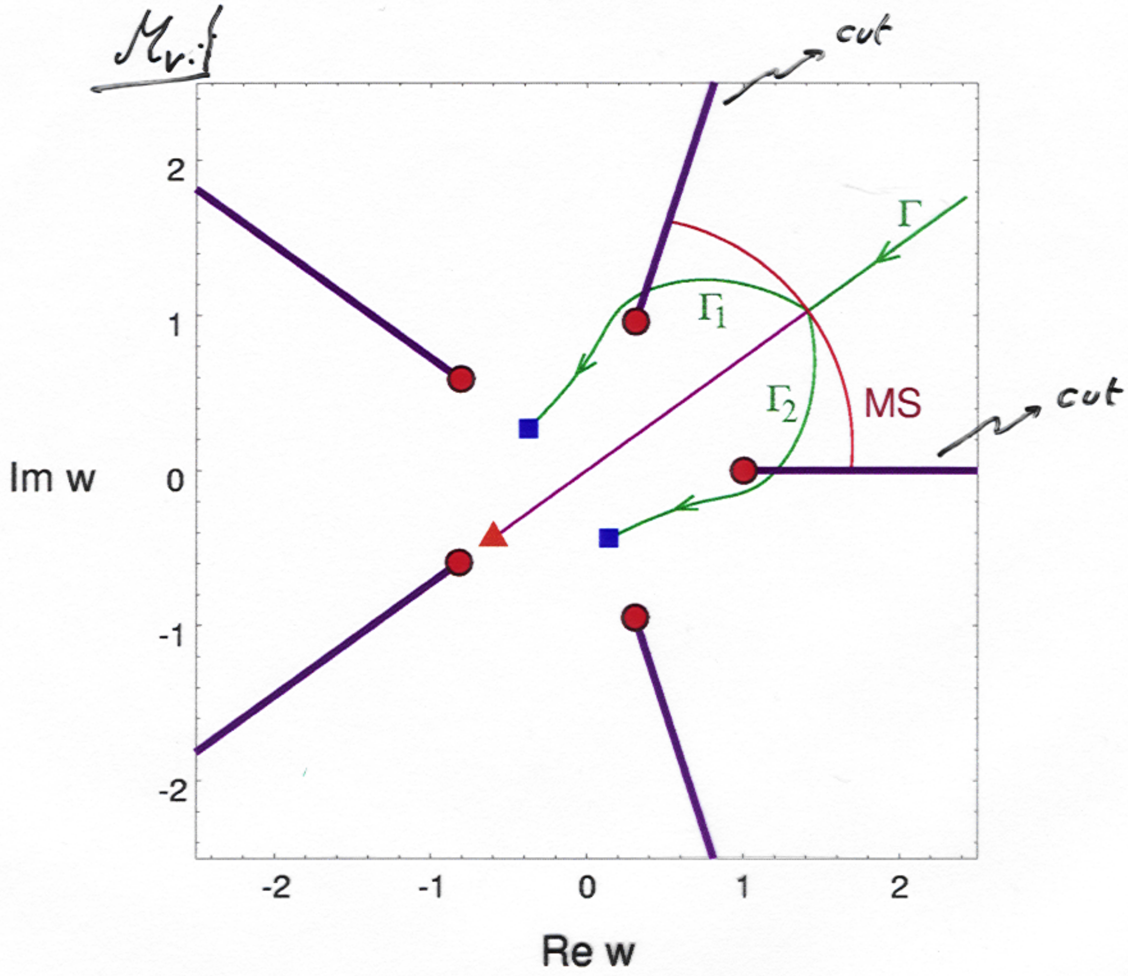
(~Hanany - Witten effect)



Corresponding force potentials for test particle with charge $\epsilon(1,0,0,0)$:

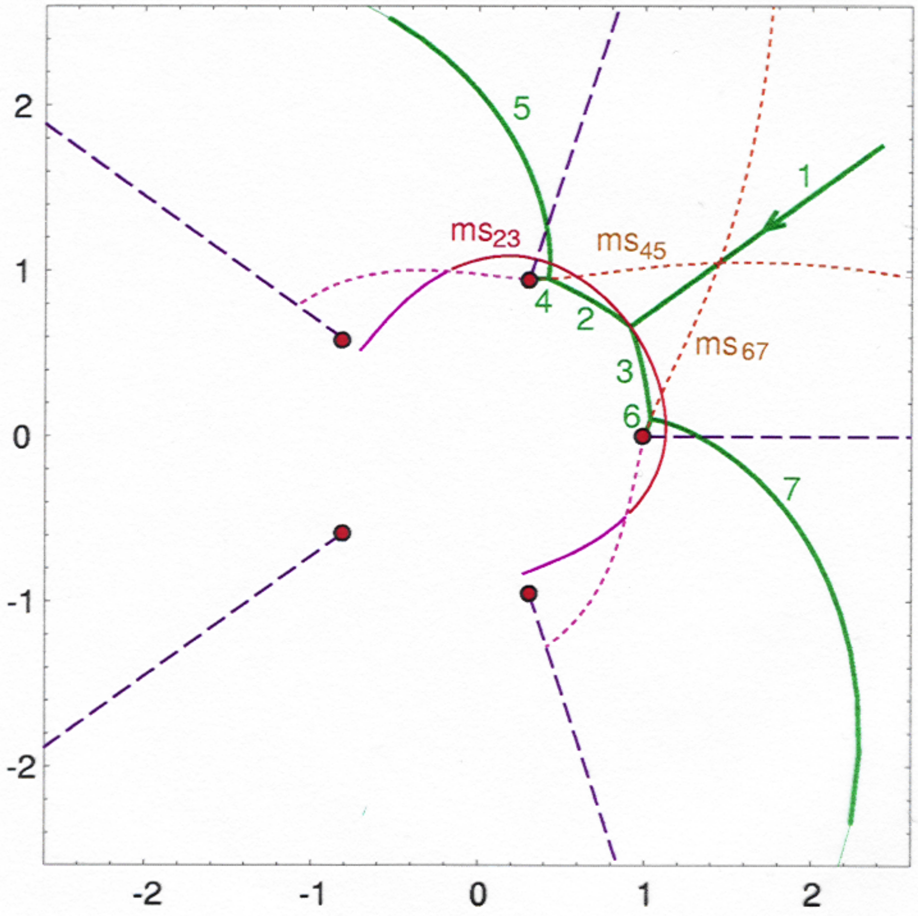


Example of a bound state of two black holes that cannot be realized as a single center black hole, and decays when going from $\psi = \infty$ to $\psi = 0$:



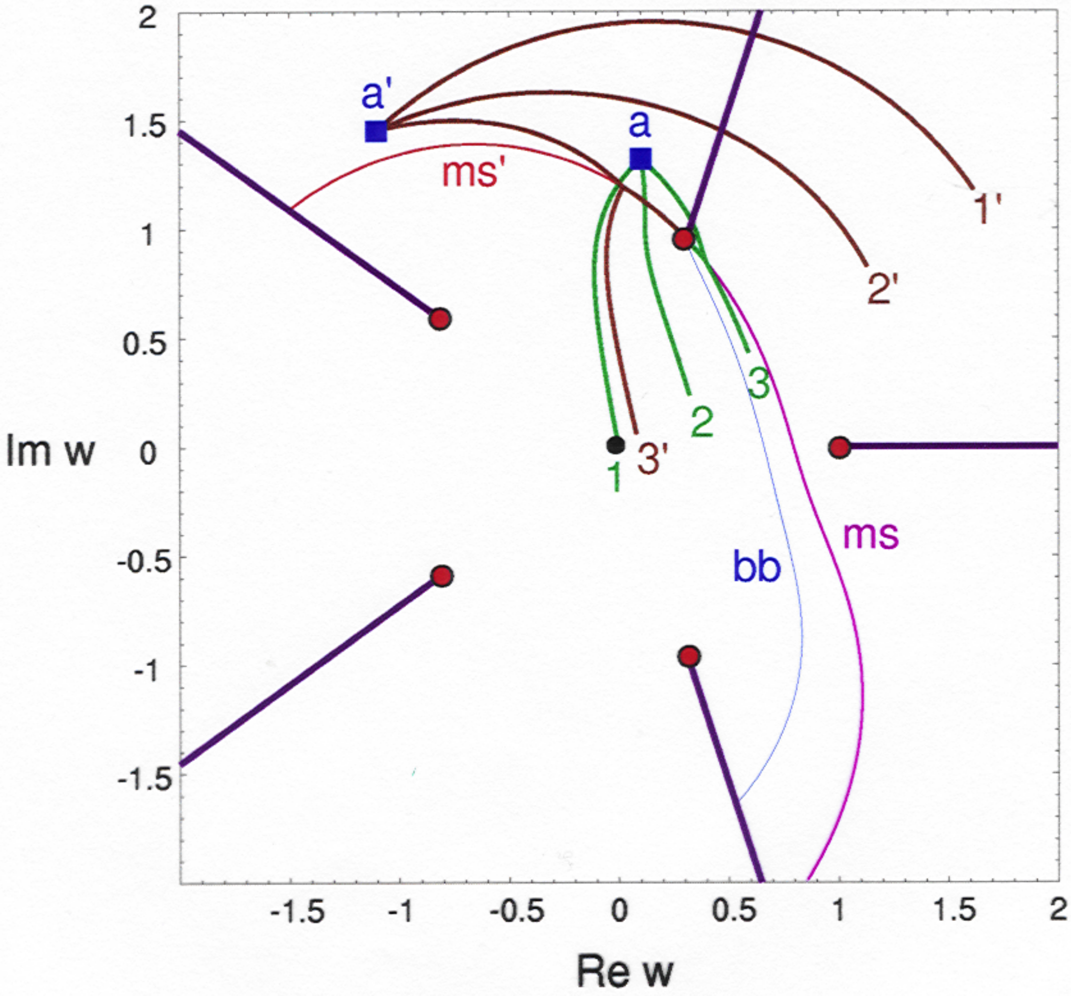
$Q = (0, 3, 9, -8)$. Decay products: $(-1, 1, 4, -1)$ and $(1, 2, 5, -7)$.

There exist an alternative split flow for this charge (only one in the region of moduli space displayed here, out of 6,765,200 candidates!):



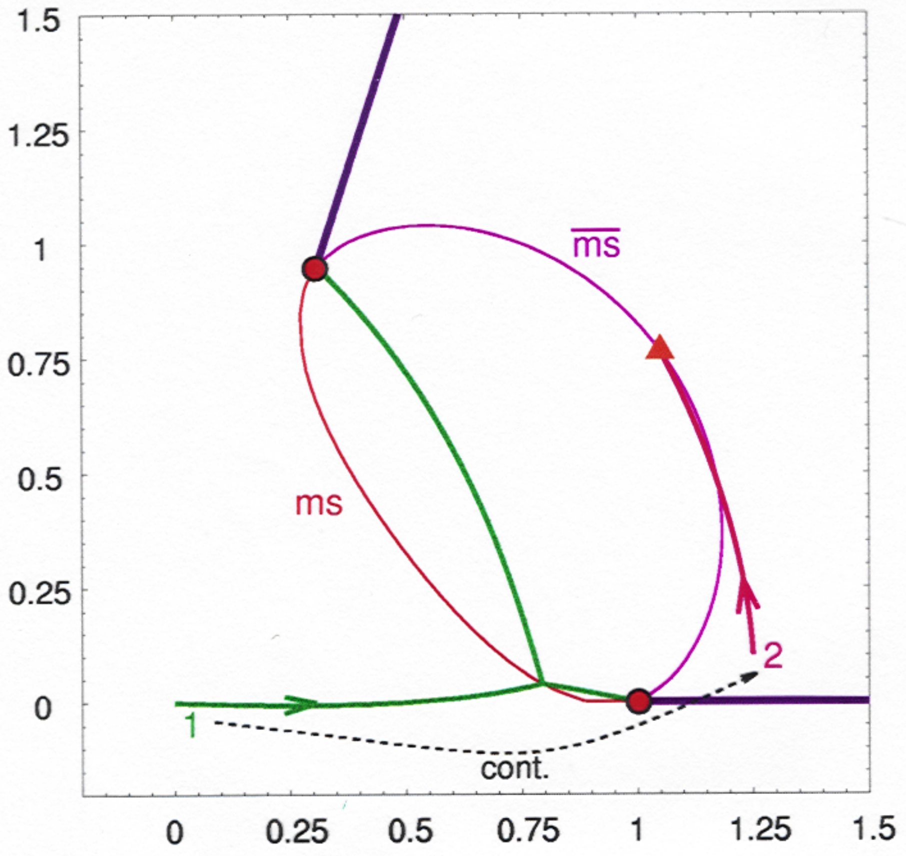
Flow structure: $1 = (0, 3, 9, -8)$, $2 = (3, 3, 10, -11)$, $3 = (-3, 0, -1, 3)$, $4 = 3(1, 0, 0, 0)$, $5 = (0, 0, 1, 4)$, $6 = 3(-1, 0, 0, 0)$, $7 = (0, 0, -1, 3)$. [All legs are of D6 and D2-D0 type.]

Multiple basins of attraction and conifold branch creation as the solution to an apparent paradox:



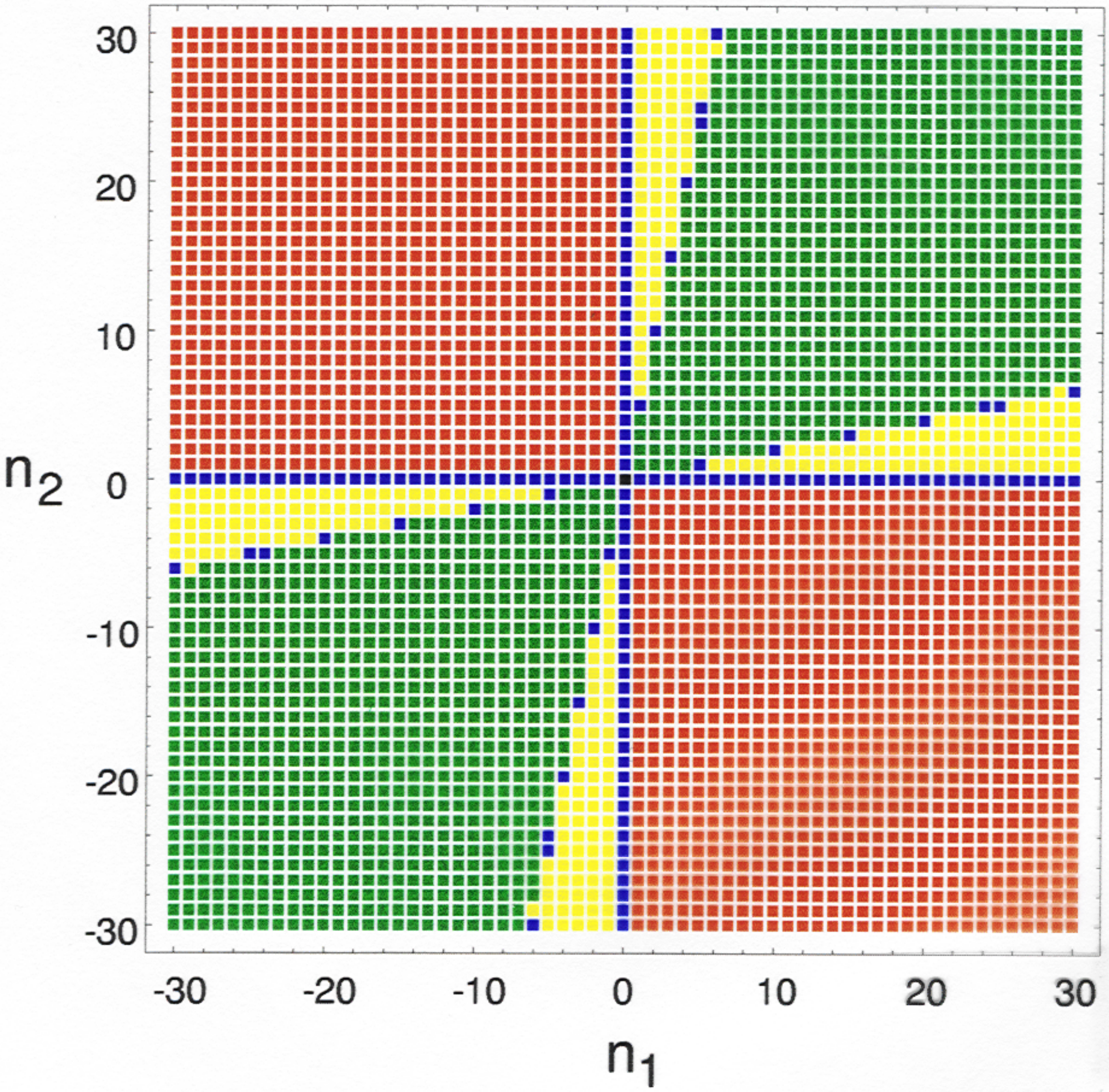
$Q = (0, -3, 10, 18)$. $|Z_m|_a \approx 6.045 \neq |Z_m|_b \approx 5.603$.

The monodromy stability problem (or “s-rule” problem): some split flows can cease to exist *without* crossing MS-line:



In analogous QFT cases, such charges are known not to be in the BPS spectrum despite existing as a string web (→ s-rule). Can this be seen here at effective field theory level??? (...)

Condition for stability under monodromies around MS-ellipse depends on intersection product of the two charges involved. Here $= 5$, giving for stable spectrum at $\psi = 0$ side:



Other topics:

- minimal area in which sources can be localized for *all* states; black holes, empty holes or composites.

- D6-D2 systems and comparison numerical results to LCSL approximation.

- near horizon flow fragmentation.

- (in)validity of supergravity.

4. Open problems

- Why does this picture work so well, apparently also beyond naive domain of validity?

- Proper full treatment of conifold-like charges possible within low energy effective field theory framework? Maybe by introducing additional charged hypermultiplet fields?

- s-rule?

- Precise relation to stability criteria derived in microscopic D-brane picture?

- Low energy dynamics (scattering) of multiparticle states?

- ...