A RELATION BETWEEN APPROACHES TO INTEGRABILITY IN SUPERCONFORMAL YANG-MILLS THEORY

hep-th 0308089

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INFINITE-DIMENSIONAL NON-ABELIAN SYMMETRY ALGEBRA :

$J^A, Q^A, \dots J_n^A$ GENERATORS $n = 0, 1, 2, \dots$

$[J^A, J^B] = f_{ABC} J^C$ $PSU(2,2|4)$

$[J^A, Q^B] = f_{ABC} Q^C$

....

COMMUTES WITH THE PLANAR ONE-LOOP ANOMALOUS DIMENSION OPERATOR, ie THE SPIN CHAIN HAMILTONIAN.

SPIN CHAIN HAMILTONIAN — hep-th/0212208 J. Minahan, K. Zarembo

NON-LOCAL SYMMETRIES OF THE $AdS_5 \times S^5$ SUPERSTRING — hep-th/0305116 I. Bena, J. Polchinski, R. Roiban

NON-LOCAL SYMMETRIES OF 2-DIMENSIONAL MODELS:

LUSCHER - POHLMEYER 1978

RELATION TO 4-DIMENSIONAL YANG-MILLS THEORY

POLYAKOV "RINGS OF GLUE" 1975-1980

$$\psi[\xi_\mu(s)] = P e^{\oint A^a_\mu T^a \cdot d\xi^\mu} \qquad 0 \le \mu \le 3$$

$$\frac{\delta}{\delta\xi_\mu}\left(\psi^{-1}\frac{\delta}{\delta\xi^\mu}\psi\right) \sim D^\mu F^a_{\mu\nu}T^a \sim 0$$

$$\Updownarrow$$

$\underline{2D}$: $\partial_\mu(g^{-1}\partial^\mu g) = 0 \qquad g_{ij}(x,t) \in G$

FLAT CONNECTION

$$\partial_\mu j_\nu - \partial_\nu j_\mu + [j_\mu, j_\nu] = 0$$

$$j_\mu(x,t) = g^{-1}\partial_\mu g \quad , \quad \partial^\mu j_\mu = 0$$

$$= j^A_\mu(x,t)\,T^A$$

CONSERVED CHARGES:

$$J^A = \int_{-\infty}^{\infty} dx \; j^{0A}(x,t) \qquad 1 \le A \le \dim G$$

$$Q^A = f^A{}_{BC}\int_{-\infty}^{\infty} dx \int_x^{\infty} dy\, j^{0B}(x,t)\, j^{0C}(y,t) - 2\int_{-\infty}^{\infty} dx\, j_1^A(x,t)$$

... etc.

INFINITESIMAL FIELD TRANSFORMATIONS :

$$g(x,t) \to g(x,t) + \delta^{A}_{(n)} g(x,t)$$

$$M^{A}_{n} \equiv - \int d^2x \, \delta^{A}_{(n)} g(x) \frac{\delta}{\delta g(x)}$$

$$[M^{A}_{n}, M^{B}_{m}] = f_{ABC} M^{C}_{n+m}$$

KAC-MOODY LOOP ALGEBRA

LD 1981

$$\{Q^{A}, g(x,t)\} = -\delta^{A}_{(1)} g(x,t) + \tfrac{1}{2} f_{ABC} J^{B} \delta^{C}_{(0)} g$$

field dept. parameter

$$\Rightarrow [Q^{A}, Q^{B}] = f_{ABC} M^{(C)}_{2} + \dots$$

$J^{A}, Q^{A} \dots$ GENERATE YANGIAN ALGEBRA

DRINFELD 1985
D. BERNARD 1991 - 1993
...

KMA AND YANGIAN GENERATE THE SAME EQUIVALENCE RELATION : FIELD TRANSF'S ARE LINEAR COMBINATIONS

NON-LOCAL SYMMETRIES OF 2-DIMENSIONAL WORLDSHEET THEORIES :

WADIA, MANDAL, SURYANARAYANA 2002

TARGET SPACE OF TYPE IIB SUPERSTRING : BOSONIC COSET

$$AdS_5 \times S^5 \sim \frac{SO(4,2)}{SO(4,1)} \times \frac{SO(6)}{SO(5)}$$

$$G \to PSU(2,2|4)$$

1 NEGLECTING FOR THE MOMENT CLOSED STRING BC, NON-LOCAL CHARGES FORM INFINITE-DIMENSIONAL EXTENSION OF $PSU(2,2|4)$.

2 AdS/CFT CORRESPONDENCE IMPLIES REALIZATION OF YANGIAN ON CHAIN OF PARTONS

$D=4, \mathcal{N}=4$ SYM, $SU(N)$:

IIB SUPERSTRING ON $AdS_5 \times S^5$:

LARGE N $\longleftrightarrow$ STRING TREE GRAPHS

LARGE $g^2 N$ — SG LIMIT $\alpha' \to 0$ BENA, POLCHINSKI ROIBAN

SMALL $g^2 N$ — LARGE α'

DNW

MINAHAN ZAREMBO

BEISERT, STAUDACHER, KRISTJANSEN ...

SUPERCONFORMAL GAUGE THEORY: $D=4$, $\mathcal{N}=4$ SYM SU(N)

[LARGE N — 't HOOFT 1974

PLANAR GRAPHS

RADIAL QUANTIZATION ON $R \times S^3$:

- DILATATION GENERATOR IS HAMILTONIAN $D \sim \frac{P^0 + K^0}{2}$
- STATES IN $1 \leftrightarrow 1$ CORRESPONDENCE WITH LOCAL OPERATORS $\mathcal{O}(x)$

$$\lim_{|x| \to 0} \mathcal{O}(x)|0\rangle \sim |\mathcal{O}\rangle$$

- LOCAL OPERATORS ~ TRACE OF PRODUCT OF "LETTERS" $\uparrow$ ELEMENTARY FIELDS

SINGLE TRACE - LARGE N

$$\mathcal{O}(x) = \Phi^{(1)}_{ij}(x)\,\Phi^{(2)}_{jk}(x)\ldots$$

$$= \mathrm{Tr}\,\Phi^{(1)}(x)\,\Phi^{(2)}(x)\ldots\Phi^{(L)}(x)$$

$$\Phi^{I,\alpha,\mu\nu}_{ij}(x) \begin{cases} \phi^I = \Phi^{Ia}(x)\,T^a_{ij} \\ \psi_\alpha = \psi^a_\alpha(x)\,T^a_{ij} \\ F_{\mu\nu} = F^a_{\mu\nu}(x)\,T^a_{ij} \end{cases}$$

REPRESENTS A STATE OF A "CHAIN" OF L "SPINS"

$\uparrow$ i.e. PARTONS

AT $g^2 N = 0$:

$$\boxed{J^A = \sum_{i=1}^{L} J_i^A}$$

$$[J_i^A, J_j^B] = f_{ABC} J_j^C \delta_{ij}$$

$$[J^A, J^B] = f_{ABC} J^C \qquad : \quad PSU(2,2|4)$$

WHAT ABOUT Q^A ?

$$\boxed{Q^A = f^A{}_{BC} \sum_{i<j} J_i^B J_j^C}$$

THEN $[J^A, Q^B] = f_{ABC} Q^C$

YANGIAN GENERATORS : $\mathcal{J}_n^A$

where $\mathcal{J}_0^A \equiv J^A$

$\mathcal{J}_1^A \equiv Q^A$

$\mathcal{J}_2^A$ . . . from COMMUTATORS OF $[Q^A, Q^B]$ . . .

. . . .

$n = 0, 1, 2, 3, \ldots$

AS AN EXPANSION IN $g^2 N$:

$$\tilde{J}^A = J^A + g^2 N\, \delta J^A + O((g^2 N)^2)$$
$$\tilde{Q}^A = Q^A + g^2 N\, \delta Q^A + \text{''}$$

ONE-LOOP CORRECTIONS

WE WILL ASSUME THAT $\mathcal{N}=4$ SYM HAS YANGIAN SYMMETRY IN PLANAR LIMIT FOR ALL $g^2 N$:

$$[\tilde{J}^A, \tilde{J}^B] = f_{ABC}\, \tilde{J}^C$$
$$[\tilde{J}^A, \tilde{Q}^B] = f_{ABC}\, \tilde{Q}^C$$

EXPANDING TO ONE LOOP: FOR EG.

$$[\delta J^A, Q^B] + [J^A, \delta Q^B] = f_{ABC}\, \delta Q^C$$

FOR $J^A = D$

$$[\delta D, Q^B] + [D, \delta Q^B] = \lambda^B\, \delta Q^B$$

bare conformal dimensi

$$\Rightarrow \quad [\delta D, Q^B] = 0$$

1-LOOP HAMILTONIAN SPINCHAIN

OUR REQUIREMENT PARALLELS ONE USED BY BEISERT hep-th/0307015 :

$$[\delta D, J^B] + [D, \delta J^B] = \lambda^B \delta J^B$$

$$\downarrow$$

$$0$$

THEREFORE :

δD MUST COMMUTE WITH THE $g^2N = 0$ LIMIT OF THE WHOLE YANGIAN.

THE OPERATOR δD IS A SUM OF OPERATORS LOCAL ALONG THE CHAIN

ITS EIGENVALUES ARE THE (ONE-LOOP) ANOMALOUS DIMENSIONS :

Berenstein, Maldacena, Nastase
Minahan, Zarembo
Beisert, Staudacher, Kristjansen
Belitsky, Gorsky, Korchemsky

OPERATORS OF THIS TYPE THAT COMMUTE WITH THE ($g^2N=0$) YANGIAN $\left\{ \begin{matrix} J^A \\ Q^A \end{matrix} \right\}$ ARE CALLED THE HAMILTONIANS OF THE INTEGRABLE SPIN CHAIN.

THUS

FROM OUR CONSTRUCTION OF THE FREE FIELD LIMIT ($g^2N=0$) OF THE YANGIAN GENERATORS, WHICH WAS MOTIVATED BY THE NON-LOCAL SYMMETRIES FOUND BY BENA, POLCHINSKI & ROIBAN, WE HAVE DEDUCED THE CONCLUSION OF BEISERT & STAUDACHER hep-th/0307042 SHOWN EARLIER IN A SPECIAL CASE BY MINAHAN & ZAREMBO THAT δD IS A HAMILTONIAN OF AN INTEGRABLE SPIN CHAIN.

FOR THE REMAINDER OF THE TALK, WE WILL VERIFY THIS PICTURE BY USING FORMULAS FROM

BEISERT hep-th/0307015

BEISERT & STAUDACHER hep-th/0307042

THAT $\longrightarrow$ $[\delta D, Q^A] = 0$

$$[\delta D, J^A] = 0. \quad \checkmark$$

COMMUTATION OF Q^A WITH THE PLANAR ONE-LOOP HAMILTONIAN:

$$H \equiv \delta D$$

$$H = \sum_{i=1}^{L-1} H_{i,i+1}$$

A SUM OF OPERATORS ACTING ON NEAREST NEIGHBORS

LATTICE VERSION OF A TOTAL DERIVATIVE:

$$q^A = \sum_{i=1}^{L-1} (J_i^A - J_{i+1}^A) = J_1^A - J_L^A$$

THEN USING THE SPECIFIC FORM OF H DETERMINED BY BEISERT, WE CAN SHOW

$$[H, Q^A] = q^A$$

SO, FOR AN INFINITE CHAIN, ASSUMING TERMS AT INFINITY CAN BE DROPPED (NO SPONTANEOUS SYMMETRY BREAKING)

$$\Rightarrow [H, Q^A] = 0$$

FOR A FINITE CHAIN WITH PERIODIC BOUNDARY CONDITIONS, WHERE A TOTAL DERIVATIVE WILL SUM TO ZERO $\Rightarrow$ $[H, \mathcal{C}(Q^A)] = 0$

WELL DEFINED FOR PER. BC $\longrightarrow$ YANGIAN CASIMIR

TO SHOW $[H, Q^A] = q^A$:

CONSIDER $[H_{12}, Q^A_{12}]$

$$J^2_{12} = \sum_A (J_1^A + J_2^A)(J_1^A + J_2^A)$$

$$Q^A_{12} = f^A{}_{BC} J_1^B J_2^C$$

$$q^A_{12} = J_1^A - J_2^A$$

H_{12} ACTS ON A 2-PARTICLE SYSTEM

V_F SPACE OF 1-PARTICLE STATES IN FREE SYM$_4$ ON $R \times S^3$

$$V_F \otimes V_F = \bigoplus_{j=0}^{\infty} V_j$$

$j=0$	$j=1$	$j=2$	$j=3$
v_0	v_1	v_2	v_3
$P_\mu v_0$	$P_\mu v_1$	$\vdots$	$\vdots$
$P_\mu P_\nu v_0$	$\vdots$	$\vdots$	$\vdots$
$\vdots$			

$K_\mu v_j = 0$

$S_\alpha v_j = 0$

$J^A_{12} v_j$ $\updownarrow$

$Q^A_{12} v_j \to \in V_j$

$$\left[\begin{array}{l} q^A_{12} V_j \in V_{j-1} \oplus V_{j+1} \\ J^2_{12} V_j = j(j+1) V_j \end{array} \right.$$

$$H_{12} = \sum_{j=0}^{\infty} 2h(j)\, P_{12,j} \quad \text{— BEISERT, et al}$$

$$h(j) = \sum_{n=1}^{j} \frac{1}{n}$$

$$Q^A_{ij} = \frac{1}{4}\left[J^2_{ij}, q^A_{ij} \right] \quad \text{— IDENTITY}$$

$$[H_{12}, Q^A_{12}]\, |\lambda(j)\rangle$$

$$= \frac{1}{4}\left[H_{12}, [J^2_{12}, q^A_{12}] \right] |\lambda(j)\rangle$$

$$= \frac{1}{4}\left[H_{12} J^2_{12} - j(j+1) H_{12} - 2h(j) J^2_{12} + 2h(j) j(j+1) \right] \cdot q^A_{12}\, |\lambda(j)\rangle$$

$*$

$$= j\,(h(j) - h(j-1))\, |\chi^A(j-1)\rangle + (j+1)(h(j+1) - h(j))\, |\rho^A(j+1)\rangle$$

$$= |\chi^A(j-1)\rangle + |\rho^A(j+1)\rangle$$

$$= q^A_{12}\, |\lambda(j)\rangle$$

$\lambda(j) \in V_j$

$*$ USE $q^A_{12}|\lambda(j)\rangle = |\chi^A(j-1)\rangle + |\rho^A(j+1)\rangle$

NON-LOCAL CHARGE AS NOETHER CURRENTS :

SYM_4 :

$$\mathcal{L} = \frac{1}{g_{YM}^2} \mathrm{Tr} \left(\tfrac{1}{2} F_{\mu\nu} F^{\mu\nu} + D_\mu \varphi^I D^\mu \varphi^I - \tfrac{1}{2} [\varphi^I, \varphi^J][\varphi^I, \varphi^J] + \text{fermions} \right)$$

NOETHER CURRENTS FOR $SO(2,4)$:

CLASSICALLY : $j^{\mu A}(x) = K_\nu{}^A \Theta^{\mu\nu}(x)$

$K_\nu{}^A$ CONFORMAL KILLING VECTORS

$$\Theta^{\mu\nu} = 2 \mathrm{Tr}\, F^{\mu\rho} F^\nu{}_\rho + 2 \mathrm{Tr}\, D^\mu \varphi^I D^\nu \varphi^I - g^{\mu\nu} \mathcal{L} - \tfrac{1}{3} \mathrm{Tr} \left(D^\mu D^\nu - g^{\mu\nu} D^\rho D_\rho \right) \varphi^I \varphi^I + \text{fermions}$$

$\partial_\mu j^{\mu A}(x) = 0$ FOR ANY $g^2 N$.

IF WE SET $g^2 N = 0$, THEN THE UNTRACED MATRIX $(\Theta^{\mu\nu})^i{}_j = 2 F^{\mu\rho} F^\nu{}_\rho + 2 F^{\nu\rho} F^\mu{}_\rho + \text{scalars} + \text{fermions}$

IS ALSO CONSERVED, AS IS $K_\nu{}^A (\Theta^{\mu\nu})_{ij}$

$$\Rightarrow \quad Q^{AB} = \int_M K_\mu^A (\Theta^{0\mu})_i{}^j \int_M K_\nu^B (\Theta^{0\nu})_j{}^k \cdots \qquad \text{AT } g^2 N = 0.$$

- THE APPEARANCE OF A HAMILTONIAN δD THAT COMMUTES WITH THE YANGIAN $J^a, Q^A \ldots$ DEPENDS ON EXPANDING TO FIRST ORDER NEAR $g^2N = 0$.

- IN THE EXACT THEORY (AT NONZERO g^2N) THE EXACT DILATATION OPERATOR D (WHICH DEPENDS ON g^2N) IS A YANGIAN GENERATOR.

- IN THE EXACT THEORY IT IS NOT THE CASE THAT THERE IS A YANGIAN ALGEBRA <u>AND</u> A DILATATION OPERATOR THAT COMMUTES WITH IT.

- IN STRING THEORY, OR SYM_4, WANT TO COMPACTIFY THE STRING (OR THE SPIN CHAIN) WITH PERIODIC B.C. SINCE THE STRING IS CLOSED.

PERIODIC BC MAKE IT IMPOSSIBLE TO DEFINE THE YANGIAN BECAUSE RESTRICTION TO INTEGRATION REGION $x<y$ DOES NOT MAKE SENSE.

THE $PSU(2,2|4)$ GENERATORS J^A STILL MAKE SENSE AS DO SOME TRACES OF HOLONOMIES (WHICH ARE LIKE CASIMIR OPERATORS OF THE YANGIAN). THESE CASIMIRS WHICH COMMUTE WITH $PSU(2,2|4)$ MAY BE USEFUL IN COMPUTING THE SPECTRUM OF SYM_4 IN THE PLANAR LIMIT.

SOME OF THEM ARE ODD UNDER CHARGE CONJUGATION (THE SYMMETRY THAT REVERSES THE ORDER OF THE SPIN CHAIN), SO THEIR COMMUTATION WITH $PSU(2,2|4)$ LEADS TO DEGENERACIES AMONG STATES OF OPPOSITE CHARGE CONJUGATION PROPERTIES AS FOUND BY BEISERT, KRISTJANSEN, STAUDACHER.

TENSOR PRODUCT OF TWO V_F's IS GIVEN BY

$$V_F \otimes V_F = \bigoplus_{j=0}^{\infty} V_j$$

WHERE V_j ARE THE MODULES WITH PRIMARY WEIGHTS

$$v_0 = [2;0,0;020;2] \sim \phi^I\phi^J + \phi^J\phi^I - \tfrac{1}{3}\delta^{IJ}\phi^E\phi^E$$

$$v_1 = [2;0,0;101;2] \sim \phi^I\phi^J - \phi^J\phi^I$$

$$v_j = [j;j-2,j-2;000;2] \sim \sum_{I=1}^{6}\sum_{k=0}^{j-2}\Big(C_k^{(j-2)}\,\partial^k\phi^I\,\partial^{j-2-k}\phi^I + \cdots \Big)$$

WHERE $C_k^{(j-2)} = (-1)^j\, C_{j-2-k}^{(j-2)}$

$$v \equiv [\Delta_0;\ \underbrace{S_1,S_2}_{SO(4)\subset SO(2,4)};\ SU(4)_R\ \text{Dynkin label};\ L]$$

Δ_0: eigenvalue of D

L: length of chain = number of fields

ACTION OF QUADRATIC CASIMIR :

$$J^2 \, V_j = j(j+1) \, V_j \quad :$$

AS IF THE GROUP WERE $SU(2)$

INSTEAD OF $PSU(2,2|4)$:

$$J^2 = \tfrac{1}{2} D^2 + \tfrac{1}{2} L^{\gamma}{}_{\delta} L^{\delta}{}_{\gamma} + \tfrac{1}{2} \dot{L}^{\dot\gamma}{}_{\dot\delta} \dot{L}^{\dot\delta}{}_{\dot\gamma} - \tfrac{1}{2} R^{c}{}_{d} R^{d}{}_{c}$$
$$- \tfrac{1}{2} [Q^{c}_{\gamma}, S_{c}{}^{\gamma}] - \tfrac{1}{2} [\dot{Q}^{c}{}_{\dot\gamma}, \dot{S}_{c}^{\dot\gamma}] - \tfrac{1}{2} \{ P_{\gamma\dot\gamma}, K^{\gamma\dot\gamma} \}$$

$$P_{\alpha\dot\beta} \equiv P_{\mu} \sigma^{\mu}_{\alpha\dot\beta} \quad \ldots$$

$$J^2 |\psi(j)\rangle = \Big(\tfrac{1}{2} D^2 + \tfrac{1}{2} L^{\gamma}{}_{\delta} L^{\delta}{}_{\gamma} + \tfrac{1}{2} \dot{L}^{\dot\gamma}{}_{\dot\delta} \dot{L}^{\dot\delta}{}_{\dot\gamma} - \tfrac{1}{2} R^{c}_{d} R^{d}_{c}$$
$$+ \tfrac{1}{2} \{ S_{c}{}^{\gamma}, Q^{c}_{\gamma} \} + \tfrac{1}{2} \{ \dot{S}^{\dot\gamma}_{c}, \dot{Q}_{\dot\gamma c} \}$$
$$- \tfrac{1}{2} [K^{\gamma\dot\gamma}, P_{\gamma\dot\gamma}] \Big) |\psi(j)\rangle$$

$$J^2 |\psi(j)\rangle = \Big(\tfrac{1}{2} D^2 + s_1(s_1+1) + s_2(s_2+1) - \tfrac{1}{2} R^{c}{}_{d} R^{d}{}_{c} + 2D \Big) |\psi(j)\rangle$$

e.g. for $j \geq 2$, $R^{c}{}_{d} |\psi(j)\rangle = 0$

$$J^2 |\psi(j)\rangle = \Big(\tfrac{1}{2} j^2 + (\tfrac{j}{2} - 1) \tfrac{j}{2} \cdot 2 + 2j \Big) |\psi(j)\rangle$$
$$= j(j+1) \, |\psi(j)\rangle .$$

DECOMPOSITION OF $q^A_{12} V_j$:

WISH TO SHOW $q^A_{12} V_j \in V_{j+1} \oplus V_{j-1}$

DO THIS BY PROVING :

1) $q^A_{12} V_j \in \bigoplus_{k-j \text{ ODD}} V_k$

2) $q^A_{12} V_j \in \bigoplus_{|k-j| \leq 1} V_k$.

PROVE 1) : $q^A_{12} = -q^A_{21}$

$$V_j \sim \sum_k c_k^{(j-2)} \partial^k \phi^I \partial^{j-2-k} \phi^I$$

$$\sim (-1)^j \sum_{k'} c_{k'}^{(j-2)} \partial^{j-2-k'} \phi^I \partial^{k'} \phi^I$$

FOR EG., FOR j EVEN , $q^A_{12} V_j$ IS ODD UNDER THE INTERCHANGE OF THE TWO SPINS

THEN $q^A_{12} V_j \in \bigoplus_{k \text{ ODD}} V_k$.

PROVE 2) :

FIRST SHOW $q^{A}_{12} |\psi(j)\rangle \in \bigoplus_{k-j \leq 1} V_k$

↑PRIMARY

$$q^{A}_{12} |\psi(j)\rangle \subset \bigoplus_{k \leq j+1} V_k$$

FOR $j \geq 2$

SINCE $\text{DIM } |\psi(j)\rangle = j$

$\text{DIM } q^{A} = 1$

$$q^{A}_{12} |\psi(1)\rangle \subset \bigoplus_{k \leq 3} V_k \in V_0 + V_1$$

FOR $j = 1$

$\text{DIM } |\psi(1)\rangle = 2$

$$q^{A}_{12} |\psi(0)\rangle \subset \bigoplus_{k \leq 3} V_k \subset V_1 + V_3 \in V_1$$

FOR $j = 0$

$\text{DIM } |\psi(0)\rangle = 2$

SINCE ONLY STATE IN V_3 WITH DIM ≤ 3 IS $|\psi(3)\rangle$ WHICH IS $SU(4)_R$ INVARIANT. BUT $q^{A} |\psi(0)\rangle \sim q^{A} (\phi^{I} \phi^{J} + \phi^{J} \bar{\phi} + \ldots)$ IS NOT.

AND SHOW $q^{A}_{12} |\psi(j)\rangle \bigoplus_{k \geq j-1} V_k$

$$\langle x | q^A | \psi(j) \rangle \overset{?}{=} 0 \qquad \text{FOR } k < j-1$$

$$\langle x | q^A | \psi(j) \rangle = \left(\langle \psi(j) | q^{A\dagger} | x \rangle \right)^\dagger$$

$$\langle \psi(j) | q^{A\dagger} | x \rangle = 0 \qquad \text{FOR } j > k+1$$

IE. $k < j-1$

RADIAL QUANT. $P_\mu^\dagger = K_\mu$ . . .

THEN $q^{A\dagger}$ LINEAR COMB. OF q^A.

IF $q_{12}^A |\psi(j)\rangle \in \bigoplus_{|k-j| \le 1} V_k$

THEN $q_{12}^A V_j \in \bigoplus_{|k-j| \le 1} V_k$

SINCE $V_j = L_1 L_2 \ldots |\psi(j)\rangle$

↑ LOWERING OP'S IN $PSU(2,2|4)$ $\qquad$ P_μ, Q^a_α

MOM., GLOBAL SUPERCH.

AND $[L, q^A] \sim q^A$

SINCE $([J^B, q^A] = f_{BAC}\, q^C)$.