# DIPOLES, TWISTS, NONCOMMUTATIVE GEOMETRY AND PINNED BRANES

December 2000

Ori Ganor

Princeton University

BASED ON:

- "Vector Deformations of $\mathcal{N} = 4$ SYM, Pinned Branes and Arched Strings," [hep-th/0010072], K. Dasgupta, G. Rajesh and OG
- "Pinned Branes and New Non-Lorenz Invariant Theories," [hep-th/0002175], S. Chakravarty, K. Dasgupta, G. Rajesh and OG
- "Dipoles, Twists and Noncommutative Geometry," [hep-th/0008030], A. Bergman and OG

## GENERAL

Nowadays, a lot of effort is devoted by string-theorists to noncommutative-geometry and field-theories on noncommutative-spaces. These are field-theories with higher-derivative interactions that break Lorentz-invariance.

In the framework of string-theory, can we find other simple theories that break Lorentz-invariance?

We studied theories that contain fundamental dipoles.

## THREE QUESTIONS

- The transverse fluctuations of $N$ D-branes are described by scalar fields in the adjoint representation $N \otimes \overline{N}$ of a $U(N)$ gauge-theory.

  Can we construct examples in which the quanta of the transverse fluctuations are $(N, \overline{N})$ dipoles of the $U(N)$ gauge group?

- Noncommutative $\mathcal{N} = 4$ $U(N)$ SYM is a deformation of the ordinary $\mathcal{N} = 4$ SYM by a tensor operator of conformal dimension-6. The theory also has a vector operator of conformal dimension-5. Can we find a simple theory that at low-energies is described by the vector deformation?

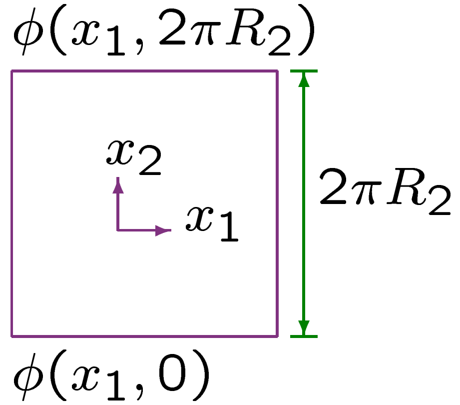
## THREE QUESTIONS (cont.)

- Field-theories on a noncommutative $T^2$ have a T-duality that acts as $\rho \to 1/\rho$, where $\rho$ is a dimensionless parameter that measures the area of the $T^2$ in units of the noncommutativity length-scale.

  Here all the fields are assumed to have periodic boundary conditions. What happens if we introduce phases such as

$$\phi(x_1, 2\pi R_1) = e^{i\alpha}\phi(x_1, 0).$$

  How do the phases affect the T-duality?



# OUTLINE

- Noncommutative geometry – brief review.
- Deformations of $\mathcal{N} = 4$ SYM – brief review.
- Definition of dipole theories.
- Applications:
  - NonCommutative Geometry: T-duality.
  - M(atrix)-Theory: Twists.
- Realization in String-Theory: Pinned Branes.
- Open Questions.

# OUTLINE (cont.)

* GENERALIZATIONS TO (2,0)-THEORY
* GRAVITY DUAL

# NCG AND STRING THEORY – i

- $N$ coincident $p$-dimensional D-branes are described at low energies $E \ll M_s$ by $(p+1)$-dimensional Supersymmetric Yang-Mills theory (Witten)

- The scalar fields are: a $U(N)$ gauge field $A_\mu$ and $(9-p)$ scalars $\Phi^I$ that are $N \times N$ matrices in the adjoint representation.

- The scalars have a potential $\mathrm{tr}\{\sum_{I<J}[\Phi^I, \Phi^J]^2\}$ that attains a minimum of 0 when all $(9-p)$ $N \times N$ matrices are diagonal.

- The $N$ eigenvalues of $\Phi^I$ at the minimum of the potential describe the $I^{th}$ transverse coordinates of the $N$ branes.

## NCG AND STRING THEORY – ii

- The description in terms of a $U(N)$ gauge theory with Lagrangian:

$$\mathcal{L}_{YM} \equiv \frac{1}{4g^2}\mathrm{tr}\{F_{\mu\nu}F^{\mu\nu}\} + \frac{1}{2g^2}\mathrm{tr}\{\sum_I D_\mu\Phi^I D^\mu\Phi^I\} + \frac{1}{2g^2}\mathrm{tr}\{\sum_{I<J}[\Phi^I,\Phi^J]^2 + \cdots\},$$

is valid when $|F_{\mu\nu}| \ll M_s^2 = \alpha'^{-1}$ and $|\Phi| \ll M_s$ and $|\partial_\mu\Phi| \ll M_s^2$.

- When $\alpha' F \sim 1$ and $|\partial_\mu\Phi| \sim M_s$ but $\partial_\mu F$ and $\partial_\mu\partial_\nu\Phi$ can be neglected ($|\partial_\mu F| \ll M_s|F|$), the correct Lagrangian is the Dirac-Born-Infeld (DBI). For $U(1)$ it is:

$$\mathcal{L}_{DBI} \equiv \frac{M_s^{(p+1)}}{g^2} \sqrt{\det(\eta_{\mu\nu} + M_s^{-2}F_{\mu\nu} + M_s^{-4}\sum_I \partial_\mu\Phi^I\partial_\nu\Phi^I)}.$$

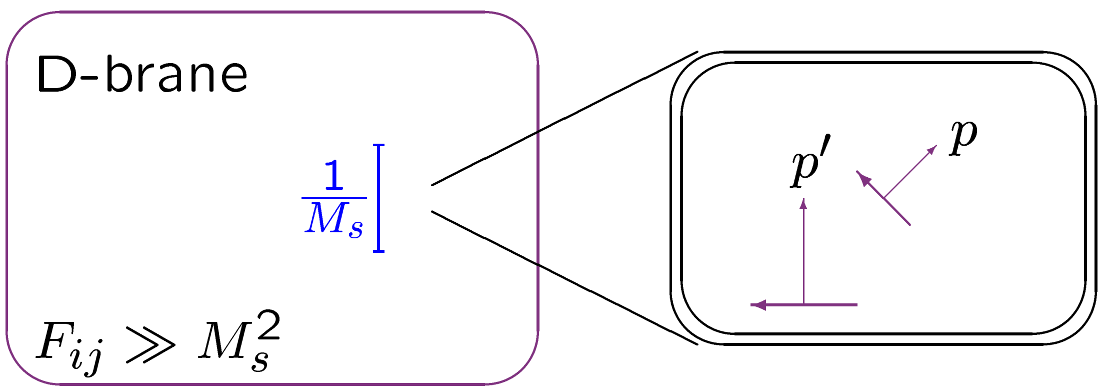
## NCG AND STRING THEORY – iii

Noncommutative geometry is related to the region $|F_{\mu\nu}| \gg M_s^2$ and the derivatives $|\partial_\mu F|$ are not small (i.e. $|\partial_\mu F|^2 \sim |F|^3$).

We turn on a strong magnetic field $F_{ij} \gg M_s^2$ and examine the dynamics on a length scale of the order of $|F|^{-1/2} \ll M_s^{-1}$. (Douglas&Hull, Connes&Douglas&Schwarz, Seiberg&Witten, ...)

THIS IS WHAT WE SEE:

# NCG AND STRING THEORY – iv




Every particle with momentum $p$ becomes an extended dipole of length $\theta^{ij} p_j$.

$$\theta^{ij} = (F^{-1})^{ij}.$$

(Bigatti& Susskind [hep-th/9908056])

# D-BRANES AND GAUGE THEORY (table)

| Description | $\frac{\lvert F \rvert}{M_s^2}$ | wave-length |
|---|---|---|
| SYM | small | $\gg M_s^{-1}$ |
| DBI | any | $\gg M_s^{-1}$ |
| NCG | large | $\ll M_s^{-1}$ |

## NCG

A field theory or gauge theory on a noncommutative space is defined as follows:

- Take the Lagrangian of the ordinary field theory and replace every product of fields with a noncommutative $\star$-product.

- The $\star$-product is defined in momentum space as:

  $$\Phi_1(p) \star \Phi_2(q) \equiv \mathsf{exp}\left\{\frac{i}{2}\theta^{kl} p_k q_l\right\} \Phi_1(p)\Phi_2(q).$$

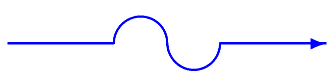
- Alternatively, the fields are functions of coordinates that are noncommutative. $[x_k, x_l] = i\theta_{kl}$ and then:

  $$e^{ip\cdot x} \star e^{iq\cdot x} \equiv \mathsf{exp}\left\{\frac{i}{2}\theta^{kl} p_k q_l\right\} e^{i(p+q)\cdot x}.$$

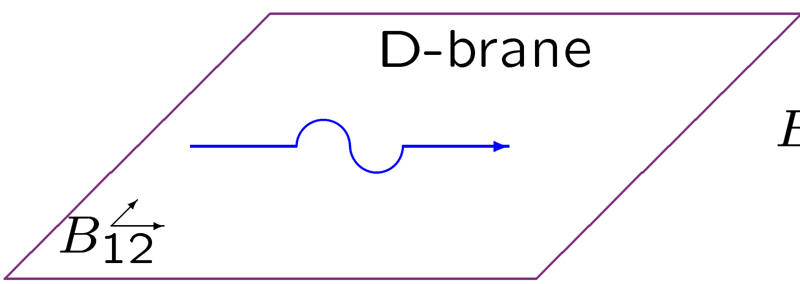
## RESCALING OF METRIC - review

In the presence of a $B$-field (or $F$-field) distances become longer close to the D-brane. (Seiberg & Witten)

A massless state with momentum $p$ has energy $E = |p|$ far away from the D-brane but only $E = |p| / \det(I + M_s^{-2} B)^{\frac{1}{2}}$ on the D-brane.



$$E = |p|$$




$$E = \frac{|p|}{\sqrt{1 + \frac{B^2}{M_s^4}}}$$

# DEFORMATIONS OF $\mathcal{N}=4$ SYM

From the list of ( Ferrara& Fronsdal& Zaffaroni [hep-th/9802203], Intriligator [hep-th/9811047]):

| $p$ | $SO(4)$ | $SU(4)_R$ | Dim | Realization |
|---|---|---|---|---|
| 2 | Scalar | 1 | 4 | $\delta g_{\text{YM}}$ |
| 2 | Scalar | $10 \oplus \overline{10}$ | 3 | Mass |
| 3 | Tensor | 1 | 6 | NCSYM |
| 3 | Vector | 15 | 5 | ??? |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

| | |
|---|---|
| $SO(4)$ | Lorentz representation |
| $SU(4)_R$ | R-symmetry representation |
| $p$ | Chiral primary is $\text{tr}\{\Phi^{(I_1}\Phi^{I_2}\ldots\Phi^{I_p)}\}$ |
| Dim | The conformal dimension |

## DEFORMATIONS OF $\mathcal{N} = 4$ SYM (cont.)

$$\mathcal{L} = \frac{1}{4g^2}\text{tr}\{F_{\mu\nu}F^{\mu\nu}\} + \frac{1}{2g^2}\sum_{I=1}^{6}\text{tr}\{D_\mu\Phi^I D^\mu\Phi^I\} + \frac{1}{2g^2}\sum_{I<J}^{6}\text{tr}\{[\Phi^I, \Phi^J]^2\} + \text{fermions}$$

At low energies, the Lagrangian for the $N = 4$ SYM on a noncommutative space looks like:

$$\mathcal{L}_{NC} = \mathcal{L} + \theta^{\mu\nu}\mathcal{O}_{\mu\nu} + \cdots.$$

$$\mathcal{O}_{\mu\nu} = \frac{1}{2g^2}\text{tr}\{F_{\nu\rho}F^{\rho\tau}F_{\tau\mu} - F_{\mu\nu}F^{\rho\tau}F_{\rho\tau}\} + \frac{1}{g^2}\text{tr}\{F_{\mu\rho}\sum_{I=1}^{6}\partial_\nu\phi^I\partial^\rho\phi^I - \frac{1}{4}F_{\mu\nu}\sum_{I=1}^{6}\partial_\rho\phi^I\partial^\rho\phi^I\} + \text{fermions}.$$

(Ferrara & Lledo & Zaffaroni [hep-th/9805082])

## VECTOR OPERATOR

The vector operator is given by:

$$\mathcal{O}_\mu \;=\; i\mathrm{tr}\{F_{\mu\nu}(D^\nu\phi^I\phi^J - D^\nu\phi^J\phi^I)\}$$

Note that there is another vector operator with the same quantum numbers where $F$ is replaced by the dual $\widetilde{F}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu}{}^{\rho\tau}F_{\rho\tau}$.

QUESTION

Can we find a simple theory that

at low-energies is described as

the deformation of $SU(N)$ $N = 4$ SYM

by the vector operator?

## DEFINITION OF DIPOLE THEORIES

A generalization of field-theories on commutative or noncommutative spaces:

- To each field $\Phi(x)$ assign a dipole-vector $L^\mu$.

- The complex conjugate field $\Phi(x)^\dagger$ is assigned $-L^\mu$.

- Define the dipole-product to be:

$$(\Phi_1 \tilde{\star} \Phi_2)_{(x)} \equiv \Phi_1(x - \frac{L_2}{2})\Phi_2(x + \frac{L_1}{2}).$$

- Associativity requires that $\Phi_1 \tilde{\star} \Phi_2$ is assigned $L_1 + L_2$.

# DEFINITION OF DIPOLE THEORIES (cont.)

- To insure associativity, we can pick a global additive charge and a global vector $\vec{L}$. A field $\Phi_a$ with charge $Q_a$ will have a dipole-vector $Q_a\vec{L}$.

- More generally, in $D$-dimensions, we can pick $n$ global charges, and a $D \times n$ matrix $\Theta^{\mu I}$ ($\mu = 1 \dots D$) and ($I = 1 \dots n$). A field $\Phi_a$ with charges $Q_{Ia}$ will be assigned a dipole-vector $L^\mu = \sum_I \Theta^{\mu I} Q_{Ia}$.

- Ordinary noncommutative geometry is a special case with $Q_I \equiv p_I$, the momentum, and $\Theta$ anti-symmetric.

## DEFINITION OF DIPOLE THEORIES (cont.)

Gauge fields are assigned dipole-vector 0.

The covariant derivative becomes:

$$
\begin{aligned}
D_\mu \Phi(x) &= \partial_\mu \Phi(x) - iA_\mu(x)\tilde{\star}\Phi(x) + i\Phi(x)\tilde{\star}A_\mu(x) \\
&= \partial_\mu \Phi(x) \\
&- iA_\mu(x - \frac{L}{2})\Phi(x) + i\Phi(x)A_\mu(x + \frac{L}{2}).
\end{aligned}
$$

$\Phi$ represents a dipole that is charged under $U(N)_{x-\frac{L}{2}} \times U(N)_{x+\frac{L}{2}}$.

## COMPARISON WITH NCYM

Several properties of NCYM have simpler analogs for dipole-theories. The dipole-theories have:

- A map to "local" variables (analogous to Seiberg-Witten map for NCYM):

$$\widetilde{\Phi}(x) \equiv Pe^{i\int_{x-\frac{L}{2}}^{x} A_\mu dx^\mu}\Phi(x)Pe^{i\int_{x+\frac{L}{2}}^{x} A_\mu dx^\mu}.$$

- Compactification on $S^1$ with "rational" $L = \frac{2\pi p}{q}$Radius is equivalent to a local $U(N)^q$ theory. (Analogous to compactification of NCYM on $T^2$ with rational $\theta^{12} = \frac{p}{q}$Area, Bigatti [hep-th/9804120], Seiberg & Witten [hep-th/9908142].)

## T-DUALITY IN NCG

On a noncommutative $T^2$ of size $(2\pi R_1) \times (2\pi R_2)$, with:

$$[x_1, x_2] = i\theta,$$

define $\rho \equiv \frac{2\pi R_1 R_2}{\theta}$.

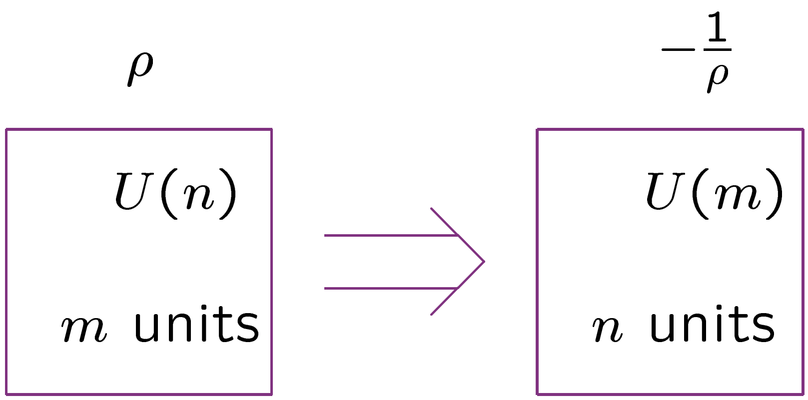
$U(n)$ gauge theory on a noncommutative $T^2$ with parameter $\rho$ and with $m$ units of magnetic flux is equivalent to $U(m)$ gauge theory on a noncommutative $T^2$ with parameter $-1/\rho$ and $n$ units of magnetic flux.

The area and Yang-Mills coupling constant transform as:

$$A \to \rho^{-2} A, \qquad g_{\text{YM}} \to \rho^{-1/2} g_{\text{YM}}$$

(Connes&Douglas&Schwarz, Rieffel&Schwarz, Brace&Morariu&Zumino, ..., Seiberg&Witten, ...)

## T-DUALITY (drawing)



## T-DUALITY (cont)

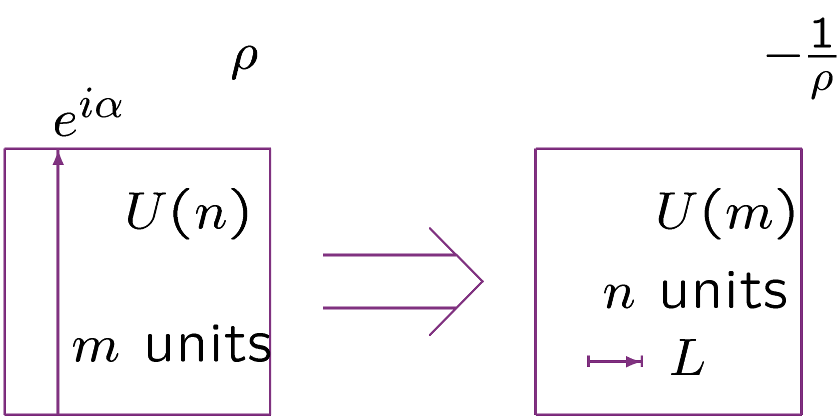
We introduce scalar fields with twisted boundary conditions:

$$\phi(x_1, 2\pi R_2) = e^{i\alpha}\phi(x_1, 0).$$

After T-duality $\phi$ is periodic:

$$\phi(x_1, 2\pi R_2) = \phi(x_1, 0),$$

but becomes a dipole along the $1^{st}$ direction with dipole-length: $\boxed{L = \alpha R_1}$.



## APPLICATION TO MATRIX-THEORY

- M(atrix)-theory is a (conjectured) formalism for calculating amplitudes in various theories of supergravity.

  (Banks&Fischler&Shenker&Susskind)

- A supergravity background is associated with a gauge field-theory and scattering amplitudes in supergravity are calculated from a large $N$ limit of amplitudes in the field theory.

- For 11D M-theory the gauge theory is 0+1D SYM with 16 supersymmetries.

  (Claudson&Halpern, Baake&Reinicke&Rittenberg, Flume, deWitt&Hoppe&Nicolai, Witten)

## APPLICATION TO MATRIX-THEORY (cont)

- For 11D M-theory the gauge theory is 0+1D SYM with 16 supersymmetries.

$$L = \frac{1}{2g^2}\mathrm{tr}\{\sum_{I=0}^{9} \dot{X}_I^2 + \sum_{I<J} [X_I, X_J]^2\} + \text{fermions.}$$

- For M-theory on $T^3$ the M(atrix)-model is $N = 4$ SYM compactified on a dual $T^3$.

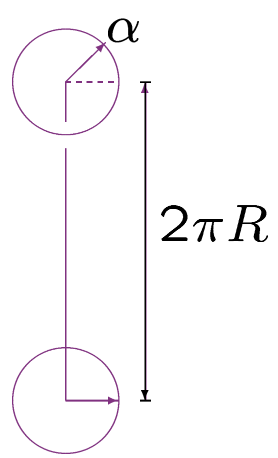
## DO DIPOLE-THEORIES PROVIDE ANY USE-FULL MATRIX MODELS?

## APPLICATION TO MATRIX-THEORY (cont)

Compactify on a circle with a twist:

$$(x_1, x_2 + ix_3) \sim (x_1 + 2\pi R, e^{i\alpha}(x_2 + ix_3)).$$

The M(atrix)-model is 1+1D SYM with $N = 16$ compactified on a circle of radius $r$ (such that $g_{\mathrm{YM}} r = \frac{1}{M_p R}$) and two scalar fields become dipoles of length $\alpha R$.




(Witten [hep-th/9710065], Cheung & Krogh & Mikhailov & OJG [hep-th/9812172])

# TWISTED COMPACTIFICATIONS

- We can generalize this twisted compactification of type-II string-theory for any embedding of $U(1) \subset Spin(8)$. The boundary conditions are twisted by a $Spin(8)$ rotation of the transverse directions.

- We can compactify on $T^d$ with twists in a $U(1)$ subgroup of $Spin(9-d)$. They are parameterized by $d$ phases $\alpha_1 \dots \alpha_d$.

- T-duality compels us to add an option for $d$ dual twists: $\beta_1 \dots \beta_d$.

- A state with charge $Q$ under $U(1)$ has fractional Kaluza-Klein momentum, related to $\alpha_i$. It also has fractional string winding number, related to $\beta_i$!

# TWISTED COMPACTIFICATIONS-cont

- What is the low-energy description of $N$ D$d$-brane probes for large $T^d$?

- What is the generalization to M-theory with U- instead of T-duality and M5-brane probes?

The answer to the first question is $(d+1)$-dimensional $U(N)$ SYM with $(9-d)$ scalars. They decompose into charged scalars under the $U(1) \subset Spin(9-d)$. A scalar $\Phi$ with charge $Q$ has boundary condions twisted by:

$$\Phi(x_k + 2\pi R_k) = e^{iQ\alpha_k}\Phi(x_k).$$

It also has a dipole-vector:

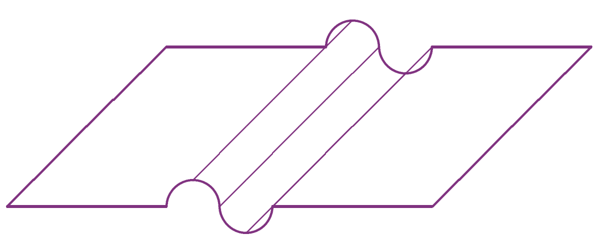
$$L = (Q\beta_1 R_1, Q\beta_2 R_2, \ldots, Q\beta_d R_d).$$

PINNED-BRANES

- Can we find a background where transverse fluctuations of D-branes are described by dipole-fields?

- Backgrounds where D-branes are pinned and transverse fluctuations of D-branes are massive fields?

- Generalize to M5-branes and M2-branes?

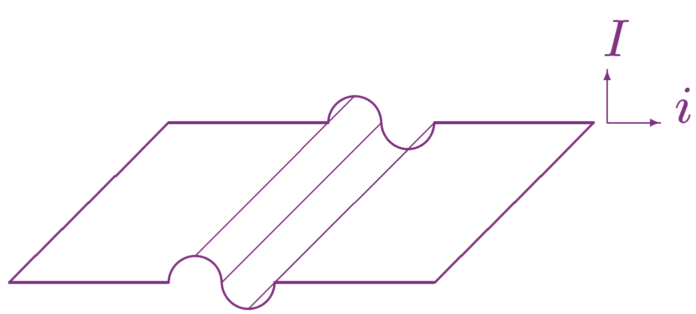


PINNED-BRANES (cont.)

We would like to see what happens when we turn on:

- $B_{Ii}$ with one component transverse to the brane, or

- $B_{IJ}$ with both components transverse to the brane.




But such components can be completely gauged away!

## Taub-NUT SPACE

A Taub-NUT space is a 4D manifold with metric:

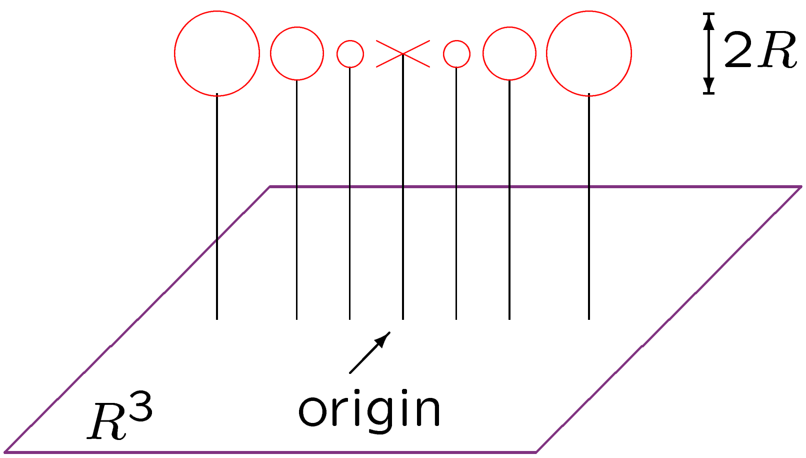
$$ds^2 = R^2 U \, (dy - A_i dx^i)^2 + U^{-1} \, d\vec{x}^2, \qquad i = 1 \ldots 3,$$

where,

$$U = \left(1 + \frac{R}{|\vec{x}|}\right)^{-1},$$

and $A_i$ is the gauge field of a monopole centered at the origin.

## Taub-NUT SPACE (drawing)




Taub-NUT space is a circle fibration with a base $R^3$ (only $R^2$ is shown in the picture).

## Taub-NUT SPACE (properties)

This metric has a few properties that we will utilize.

- It is a circle fibration over $R^3$ when the origin is excluded.

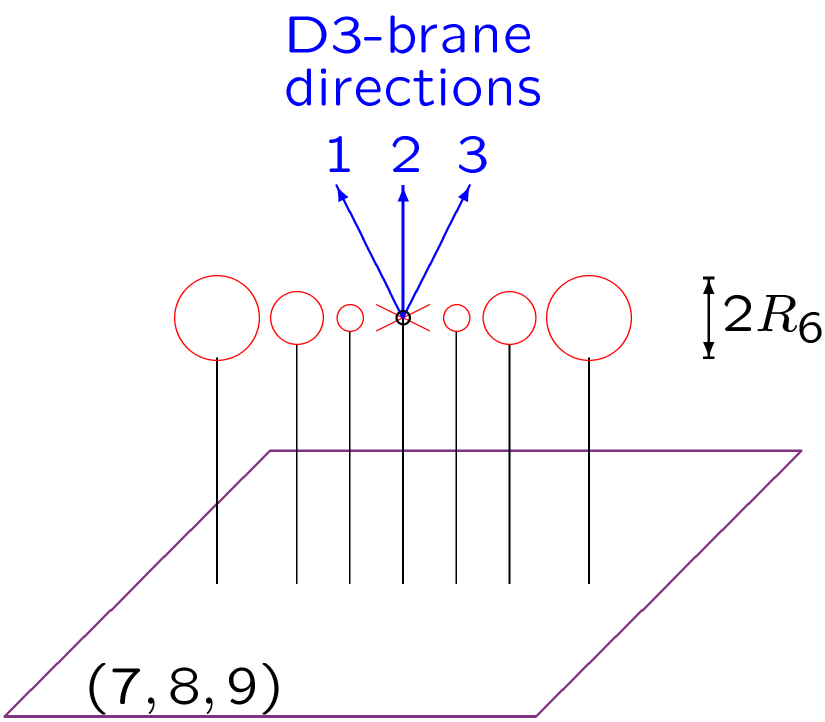
- The radius of the fiber shrinks to zero as we approach the origin and becomes a constant $R$ as we approach infinity.

- If we restrict to $|\vec{x}| = r$ with constant $r > 0$. the circle fibration is equivalent to the Hopf fibration of $S^1$ over $S^2$.

- There is a $U(1)$ isometry $y \to y + \epsilon$. It has one fixed point at the origin.

- The $U(1)$ isometry acts nontrivially on the tangent space to the point at the origin.

# TURNING ON $B$-FLUX




We put D3-branes at the center of the Taub-NUT space and turn on a $B_{16}$-flux at $\infty$.

## SUPERGRAVITY SOLUTION - TAUB-NUT WITH FLUX

Without the D3-branes the solution of a Taub-NUT with $B$-field is:

$$\begin{aligned} B &= \frac{b}{1+\frac{R_6}{(1+b^2)r}} dx^5 \wedge (dx^6 + \sum_7^9 A_i dx^i), \\ e^{\phi-\phi_0} &= \sqrt{\frac{1+\frac{R_6}{r}}{1+\frac{R_6}{(1+b^2)r}}}, \end{aligned}$$

The metric is:

$$\begin{aligned} ds^2 &= dx_0^2 + \cdots dx_2^2 + \cdots dx_5^2 \\ &+ \frac{1+\frac{R_6}{r}}{1+\frac{R_6}{(1+b^2)r}} dx_1^2 \\ &+ \frac{1}{1+\frac{R_6}{(1+b^2)r}} (dx_6 - \sum_7^9 A_i dx_i)^2 \\ &+ \left(1+\frac{R_6}{r}\right)(dx_7^2 + dx_8^2 + dx_9^2). \end{aligned}$$

## SUPERGRAVITY SOLUTION (cont)

Note that near the origin the good coordinates are $\sqrt{r}dx_6$ and $\frac{1}{\sqrt{r}}dx_i$ ($i = 7, 8, 9$).

The origin is smooth with a finite $H = dB$ field-strength and a finite curvature.

## EFFECT OF B-FLUX ON BRANES

In general, a strong $B$-flux with one direction along the Taub-NUT circle (direction $6^{th}$) can have two kinds of effects on the branes at the center of the Taub-NUT space, according to the direction of the other index in $B$ at $\infty$:

- Transverse to the brane (e.g. $B_{56}$) – pinning.
- Parallel to the brane (e.g. $B_{16}$) – dipole-theory.

In the last case we have to rescale the direction along the brane that is parallel to the $B$-field by $\sqrt{1+\frac{B^2}{M_p^4}}$.

## EFFECT OF B-FLUX - PINNING

1. The D3-branes are pinned to the origin by a gravitational potential.

2. The transverse fluctuations are described by massive fields.

3. The low-energy description is $U(n)$ SYM with $\mathcal{N} = 2$ supersymmetry and a massive adjoint hypermultiplet.

## EFFECT OF B-FLUX - DIPOLES

1. The D3-branes are not pinned to the origin and are free to move.

2. The transverse fluctuations are described by dipole-fields.

3. The low-energy description is $SU(n)$ SYM with $\mathcal{N} = 2$ supersymmetry and a massless dipole hypermultiplet.

## THE OVERALL $U(1)$

The $U(1)$ factor probably becomes massive

Recall that for the quiver-theories on D3-branes at $A_q$ singularities the gauge group is at first sight:

$$U(N) \times U(N) \times \cdots \times U(N)$$

But the relative $U(1)$ factors become massive through an interaction with an RR 2-form (Douglas & Moore, [hep-th/9603167]):

$$\frac{1}{2}\int |dC^{(RR)}|^2 + \int C^{(RR)} \wedge F \Longrightarrow \int (A_\mu + \partial_\mu \widetilde{C})^2.$$

We expect a similar effect in our case. There is probably an interaction of the form:

$$\int \epsilon^{\mu\nu\sigma\tau}(C^{(RR)}_{\mu\nu} \tilde{\star} F_{\sigma\tau} - F_{\sigma\tau} \tilde{\star} C^{(RR)}_{\mu\nu}).$$

which leads to:

$$\int d^4x (A_\mu(x+L) - A_\mu(x) + \partial_\mu C)^2,$$

where $C$ is the dual of an RR 2-form.

## EFFECT OF B-FLUX ON BRANES (table)

| Dir | Effect | SUSY | symmetry |
|---|---|---|---|
| $B_{56}$ | pinning | $N = 2$ | $SU(2) \times U(1)$ |
| $B_{16}$ | dipoles | $N = 2$ | $U(1) \times SU(2) \times U(1)$ |
| $B_{12}$ | NCG | $N = 4$ | $SU(4)$ |
| $B_{15}$ | no effect | $N = 4$ | $SU(4)$ |
| $B_{45}$ | no effect | $N = 4$ | $SU(4)$ |

Dir $\equiv$ Direction.

symmetry $\equiv$ Unbroken symmetry $\subset SU(4)_R$

BPS FORMULAS

In this setting, it is easy to calculate the mass of BPS objects from BPS formulas of string theory on $T^6$.

We compactify all the directions along the D3-branes so that they become particles.

The BPS formulas give us the mass of a particle as a function of the charges and the fundamental masses of the objects:

$$M_{TN} = \frac{1}{g_s^2} M_s^8 R_1 R_2 R_3 R_4 R_5 R_6^2,$$
$$M_{D3} = \frac{N}{g_s} M_s^4 R_1 R_2 R_3.$$

and the $B$-flux.

We will always take the limit $R_5 \to \infty$.

## TRANSVERSE FLUCTUATIONS

We can also look for objects with nontrivial R-symmetry quantum numbers. These would be the quanta of fluctuations of the D3-branes in the transverse directions.

They transform nontrivially under rotations in the directions $4 \ldots 9$ transverse to the D3-branes.

Since translations of the coordinate of the $6^{th}$ circle at $\infty$ get mapped to rotations at the origin of the Taub-NUT space, we can simply look for states with Kaluza-Klein momentum in the $6^{th}$ direction!

PINNING

For a $B$-flux transverse to the D3-branes, we set $b \equiv M_s^{-2} B_{56}$ and

$$M = M_{TN} + \frac{1}{\sqrt{1+b^2}} M_{D3}.$$

The tension of the D3-brane is smaller at the center of the Taub-NUT space and it is attracted to the origin.

$\Longrightarrow$Confirmed from the supergravity solution!

4 out of the 6 modes of the transverse fluctuations are massive with a mass:

$$\boxed{M = \frac{b}{\sqrt{1+b^2}\, R_6}}$$

DIPOLES

For a $B$-flux with one direction along the D3-branes, we set $b \equiv M_s^{-2} B_{16}$ and

$$M = M_{TN} + M_{D3}, \qquad \text{NO PINNING}$$

The transverse fluctuations have no mass but 4 out of 6 modes are dipoles (as we shall soon confirm).

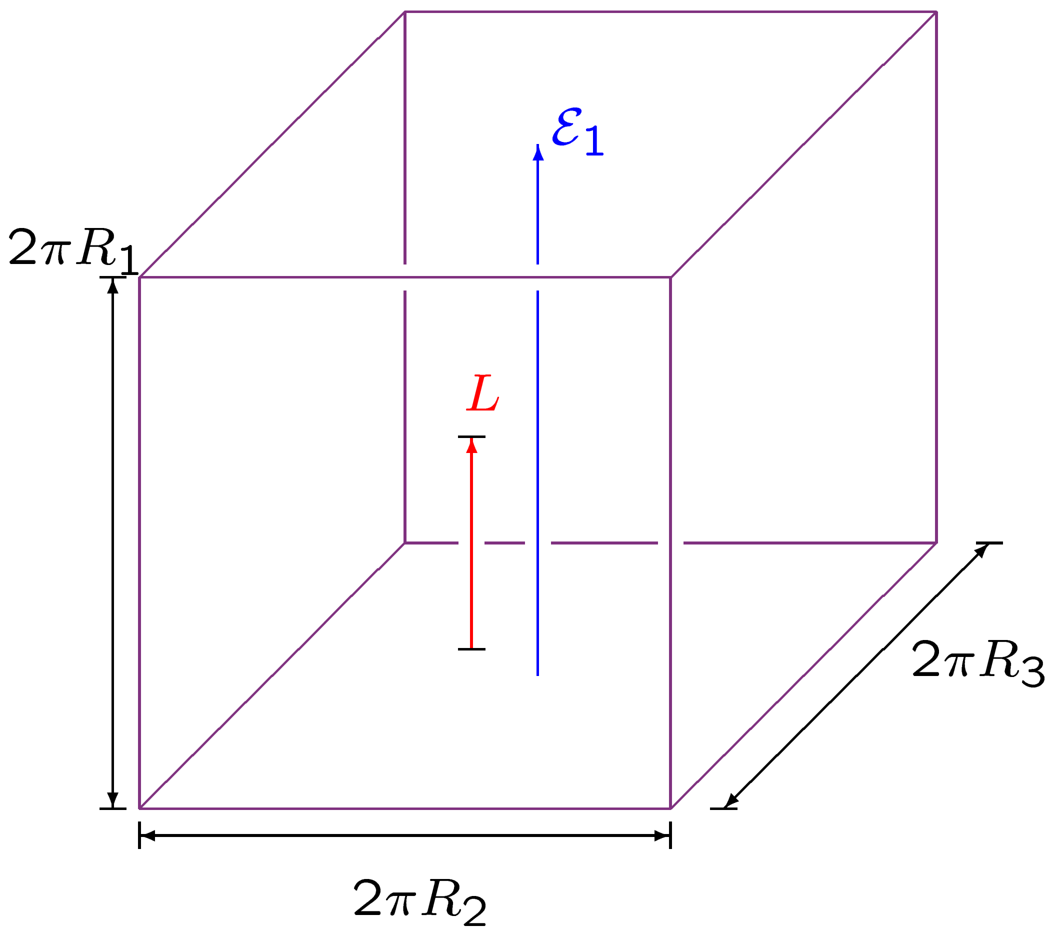
A Kaluza-Klein excitation along the $1^{st}$ direction has mass:

$$M_{KK} = \frac{1}{\sqrt{1+b^2}\, R_1}$$

$\Longrightarrow$Requires rescaling:

$$\boxed{\widetilde{R}_1 \equiv \sqrt{1+b^2} R_1}$$

# ELECTRIC FIELDS - THE SETTING




The electric flux $\mathcal{E}_1$ and the dipole $\vec{L}$ are in the same direction.

# ELECTRIC FIELDS

How do the dipoles behave in an electric field?

$U(N)$ $\mathcal{N}=4$ supersymmetric gauge-theory in a box of size $(2\pi R_1) \times (2\pi R_2) \times (2\pi R_3)$ has BPS sectors of different electric fluxes.

With $k$ units of electric flux along the $1^{st}$ direction, the energy is:

$$E_{\text{flux}} = \frac{g^2 k^2 R_1}{4\pi N R_2 R_3}.$$

For $U(1)$ this would correspond to an electric field of

$$\mathcal{E}_1 = \frac{k}{4\pi^2 R_2 R_3}.$$

The BPS mass formula for $d$ dipoles in a sector of $k$ units of electric flux is:

$$\begin{aligned} E &= \frac{g^2 (k R_1 + \frac{L}{2\pi})^2}{4\pi N R_1 R_2 R_3} \\ &= E_{\text{flux}} + g^2 \mathcal{E}_1 L + \frac{g^2 L^2}{8\pi^3 R_1 R_2 R_3}. \end{aligned}$$

## ELECTRIC FIELDS - DETAILS

In the context of string-theory we set:

$$
\begin{aligned}
x &= \frac{1}{g_s^2} M_s^8 R_1 R_2 R_3 R_4 R_5 R_6^2, \\
y &= \frac{N}{g_s} M_s^4 R_1 R_2 R_3, \\
w &= \frac{d}{R_6}, \; u = M_s^2 R_1, \; b \equiv M_s^{-2} B_{16}.
\end{aligned}
$$

The BPS mass formula is:

$$
\begin{aligned}
m^2 &= (1+b^2)x^2 + u^2 + w^2 \\
&\quad + 2\sqrt{(1+b^2)x^2y^2 + (u+bw)^2x^2}.
\end{aligned}
$$

For $R_4, R_5 \to \infty$ and $M_s R_i \to \infty$ (for $i = 1,2,3$) this becomes:

$$
\begin{aligned}
M &= \frac{M_s^8 R_1 R_2 R_3 R_4 R_5 R_6^2}{g_s^2} + \frac{N M_s^4 R_1 R_2 R_3}{g_s} \\
&\quad + \frac{g_s}{N(1+b^2) R_1 R_2 R_3} \left( k R_1 + \frac{bd}{R_6} \right)^2 .
\end{aligned}
$$

## ELECTRIC FIELDS - DETAILS (cont.)

By studying KK excitations we learned that we need to rescale:

$$\widetilde{R}_1 \equiv \sqrt{1+b^2} R_1, \qquad \widetilde{g}_s \equiv \frac{g_s}{\sqrt{1+b^2}}.$$

Therefore, the dipole length is:

$$L = \frac{b}{\sqrt{1+b^2}\, M_s^2 R_6}.$$

## ELECTRIC FIELDS - CONCLUSION

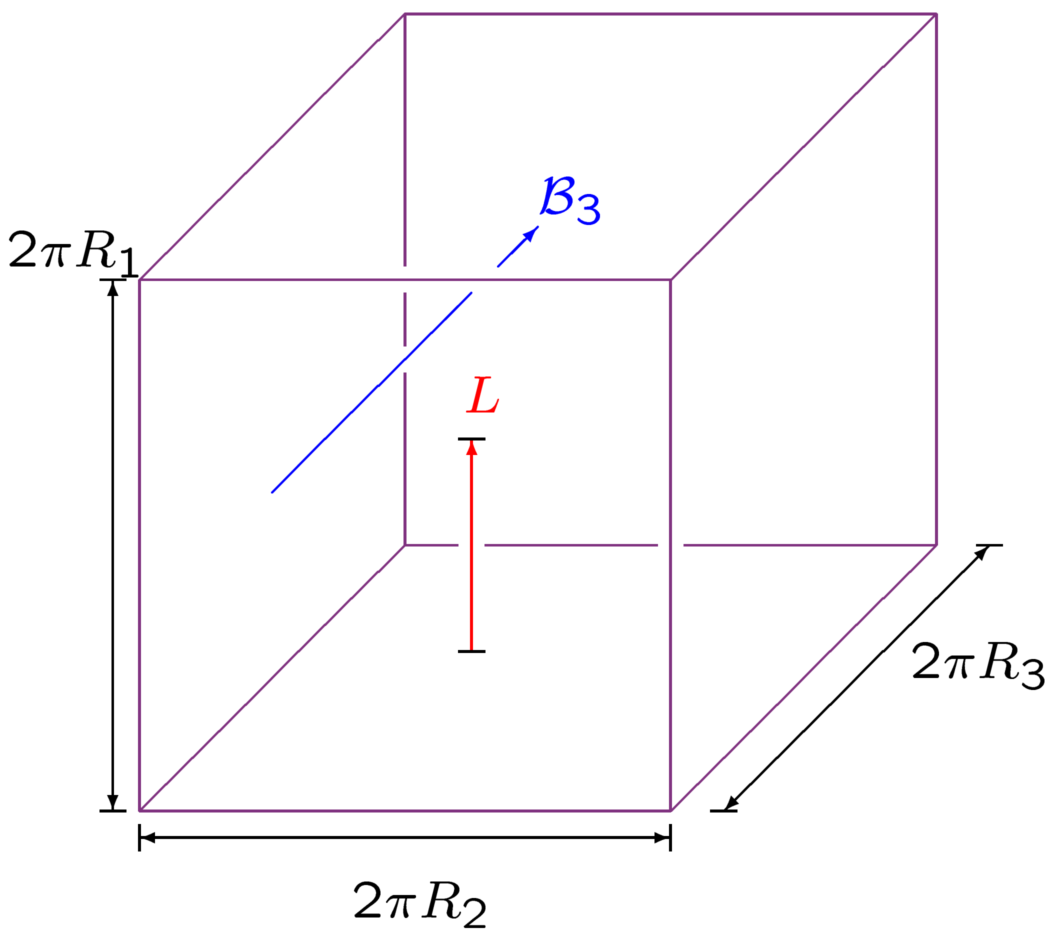
The dipole length is given by:

$$L = \frac{b}{\sqrt{1+b^2}\, M_s^2 R_6}.$$

Note that $L \gg M_s^{-1}$ requires $R_6 \gg M_S^{-1}$.

The T-dual picture would then be more suitable. (It is an NS5-brane with a transverse circle and D4-branes that wrap around it with a shift reminiscent of the "elliptic" brane configurations of Witten, [hep-th/9703166].)

## MAGNETIC FIELDS - THE SETTING




The magnetic flux $\mathcal{B}_3$ is orthogonal to the dipole $\vec{L}$ direction.

## MAGNETIC FIELDS

How do the dipoles behave in a magnetic field?

$U(N)$ $N = 4$ supersymmetric gauge-theory in a box of size $(2\pi R_1) \times (2\pi R_2) \times (2\pi R_3)$ has BPS sectors of different magnetic fluxes.

With $k$ units of magnetic flux along the $3^{rd}$ direction, the energy is:

$$E_{\text{flux}} = \frac{\pi k^2 R_3}{g^2 N R_1 R_2}.$$

For $U(1)$ this would correspond to a magnetic field of:

$$\mathcal{B}_3 = \frac{k}{4\pi^2 R_1 R_2}.$$

The boundary conditions for $x_2 \to x_2 + 2\pi R_2$ contain an extra gauge transformation with $\Lambda = e^{\frac{ik}{NR_1} x_1 \tau}$ ($\tau$ is a generator of $U(N)$).

## MAGNETIC FIELDS (cont.)

In the presence of a magnetic flux the dipole-fields are no-longer periodic. They acquire an extra phase:

$$\begin{aligned}\Phi(x_1, x_2 + 2\pi R_2) &= e^{\frac{ik}{NR_1}(x_1+L)}\Phi(x_1,x_2)e^{-\frac{ik}{NR_1}x_1} \\ &= e^{\frac{ikL}{NR_1}}\Phi(x_1,x_2).\end{aligned}$$

There will be BPS states corresponding to Kaluza-Klein excitations with mass:

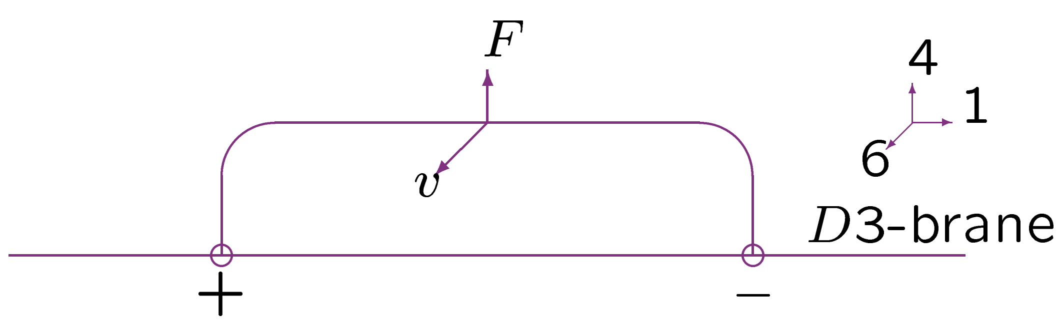
$$M = \frac{kL}{2\pi N R_1 R_2}.$$

We have also confirmed that this is the case with:

$$L = \frac{b}{\sqrt{1+b^2}\, M_s^2 R_6}.$$

# DIPOLES AS ARCHED STRINGS

A large R-symmetry charge $\Longrightarrow$ Classical angular momentum in the transverse directions.




The dipole on a D3-brane is a string that arches out into the $4-6$ dimensions. The D3-brane is stretched along the $1^{st}$ direction (and directions $2,3$ that are not shown) and is at the origin of the $4-6$ plane. The generalized magnetic force $F$ is perpendicular to the velocity and the string.

## ARCHED STRINGS – ASSUMPTIONS

We have verified that the strings emerge perpendicular to the D-brane.

We have neglected:

- Attraction between the end-points.
- Gravitational radiation to the bulk.
- Other relativistic effects.

This can be justified by:

- A large rescaling factor $\sqrt{1+b^2}$.
- Small $g_s$.

## GRAVITY DUAL - NOTATION

What is the gravity dual of the dipole theories at $N \to \infty$?

$$\begin{aligned} \vec{n} &= \text{parameterizes } S^5 \text{ and } \|\vec{n}\|^2 = 1, \\ \hat{M} &\in so(6), \qquad \text{the dipole-vectors.} \end{aligned}$$

We expect that in the IR limit, $r \to \infty$, a deformation of $AdS_5 \times S^5$ by a dimension-5 operator.

GRAVITY DUAL – SOLUTION

What is the gravity dual of the dipole-theories? To get the gravity dual, we probe with D2-branes the dual of the twisted geometry. We find the supergravity solution:

$$\frac{ds^2}{\sqrt{4\pi g^2 N}} = \frac{1}{r^2}(dr^2 + dx_0^2 + dx_1^2 + dx_2^2) + \frac{1}{r^2 + \lambda \vec{n}^T \widehat{M}^T \widehat{M} \vec{n}} dx_3^2 + d\vec{n}^T d\vec{n}.$$

The NSNS $B$-field is:

$$\sum_{a=1}^{6} B_{3a} d\hat{n}_a = \frac{\vec{n}^T \widehat{M} d\vec{n}}{r^2 + \lambda \vec{n}^T \widehat{M}^T \widehat{M} \vec{n}},$$

and the dilaton is:

$$e^{\varphi} = \frac{g}{\sqrt{1 + \frac{\lambda}{r^2} \vec{n}^T \widehat{M}^T \widehat{M} \vec{n}}}, \qquad \lambda \equiv 4\pi g^2 N.$$

## THE (2, 0)-THEORY

- There exists a 5+1D mysterious superconformal field theory that is inherently strongly coupled. (Witten)

- Its existence sheds light on S-duality of $N = 4$ SYM. (Witten)

- The $U(1)$ version of the theory has, instead of a gauge field, $A_i$, a tensor field, $\widetilde{B}_{ij}$. Its field strength $H_{ijk}$ is required to be anti-self-dual:

$$H_{ijk} = -\frac{1}{6}\epsilon_{ijklmn}H^{lmn}.$$

## $(2, 0)$-THEORY (cont)

- The $U(1)$ version also has $5$ real scalars.

- It is the low-energy description of coincident M5-branes. The scalars correspond to transverse fluctuations of the M5-branes. (Strominger)

# GENERALIZATION TO (2, 0)-THEORY

- There exists a generalization of the dipole-theories to a deformation of the (2, 0)-theory.

- The theory is parameterized by a tensor $L_{IJ}$ with dimensions of area.

- If we set $L_{12} \neq 0$, The quanta of the transverse fluctuations are described not by dipoles but by "discpoles" – objects that have fixed area $L_{12}$ in the $1-2$ plane.

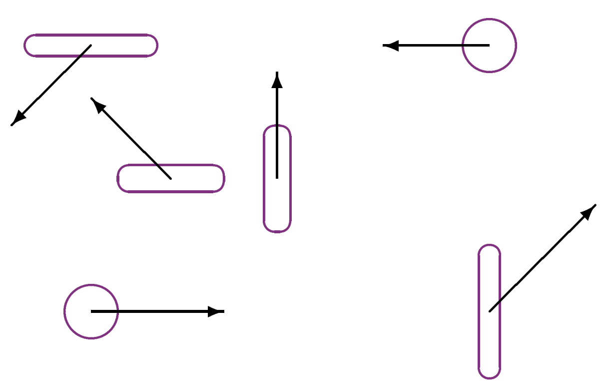
- It can be realized by placing M5-branes at the origin of a Taub-NUT space with a large $C \equiv M_p^{-3} C_{126}$ turned on.

- The discpole tensor will then be (after the necessary rescalings) $L_{12} = \frac{C}{M_p^3 R_6}$

DISCPOLES (drawing)

- The boundary of the "discpole" is charged under the tensor-field $\widetilde{B}_{ij}$.

- The boundary of the "discpole" is probably dynamical.



Discpoles in different shapes and different momenta. They all have the same area, though.

# GENERALIZED TWISTS

IN A U(1) GAUGE THEORY T WITH GAUGE FIELD $A_\mu$:

* IF WE HAVE A GLOBAL CHARGE Q $\Rightarrow$ WITH NOETHER CURRENT $J_\mu$

WE CAN ATTEMPT TO CORRELATE THE FRACTIONAL PART OF THE U(1) CHARGE WITH Q BY ADDING A TWIST $\alpha$

$\Rightarrow$ U(1) CHARGE $\in \mathbb{Z} + Q\alpha$

$\Rightarrow$ $\mathcal{L} \to \mathcal{L} + \alpha A_\mu J^\mu + \ldots$

# GENERALIZED TWISTS:

## EXAMPLES

* $\mathcal{T} = \dfrac{\text{M-THEORY}}{T^n \times R^{10-n,1}}$

geo-metrical twist:
- U(1) = TRANSLATION
- $\Phi$ = SO(10-n) ANGULAR MOMENTUM

* $\mathcal{T}$ = M-THEORY / $T^2 \times R^{10-2,1}$

M2-twist:
- U(1) ⇒ M2-BRANE CHARGE
- $\Phi$ = SO(8) ANGULAR MOME...

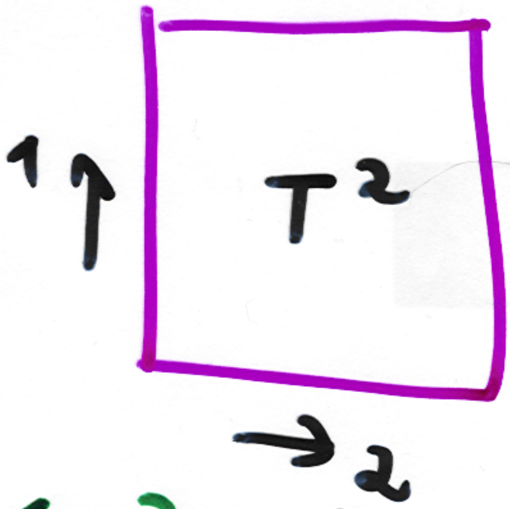
* $\mathcal{T}$ = M-THEORY / MANIFOLD W
- U(1) ⇒ ONE OF $H^2(W)$ M2-BRANE CHARGES
- .....
- ??? ⇒ PERHAPS A TORSION PART OF K-THEORY

.....

B

# MORE ON DISCPOLES

M-THEORY ON




ADD AN M2-BRANE TWIST:

$$\left(\text{12-WRAPPED M2-BRANE CHARGE}\right) \in \mathbb{Z} + \mathbb{Q}\,\alpha$$

PROBE WITH M5-BRANES:

EITHER

$U(1) \subset SO(4)$ ↑ ROTATION IN 3,4,5,6

* WRAPPED 1,2, 7,8,9 ⇒ Discpole (TENSOR) $L_{12}$
* PARTIALLY WRAPPED 1,7,8,9,10.

⇒ Pinned Massive Theory (VECTOR) $L_1$

## CONCLUSIONS

- Non Lorentz invariant theories with fundamental dipole-fields naturally appear in string-theory, M(atrix)-theory and noncommutative geometry.

- They break Lorentz invariance stronger than field-theories on Noncommutative spaces (linearly in energy rather than quadratically).

- There is a generalization to a 6D theory with disc-like objects. (Let's call them disc-poles.)

- There is another extension of the $(2,0)$ theory that is a deformation by a relevant vector operator of dimension-5.

## OPEN QUESTIONS

- What is the S-dual of the 4D $SU(N)$ dipole-theories?

- Time like dipole-vectors? Light-like dipole-vectors?

  Compare to Gomis & Mehen [hep-th/0005129] and Aharony & Gomis & Mehen [hep-th/0006236].

- Dynamics of the boundary of the disc-poles?

- Extensions to probes of other U-duals of twisted geometries.