Supersymmetric Gödel Universes in String Theory

Troels Harmark
Harvard University

February 20
UNC, Chapel Hill

Based on hep-th/0301206
with Tadashi Takayanagi.

Introduction:

Gödels solution of 4D GR:

$$ds_4^2 = ds_3^2 + dz^2 \quad \leftarrow \text{trivial direction}$$

$$ds_3^2 = -dt^2 + d\rho^2 + \frac{1}{\Omega^2}\sinh^2(\Omega\rho)\left(1-\sinh^2(\Omega\rho)\right)d\phi^2 - \frac{1}{\Omega}2\sqrt{2}\sinh^2(\Omega\rho)\,dt\,d\phi$$

$\Lambda = -2\Omega^2$, $\rho = \frac{\Omega^2}{2\pi G}$, pressure-less matter

Homogenous, trivial topology.

$\rho = t = \text{const}$: CTC for $\rho > \rho_c = \frac{\log(1+\sqrt{2})}{\Omega}$

CTCs → no causality → unphysical

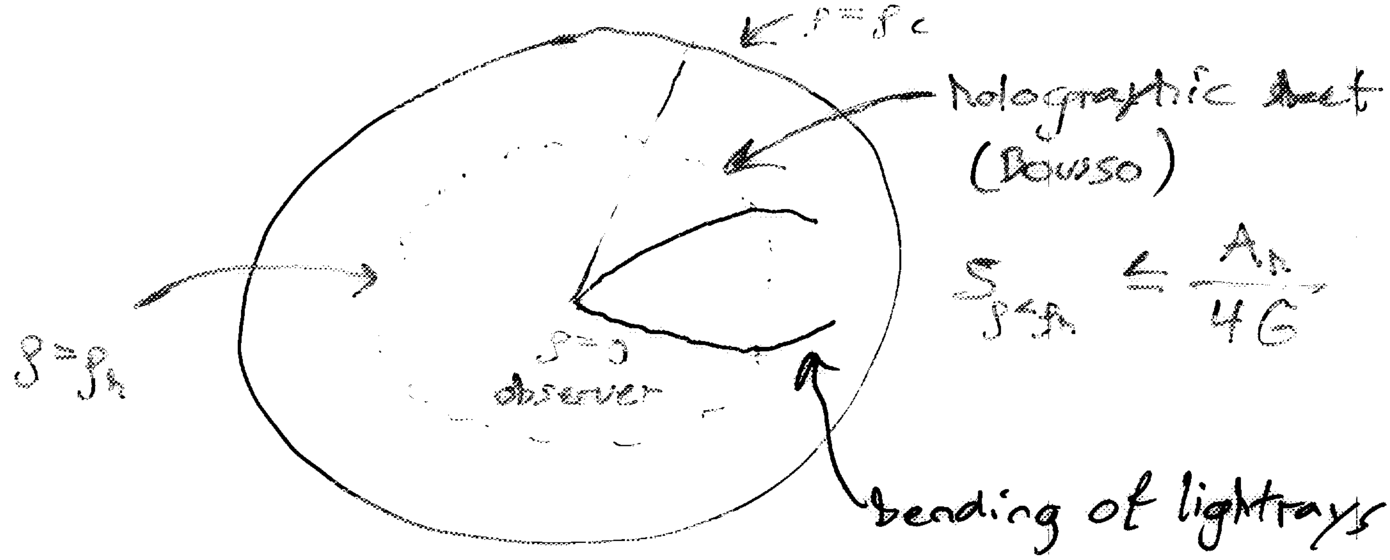
Hawking's Chronology Protection Agency:
Conjectured mechanism preventing solutions with CTCs from forming.
(similar to Penrose's Cosmic Censorship Conjecture)

However, Gauntlett, Gutowski, Hull, Pakis, Reall (GGHPR) recently found M-theory background with 20 susy of the Gödel Universe type.
SUSY protects background from Hawkings agents.
(different from naked singularity → solution is weakly curved everywhere)

SUSY protects Gödel background

⇒ Need different kind of chronology protection

Boyda, Ganguli, Horava, Varadarajan (BGHV) proposes GR can be incorrect, even for smooth backgrounds with small curvature. True d.o.f.'s given by Holography.



What about accelerated observers travelling around a CTC? Not clear

Is String / M-theory well-defined on these backgrounds?

Study of supersymmetric Gödel Universes in string and M-theory can, perhaps, provide important insights.

Outline:

- Finding supersymmetric Gödel Universes in string/M-theory
  - From T-duality
  - From S-duality
  - Examples
  - Explicit construction in M-theory
  - Examples

- String spectrum
  - Spectrum on compactified pp-wave
  - Spectrum on Gödel Universe

- Conclusions & discussion

Finding new Gödel Universes in string & M-theory:

3 ways:

- T-duality of type II pp-waves
- S-duality of M-theory pp-waves
- Solve EOMs + SUSY conditions explicitly

T-duality:

This T-duality was found by BGHV (in part also by Herdeiro)

Consider metric of type IIA/B pp-wave

$$ds^2 = -2dx^+dx^- - \beta^2 \sum_{k=1}^{n} a_k^2 \left[(\bar{x}^{2k-1})^2 + (\bar{x}^{2k})^2 \right] (dx^+)^2$$
$$+ \sum_{i=1}^{2n} (d\bar{x}^i)^2 + \sum_{i=2n+1}^{8} (dx^i)^2$$

with $a_k \neq 0$.

$$x^+ = t + y \,, \quad x^- = \frac{1}{2}(t - y)$$

assume NS-NS flux $H_{(3)} = 0$.

(Need to add RR-flux to solve EOMs)

Do now the coordinate transformation

$$\tilde{x}^{2k+1} + i\tilde{x}^{2k} = (x^{2k+1} + ix^{2k})\exp(-ia_k \beta x^+)$$

$k = 1, 2, \dots, n$

The metric is now

$$ds^2 = -dt^2 + dy^2 + \sum_{i=1}^{8} (dx^i)^2$$
$$- 2\beta \cdot \sum_{i=1}^{2n} J_{ij}\, x^i dx^j (dt + dy)$$

with

$$J_{12} = a_1 , \quad J_{34} = a_2 , \dots , J_{2n-1, 2n} = a_n$$

$$(J_{ij} = - J_{ji})$$

y-direction explicit space-like isometry.
Do T-duality along y.

After T-duality along y, we have

$$ds^2 = -\left(dt + \beta \cdot \sum_{i,j=1}^{2n} J_{ij} x^i dx^j\right)^2 + \sum_{i=1}^{8} (dx^i)^2 + dy^2$$

$$B_{iy} = \beta \sum_{j=1}^{2n} J_{ij} x^j \quad , \quad H_{ijy} = -2\beta J_{ij}$$

$$(i,j = 1, \ldots, 2n)$$

In polar coordinates

$$x^{2k-1} + i x^{2k} = \rho_k e^{i\phi_k} \quad , k = 1, \ldots, n$$

the metric is

$$ds^2 = -\left(dt + \sum_{i=1}^{n} \beta a_i \rho_i^2 d\phi_i\right)^2 + \sum_{i=1}^{n} (d\rho_i^2 + \rho_i^2 d\phi_i^2) + \sum_{i=2n+1}^{8} (dx^i)^2 + dy^2$$

$$g_{\phi_i \phi_i} = \rho_i^2 (1 - \beta^2 a_i^2 \rho_i^2)$$

All coordinates constant except ϕ_i

$\rightarrow$ CTC when $\rho_i > \dfrac{1}{\beta |a_i|}$

Solutions homogenous, trivial topology

$\rightarrow$ We call it a Gödel Universe

Note that we, in the rest of the talk, define n to be the rank of J_{ij}, which appears in $g_{0i} = \beta \sum_{j=1} J_{ij} x^j$,

i.e. we can write

$$J_{ij} = \begin{pmatrix} \begin{matrix} 0 & a_1 \\ -a_1 & 0 \end{matrix} & & & \\ & \ddots & & \\ & & \begin{matrix} 0 & a_n \\ -a_n & 0 \end{matrix} & \\ & & & 0 \end{pmatrix}$$

with $a_i \neq 0$.

S-duality:

Consider M-theory pp-wave with metric

$$ds^2 = -2dx^+dx^- - \beta^2 \sum_{k=1}^{n} a_k^2 \left[(\tilde{x}^{2k-1})^2 + (\tilde{x}^{2k})^2 \right] (dx^+)^2$$
$$+ \sum_{i=1}^{2n} (d\tilde{x}^i)^2 + \sum_{i=2n+1}^{9} (dx^i)^2$$
$$= -dt^2 + du^2 + \sum_{i=1}^{9} (dx^i)^2$$
$$-2\beta \cdot \sum_{i,j=1}^{2n} J_{ij} x^i dx^j (dt + du)$$

$x^+ = t+u$, $x^- = \frac{1}{2}(t-u)$

$J_{12} = a_1$, $J_{34} = a_2, \ldots, J_{2n-1,2n} = a_n$

Explicit space-like isometry in u-direction.

Do now IIA/M S-duality in u-direction.

$$ds_{11}^2 = ds_{10}^2 + (du + A_\mu dx^\mu)^2$$

We get the type IIA metric

$$ds_{10}^2 = -\left(dt + \beta \cdot \sum_{i,j=1}^{2n} J_{ij} x^i dx^j\right)^2 + \sum_{i=1}^{9} (dx^i)^2$$

with

$$A_i = \beta \cdot \sum_{j=1}^{2n} J_{ij} x^j \quad , \quad F_{ij} = -2\beta J_{ij}$$

Type IIA Gödel Universe background.

Strong coupling limit of IIA Gödel background is pp-wave.

(Note M-theory background also has CTCs for finite radius R_{11})

Examples of backgrounds:

$n=2$ Gödel Universes:

Consider the 24 susy IIB pp-wave from Penrose limit of $AdS_3 \times S^3 \times \mathbb{R}^4$ from intersecting D3-branes:

$$ds^2 = -2dx^+dx^- - \beta^2 \sum_{i=1}^{4} (\tilde{x}^i)^2 (dx^+)^2 + \sum_{i=1}^{4} (d\tilde{x}^i) + \sum_{i=5}^{8} (dx^i)$$

$$F_{(5)} = -2\beta dx^+ (d\tilde{x}^1 d\tilde{x}^2 + d\tilde{x}^3 d\tilde{x}^4)(dx^5 dx^6 + dx^7 dx^8)$$

Metric corresponds to $n=2$ and $a_1 = a_2 = 1$, i.e. $J_{12} = J_{34} = 1$.

After T-duality we get IIA Gödel Universe

$$ds^2 = -\left(dt + \beta \sum_{i,j=1}^{4} J_{ij} x^i dx^j\right)^2 + \sum_{i=1}^{8} (dx^i)^2 + dy^2$$

$$H_{12y} = H_{34y} = -2\beta, \quad F_{1256} = F_{1278} = F_{3456} = F_{3478} = -2\beta$$

this background has 20 supersymmetries.

Lifting this to M-theory we get

$$ds^2 = -\left(dt + \beta \sum_{i,j=1}^{4} J_{ij} x^i dx^j\right)^2 + \sum_{i=1}^{\#} (dx^i)^2$$

$$F_{1256} = F_{1278} = F_{3456} = F_{3478} = F_{129\#} = F_{349\#} = -2\beta$$

with $y = x^9$ and $x^\#$ being the "eleventh direction".
This is the 20 susy Gödel Universe found by GGHPR.

Consider again the pp-wave type IIB metric with $a_1 = a_2 = J_{12} = J_{34} = 1$ $(n=2)$, now with RR-fields

$$F_{+12} = F_{+34} = 2\beta$$

(connected to previos example by two T-dualities)
T-duality now gives

$$H_{12y} = H_{34y} = -2\beta\,, \quad F_{12} = F_{34} = -2\beta\,,$$
$$F_{012y} = F_{034y} = 2\beta$$

Another IIA Gödel Universe with 20 susy.

Uplifting this to M-theory gives $(y = x^9)$

$$ds^2 = -dt^2 + dv^2 + \sum_{i=1}^{9} (dx^i)^2 - 2\beta \sum_{i,j=1}^{4} J_{ij} x^i dx^j (dt + dv)$$

$$F_{0129} = F_{v129} = F_{0349} = F_{v349} = 2\beta$$

M-theory pp-wave with 24 susy.
Example of IIA/M S-duality between M-theory pp-waves and IIA Gödel Universes.

$$\begin{tikzcd}
 & \boxed{\text{IIA Gödel Universe}} \arrow[dl, leftrightarrow, "T(x^9)"'] \arrow[dr, leftrightarrow, "S(v)"] & \\
\boxed{\begin{array}{l}\text{IIB pp-wave} \\ F_{+12} = F_{+34} = 2\beta\end{array}} \arrow[d, leftrightarrow, "S"] & & \boxed{\begin{array}{l}\text{M-theory pp-wave} \\ F_{+129} = F_{+349} = 2\beta\end{array}} \arrow[d, leftrightarrow, "S(x^9)"] \\
\boxed{\begin{array}{l}\text{IIB pp-wave} \\ H_{+12} = H_{+34} = 2\beta\end{array}} \arrow[rr, leftrightarrow, "T(x^8)"] & & \boxed{\begin{array}{l}\text{IIA pp-wave} \\ H_{+12} = H_{+34} = 2\beta\end{array}}
\end{tikzcd}$$

The T & S dualities (i.e. $T(x^9)$ & $S(v)$) can be mapped to each other for M-theory on T^3. (i.e. x^8, x^9, v)

T & S-duality equivalent for $n \leq 3$.

$n=4$ Gödel Universes:

Consider maximally supersymmetric pp-wave of type IIB

$$ds^2 = -2dx^+dx^- - \beta^2 \sum_{i=1}^{8} (x^i)^2 (dx^+)^2 + \sum_{i=1}^{8} (dx^i)^2$$

$$F_{+1234} = F_{+5678} = 4\beta$$

corresponds to $n=4$ and $a_1^2 = a_2^2 = a_3^2 = a_4^2 = 1$

Choose $a_1 = a_2 = a_3 = 1$, $a_4 = s$, i.e.

$$J_{12} = J_{34} = J_{56} = 1, \quad J_{78} = s$$

with $s = \pm 1$.

T-dualize to $n=4$ IIA Gödel Universe.

Uplift to M-theory:

$$ds^2 = -\left(dt + \beta \cdot \sum_{i,j=1}^{8} J_{ij} x^i dx^j\right)^2 + \sum_{i=1}^{\#} (dx^i)^2$$

$$F_{129\#} = F_{349\#} = F_{569\#} = s F_{789\#} = -2\beta$$

$$F_{1234} = F_{5678} = 4\beta$$

20 susy for $s=-1$, 0 susy for $s=1$.

(Background found by BGHV)

$n = 1, 3$ Gödel Universes:

We can also find supersymmetric Gödel Universes with $n=1,3$ from dualizing pp-wave (We present here the cases with most susy)

$n = 1$:

M-theory Gödel Universe with 8 susy:

$$ds^2 = -\left(dt + \beta(x^1 dx^2 - x^2 dx^1)\right)^2 + \sum_{i=1}^{\#} (dx^i)^2$$

$$F_{1234} = F_{5678} = -F_{129\#} = 2\beta$$

$n = 3$:

M-theory Gödel Universe with 14 susy:

$$ds^2 = -\left(dt + \beta \sum_{i,j=1}^{6} J_{ij} x^i dx^j\right)^2 + \sum_{i=1}^{\#} (dx^i)^2$$

$$F_{3456} = F_{1278} = F_{129\#} = -4\beta$$

$$F_{1256} = F_{1234} = F_{3478} = F_{5678} = F_{349\#} = F_{569\#} = -2\beta$$

$$J_{12} = 2, \quad J_{34} = J_{56} = 1$$

We have found Gödel Universes with susy for $n = 1, 2, 3, 4$. What about $n = 5$?

Cannot get from T/S dualities.

Solving EOMs + susy conditions explicitly:

We want to find Supersymmetric Gödel Universes in M-theory. Consider ansatz

$$ds^2 = -\left(dt + \sum_{i=1}^{5} c_i \rho_i^2 d\phi_i\right)^2 + \sum_{i=1}^{5}\left(d\rho_i^2 + \rho_i^2 d\phi_i^2\right)$$

$$F_{\rho_i\phi_i\rho_j\phi_j} = \rho_i \rho_j a_{ij} \quad , \quad i,j = 1, \dots, 5$$

$c_1, \dots, c_5$: constants.

a_{ij} : symmetric matrix with $a_{ii} = 0$, i.e with 10 independent components.

The Einstein equations:

$$\sum_{j=1}^{5} a_{ij}^2 = 4c_i^2 + 4\sum_{j=1}^{5} c_j^2$$

The field strength EOM:

$$-2\sum_{j=1}^{5} a_{ij} c_j = \frac{1}{4}\sum_{j,k,l,m} \eta^{ijklm} a_{jk} a_{lm}$$

$$\eta^{ijklm} = \begin{cases} 1 & \text{if } i,j,k,l,m \text{ are all different} \\ 0 & \text{otherwise} \end{cases}$$

susy conditions:

killing spinor eq. :

$$D_\mu \eta + \frac{1}{288}\left(\Gamma_\mu{}^{\nu\sigma\lambda\rho} - 8\delta_\mu^\nu \Gamma^{\sigma\lambda\rho}\right)\eta F_{\nu\sigma\lambda\rho} = 0$$

similar simplifications...

A solution can be written as

$(c_1, c_2, c_3, c_4, c_5)$, $(a_{12}, a_{13}, a_{14}, a_{15}, a_{23}, a_{24}, a_{25}, a_{34}, a_{35}, a_{45})$

Previously mentioned solutions:

$(1,1,0,0,0)$, $(0,-2,-2,-2,-2,-2,-2,0,0,0)$ 20 susy

$(1,1,1,-1,0)$, $(4,0,0,-2,0,0,-2,4,-2,2)$ 20 susy

$(1,0,0,0,0)$, $(2,0,0,-2,0,0,0,2,0,0)$ 8 susy

$(2,1,1,0,0)$, $(-2,-2,-4,-4,-4,-2,-2,-2,-2,0)$ 14 susy

We found, by computer, the $n=5$ solution

$(2,1,1,1,1)$, $(-6,2,2,2,0,0,0,4,4,4)$ 18 susy

This $n=5$ Gödel Universe seems not to be dual to a pp-wave.

This solution is part of family of solutions with at least 16 susy:

$(p,p,p,p-q,sq)$, $(-4sp, 0, 2sq, 2p-2q, 0, 2sq, 2p-2q, 4sp-4sq, 2p+2q, 2p)$

$p,q \in \mathbb{R}$, $s=\pm 1$

($n=2$ & 4 with 20 susy also members)

Infinite number of $n=5$ solutions with 16 susy.

$n=2$ Gödel Universe / pp-wave mixture:

We have also found a different type of supersymmetric solutions.

Consider pp-wave with $a_1 = a_2 = J_{12} = J_{34} = 1$ (Penrose limit of $AdS_3 \times S^3 \times \mathbb{R}^4$) and

$$F_{+12} = F_{+34} = 2\beta\cos\gamma\ ,\quad H_{+12} = H_{+34} = -2\beta\sin\gamma$$

Solution has 24 susy. After T-duality along y and trivial uplift to M-theory we get

$$ds^2 = -\left(dt + \beta\sum_{i,j=1}^{4} J_{ij}x^i dx^j\right)^2 + \left(dy - \beta\sin\gamma\sum_{i,j=1}^{4} J_{ij}x^i dx^j\right)^2 + \sum_{i=1}^{9}(dx^i)^2$$

$$F_{0129} = F_{0349} = -2\beta\sin\gamma,\quad F_{y129} = F_{y349} = -2\beta$$

$$F_{1256} = F_{1278} = F_{3456} = F_{3478} = -2\beta\cos\gamma$$

with x^9 the "eleventh direction".

Solution has 16 susy for generic γ.

$\gamma = 0$: Gödel Universe with 20 susy

$\gamma = \frac{\pi}{2}$: pp-wave with 24 susy

Solution interpolates between Gödel Universe and pp-wave.

Similar $n=4$ solution with 12 susy also exists.

String spectrum on Gödel Universes:

We find string spectrum on $n=4$ Gödel Universe in type IIB with 20 susy by T-duality.

T-dual background is max. supersymmetric IIB pp-wave:

$$ds^2 = -2dx^+dx^- + \sum_{i=1}^{8}(dx^i)^2 - 2\mu \cdot \sum_{i,j=1}^{8} J_{ij}\, x^i dx^j dx^+$$

$$F_{+1234} = F_{+5678} = 4\mu$$

$$J_{12} = J_{34} = J_{56} = 1, \quad J_{78} = -1$$

10D bosonic sigma-model gives $\partial_\alpha \partial^\alpha X^+ = 0$.
We choose light-cone gauge

$$X^+(\tau,\sigma) = 2\alpha' p^+ \tau + 2\omega R \sigma \quad , \sigma \in [0,\pi[$$

The second term is there due to compactification on $y = \frac{1}{2}x^+ - x^-$ so we want

$$t(\tau,\sigma+\pi) = t(\tau,\sigma), \quad y(\tau,\sigma+\pi) = y(\tau,\sigma) + 2\pi\omega R$$

winding number

$$\Rightarrow \begin{cases} X^+(\tau,\sigma+\pi) = X^+(\tau,\sigma) + 2\pi\omega R \\ X^-(\tau,\sigma+\pi) = X^-(\tau,\sigma) - \pi\omega R \end{cases}$$

Since $p^0 = E$ and $p^y = \frac{m}{R}$ (m is KK number) we have

$$p^+ = E + \frac{m}{R}, \quad p^- = \frac{E}{2} - \frac{m}{2R}$$

The gauge-fixed bosonic action is

$$S = \frac{1}{4\pi\alpha'} \int d\tau d\sigma \Big(-\partial_\alpha X^i \partial^\alpha X^i - 4\beta\alpha' p^+ J_{ij} X^i \partial_\tau X^j + 4\beta\omega R J_{ij} X^i \partial_\sigma X^j \Big)$$

Define

$Z^1 = X^1 + i X^2$, $Z^2 = X^3 + i X^4$, $Z^3 = X^5 + i X^6$, $Z^4 = X^7 - i X^8$

Define moreover

$Z^a = z_0^a e^{i\beta X^+}$, $(X^+ = 2\alpha' p^+ \tau + 2\omega R \sigma)$

$a = 1, 2, 3, 4$

Then the EOM's are

$$(\partial_\tau^2 - \partial_\sigma^2) z_0^a + 4f^2 \cdot z_0^a = 0$$

with

$$f = \sqrt{(\beta\alpha' p^+)^2 - (\beta\omega R)^2}$$

Same EOMs as usual string theory on max. supersymmetric IIB pp-wave.

However, since $Z^a(\tau, \sigma + \pi) = Z^a(\tau, \sigma)$ we have that

$$Z_0^a(\tau, \sigma + \pi) = e^{-2\pi i \delta} Z_0^a(\tau, \sigma)$$

$$\delta \equiv \beta \iota \omega R$$

Twisted boundary conditions.

$$Z_0^a = i\sqrt{\frac{\alpha'}{2}} \cdot \sum_{n \in \mathbb{Z}} \left(\frac{\alpha_{n+\delta}^a}{\omega_n^+} e^{-2i\omega_n^+ \tau - 2i(n+\delta)\sigma} + \frac{\tilde{\alpha}_{n-\delta}^a}{\omega_n^-} e^{-2i\omega_n^- \tau + 2i(n-\delta)\sigma} \right)$$

$$\bar{Z}_0^a = i\sqrt{\frac{\alpha'}{2}} \cdot \sum_{n \in \mathbb{Z}} \left(\frac{\bar{\alpha}_{n-\delta}}{\omega_n^-} \cdot e^{-2i\omega_n^- - 2i(n-\delta)\sigma} + \frac{\tilde{\bar{\alpha}}_{n+\delta}}{\omega_n^+} e^{-2i\omega_n^+ + 2i(n+\delta)\sigma} \right)$$

$$\omega_n^+ = \begin{cases} \sqrt{(n+\delta)^2 + \varphi^2} & \text{for} \quad n \geq -\delta \\ -\sqrt{(n+\delta)^2 + \varphi^2} & \text{for} \quad n < -\delta \end{cases}$$

$$\omega_n^- = \begin{cases} \sqrt{(n-\delta)^2 + \varphi^2} & \text{for} \quad n > \delta \\ -\sqrt{(n-\delta)^2 + \varphi^2} & \text{for} \quad n \leq \delta \end{cases}$$

Quantization:

$$[\alpha^a_{n+\delta}, \bar{\alpha}^b_{m-\delta}] = 2\omega^+_n \delta_{n+m,0}\delta_{a,b}$$

$$[\tilde{\alpha}^a_{n-\delta}, \bar{\tilde{\alpha}}^b_{m+\delta}] = 2\omega^-_n \delta_{n+m,0}\delta_{a,b}$$

Spectrum (from $T_{++} + T_{--} = 0$)

$$E^2 - \frac{m^2}{R^2} - \frac{\omega^2 R^2}{(\alpha')^2}$$

$$= \frac{2}{\alpha'}\sum_{n\in\mathbb{Z}}\left(N^+_n\sqrt{(n+\delta)^2+f^2} + N^-_n\sqrt{(n-\delta)^2+f^2}\right)$$

$$+ 2\beta p^+ \cdot \sum_{n\in\mathbb{Z}}(N^+_n - N^-_n)$$

$$N^+_n = \begin{cases} \sum_{a=1}^{4}\frac{1}{2\omega^+_n}\bar{\alpha}^a_{-n-\delta}\alpha^a_{n+\delta}, & n \geq -\delta \\ \sum_{a=1}^{4}\frac{1}{2\omega^-_{-n}}\alpha^a_{n+\delta}\bar{\alpha}^a_{-n-\delta}, & n < -\delta \end{cases}$$

$$N^-_n = \begin{cases} \sum_{a=1}^{4}\frac{1}{2\omega^-_n}\bar{\tilde{\alpha}}^a_{-n+\delta}\tilde{\alpha}^a_{n-\delta}, & n > \delta \\ \sum_{a=1}^{4}\frac{1}{2\omega^+_{-n}}\bar{\alpha}^a_{n-\delta}\alpha^a_{-n+\delta}, & n \leq \delta \end{cases}$$

Level-matching (from $T_{++} - T_{--} = 0$)

$$\sum_{n\in\mathbb{Z}} n(N^+_n + N^-_n) + m\omega = 0$$

Including the fermions, substitute

$$N_n^+ \to \underbrace{N_n^+}_{\text{4 bosons}} + \underbrace{F_n^+}_{\text{4 fermions}}$$

$$N_n^- \to \underbrace{N_n^-}_{\text{4 bosons}} + \underbrace{F_n^-}_{\text{4 fermions}}$$

Total zero-point energy $= 0$ due to supersymmetry.

We found spectrum of max. supersymmetric IIB pp-wave compactified on $y = \frac{1}{2}x^+ - x^-$ with radius R.

We now take $R \to 0$ limit and express spectrum in quantities of the T-dual IIA Gödel Universe background

$m = 0$ ← no winding along y-direction on Gödel Universe background

$p_y = \dfrac{wR}{\alpha'}$ ← winding is momentum along y-direction in Gödel Universe background

this means

$p^+ = E$, $f = \sqrt{(\mu\alpha' E)^2 - (\mu\alpha' p_y)^2}$, $\delta = \mu\alpha' E$

$= \mu\alpha' \sqrt{E^2 - p_y^2}$

Spectrum on type IIA Gödel Universe:

$$E^2 - p_y^2 = \frac{2}{\alpha'} \sum_{n \in \mathbb{Z}} \Big(\mathcal{N}_n^+ \sqrt{(n + \alpha' \beta p_y)^2 + \beta^2 (\alpha')^2 (E^2 - p_y^2)}$$

$$+ \mathcal{N}_n^- \sqrt{(n - \alpha' \beta p_y)^2 + \beta^2 (\alpha')^2 (E^2 - p_y^2)} \Big).$$

$$+ \; 2\beta E \cdot \sum_{n \in \mathbb{Z}} \left(\mathcal{N}_n^+ - \mathcal{N}_n^- \right)$$

Level-matching:

$$\sum_{n \in \mathbb{Z}} n \left(\mathcal{N}_n^+ + \mathcal{N}_n^- \right) = 0$$

with

$$\mathcal{N}_n^+ = N_n^+ + F_n^+ \quad , \quad \mathcal{N}_n^- = N_n^- + F_n^-$$

$\sqrt{\alpha'}\,\beta \to \infty$. String bit limit

$$E^2 - p_y^2 = 4\beta \sum_{n\in\mathbb{Z}} \mathcal{N}_n^+$$

i.e
$$E = \frac{1}{2}p_y + \sqrt{\frac{1}{4}p_y^2 + 4\beta \sum_{n\in\mathbb{Z}} \mathcal{N}_n^+}$$

$p_y = 0$.

$$\frac{E^2}{2\beta} = \sum_{n\in\mathbb{Z}} \sqrt{E^2 + \left(\frac{n}{\beta\alpha'}\right)^2}\left(\mathcal{N}_n^+ + \mathcal{N}_n^-\right) + E\sum_{n\in\mathbb{Z}}\left(\mathcal{N}_n^+ - \mathcal{N}_n^-\right)$$

$p_y = 0$ & $|n| \ll \beta\alpha' E$.

$$E^3 = 4\beta E^2 \sum_{n\in\mathbb{Z}} \mathcal{N}_n^+ + \frac{1}{\beta(\alpha')^2}\sum_{n\in\mathbb{Z}} n^2\left(\mathcal{N}_n^+ + \mathcal{N}_n^-\right)$$

$p_y = 0$ & $|n| \gg \beta\alpha' E$

$$E = \beta\sum_{n\in\mathbb{Z}}\left(\mathcal{N}_n^+ - \mathcal{N}_n^-\right) + \sqrt{\left[\beta\sum_{n\in\mathbb{Z}}\left(\mathcal{N}_n^+ - \mathcal{N}_n^-\right)\right]^2 + \frac{2}{\alpha'}\sum_{n\in\mathbb{Z}}|n|\left(\mathcal{N}_n^+ + \mathcal{N}_n^-\right)}$$

Conclusions:

- Many supersymmetric Gödel Universes in string/M-theory.

n	1	2	3	4	5
#susy	8	20	14	20	18

- Found family with infinitely many $n=5$ Gödel Universes with 16 susys. 1 member with 18 susy.

- Gödel Universes by T & S dualities of pp-waves

- String spectrum found. D-brane spectrum analysed. D4-brane SUGRA solution.

⇒ Many different highly supersymmetric Gödel Universes in String/M-theory.

Interpretation?

String theoretic resolution of CTCs?

Holography and CTS's (BGHV)?

Large-scale modifications of GR?

Are Gödel Universes pathological (non-physical) backgrounds of string theory?

→ Problematic since they are supersymmetric and weakly curved, without singularities

String theory unitary on Gödel Universes?