D-instantons Localised in D branes

- Review of D-instanton effects in IIB

D instanton corrections to worldvolume theory of D3 branes

M.B Green & MG in progress

heterotic type I' duality and D-instanton corrections for D7 branes

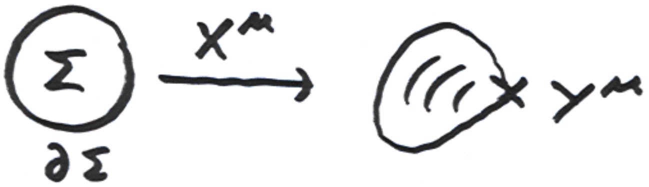
D-instanton effects and YM instantons in a $N=2$ AdS/CFT correspondence

MG hep-th/9905173

hep-th 9903010

D-instantons in Superstring theory

- D-instantons are D p=-1 branes, ie the ends of open strings are mapped to a point in space time



$$X^\mu\big|_{\partial\Sigma} = y^\mu$$

- D-instanton acts as a source for RR scalar C_0 in IIB amplitudes are weighted by $\exp\left(-\frac{1}{g} \pm i C_0\right)$ for instantons / anti instantons

- D-instanton preserves half of 32 Supersymmetries of IIB. 16 broken susy generate fermionic zero modes ε which have to be soaked up

for example 1 graviton & 4 ε

$$\langle h \rangle = h_{\mu\nu}\, k_\rho\, k_\lambda\, \bar{\epsilon}\, \gamma^{\tau\mu\rho} \epsilon\, \bar{\epsilon} \gamma^{\tau\nu\lambda}\, \epsilon$$

- Integration over fermionic collective coordinates induces

$$\int d^{16}\epsilon \, \langle h\rangle\langle h\rangle\langle h\rangle\langle h\rangle = t_8\, t_8\, R^4$$

- Collective coordinates of n D-instantons are weighted by matrix action given by dimensional reduction of SU(N) YM to 0+0 dim

$$S = \mathrm{tr}\, [\bar{\Psi}, A_\mu]\, \Gamma^\mu \Psi + \mathrm{tr}\, [A_\mu, A_\nu]^2$$

Higher dimensional terms in II B effective action depend on nonholomophic modular functions depending on $\tau = x + i e^{-\phi}$

$$\int d^{10}x \; t_8 t_8 R^4 e^{-\phi/2} \; f_{(0,0)}(\tau, \tau$$

where f is given by a nonholomophic Eistenseries or Maass wave form

$$f_{(0,0)}(\tau,\bar\tau) = \sum_{\substack{(m,n)\\ \neq (0,0)}} \frac{\tau_2^{3/2}}{|m+n\tau|^3} = \zeta(3)\, E_{3/2}(\tau,\bar\tau)$$

expansion of f in weak coupling limit

$$f_{(0,0)} = 2\zeta(3)\tau_2^{3/2} + \frac{2\pi^2}{3}\tau_2^{-1/2}$$

$$+ 4\pi^{3/2} \sum_N N^{1/2} \sum_{N|m} \frac{1}{m^2} \quad e^{2\pi i N\tau} + e^{-2\pi i N\bar\tau})$$

$$\times \left(1 + \sum_{k=1}^{\infty} (4\pi N\tau_2)^{-k} \frac{\Gamma(k-1/2)}{\Gamma(-k-1/2)\Gamma(k+1)}\right.$$

Green & Gutperle
Kiritsis Pioline

Evidence: tree + 1 loop, $SL(2,\mathbb{Z})$, D-instantons, Susy

- many other terms related to R^4 by Susy are multiplied similar functions $f_{(p,-p)}(\tau,\bar\tau)$
- $Z_N = \sum_{N|m} \frac{1}{m^2}$ can be related to Matrix partition function $SU(N)$ $(1,9) \to (0,0)$ reduction of SYM. Green, M.G.

Moore, Nekrasov, Shatashvili

D-instantons and YM-instantons in AdS/CFT

• Maldacena: IIB on $AdS_5 \times S_5$ dual to $SU(N)$ $N=4$ SYM

$$g_{string} = \frac{g_{YM}^2}{4\pi}, \quad 2\pi C_0 = \Theta_{YM}, \quad \frac{L^2}{\alpha'} = \sqrt{g_{YM}^2 N}$$

• Calculation of correlators of composite fields in CFT using supergravity and bulk to boundary propagators

Witten
Gubser Klebanov
Polyakov

$$K_\Delta = \frac{\rho^\Delta}{(\rho^2 + (x-x')^2)^\Delta}$$

• Higher derivative terms like $t_8 t_8 R^4$ provide 4 pt vertices

$$\langle TTTT \rangle = N^2 A_{sugra} + \text{const}\, N^{1/2} f^{(0,0)}(T,\bar{T})\, A_{R^4}$$ Bals & Green

• D-instanton terms in $f^{(0,0)}$ turn into YM instantons $\exp\left(-\frac{8\pi^2}{g_{YM}^2}\right)$

• Size of YM instanton ρ becomes radial position in AdS (holography)

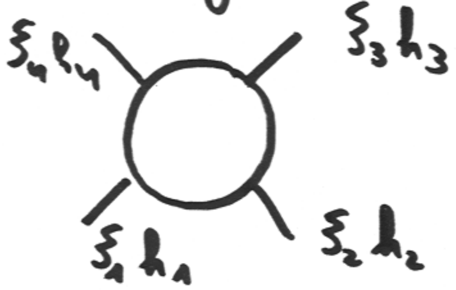
• Check predictions of IIB D-instantons using ADHM calculus Dorey Hollis Khoze Vandoren

D-Instantons localized in Dp-branes

- A Dp brane inside K Dp+4 branes (K>1) can be viewed as an Instanton in U(K) SYM of charge 1. (Douglas)
- small instanton singularity when size of Instanton goes to zero. Coulomb branch where the Instanton can leave the Dp+4 brane
- Focus on a single (K=1) D3 brane, No Higgs branch for D3 - D-1 system ⇒ Instanton cannot become 'fat'.
- Describe system using string perturbation theory in an instanton background: Polchinski, Green, Green & M. G.
  - Fermionic zero modes induce high dimensional corrections to BI action
  - Tree level & $SL(2,\mathbb{Z})$ duality suggest a modular function of $\tau = C_0 + i e^{-\phi}$ including instanton contributions.

Scattering of open string excitations on a D3 brane

For open string scattering: Superstring with $U(1)$ Chan Paton factors



$$s = -(k_1+k_2)^2$$
$$t = -(k_1+k_4)^2$$
$$u = -(k_1+k_3)^2$$

$$A_4 = K \times \left\{ \frac{\Gamma(-s/2)\,\Gamma(-t/2)}{\Gamma(1-s/2-t/2)} + (s \leftrightarrow u) + (t \leftrightarrow u) \right\}$$

Kinematic factor K

$$K = t_8^{\,m_1 n_1 m_2 n_2 m_3 n_3 m_4 n_4}\ \xi^{(1)}_{m_1} k^{(1)}_{n_1} \cdots \xi^{(4)}_{m_4} k^{(4)}_{n_4}$$

Scattering of open strings on D3 brane (by T-duality) restrict k_n to $n = 0,1,2,3$ ξ_m $m = 0,1,2,3$, gauge field ξ_m, $m = 4,5,\ldots,9$ transverse scalars.

For scalars:

$$K = -1/4 \left\{ st\, \xi_1\cdot\xi_3\ \xi_2\cdot\xi_4 + tu\, \xi_1\cdot\xi_2\ \xi_3\cdot\xi_4 + su\, \xi_2\cdot\xi_3\ \xi_1\cdot\xi_4 \right\}$$

Low energy expansion of A_4

$$\ln\Gamma(1+z) = -Cz + \sum_{k=2}^{\infty} (-1)^k \frac{z^k}{k}\, \zeta(k)$$

provides expansion of A_4 in s, t, u

$$\frac{\Gamma(-s/2)\,\Gamma(-t/2)}{\Gamma(1-s/2-t/2)} = \frac{4}{st} - \zeta(2) + \frac{\zeta(2)^2}{8}\,st - \frac{\zeta(4)}{8}\left(2t^2+2s^2+3st\right) + \alpha$$

massless pole

$\mathrm{tr}\,F^4$

Four extra derivatives $(\alpha')^2$ terms

A_4 becomes:

$$A_4 = -\frac{\pi^4}{192}\, K\,(s^2+t^2+u^2)$$

This provides higher derivative corrections to the Born-Infeld action

$$S_{BI} = \int d^4x\, e^{-\phi}\sqrt{\det(G+F)} + S_{WZ}$$

which are schematically of the form

$$S' = \int d^4x \sqrt{G}\, e^{-\phi}\left((\partial^2\phi)^4 + (\partial F)^4 + (\partial^2\phi)^2(\partial F)^4\right\}$$

An $SL(2,\mathbb{Z})$ S duality acting on $\tau = \tau_1 + i\tau_2 = C_0 + i e^{-\phi}$ as $\tau \to (a\tau+b)/(c\tau+d)$ leaves the D3 brane action 'invariant' if WV fields are transformed by an e-m duality (Zumino Gaillard, Gibbons & Rasheed, Tseytlin Green & M.G.)

S' becomes in the Einstein frame $g_{\mu\nu}^{\text{string}} = e^{\phi/2} g_{\mu\nu}^{\text{Einstein}}$

$$S' = \int d^4x \sqrt{g}\left\{\tau_2\,(\partial^2\phi)^4 + \tau_2^2\,(\partial^2\phi)^2(\partial F)^2 + \tau_2^3\,(\partial F)^4\right\}$$

Since $\tau_2 \to \tau_2/|c\tau+d|^2$ this is not invariant!

As we shall see, D-instantons will contribute to S'

this leads us to look for a nonholomorphic modular function $\tau_2 \rightarrow h(\tau\ \bar\tau$

Conjecture $\quad h(\tau\ \bar\tau) = \text{const} \times \ln \tau_2\, \eta^4(\tau$

Expansion of h in the large τ_2 limit:

$$h(\tau\ \bar\tau = \underset{\substack{\uparrow \\ \text{1 loop} \\ \text{anomaly}}}{\ln \tau_2} \quad \underset{\substack{\uparrow \\ \text{tree level}}}{\frac{\pi}{3}\tau_2} \quad 2\sum_N \underset{\substack{\uparrow \\ \text{D-instantons}}}{\sum_{N|m}} \frac{1}{m}\Big(e^{2\pi i N\tau} + e^{2\pi i N\bar\tau}$$

Additional evidence

Relation to higher curvature terms in D3 world volume $\int d^4x \sqrt{g}\, h(\tau, \bar\tau\ R^2$ Bachas, Bain, Green

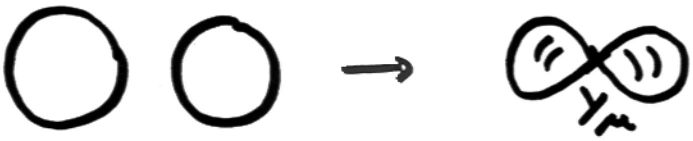
& their relation to F & M theory

similar functions appear in other D-instanton contributions to WV action of D branes (see later) Lerche Stieberger, Mukhi Dasgupta M.G., ...

D-instanton induced terms in D3 brane WV.

D instantons are defined as 'events' in spacetime, the boundary of a string worldsheet is mapped to a point

Polchinski argued that in the presence of D instantons one has to take disconnected worldsheets into account which get mapped to the same D-instanton



brane in $\mu = 0,1,2,3$ directions. Massless open string fields are classified according

$SO(6) \times SU(2)_L \times SU(2)_R$
$\overset{\shortparallel}{SU(4)}$

DD $\quad \chi_a \; (6\,1\,1) \qquad a_n' \; (1\,2\,2)$

$\lambda_A^{\alpha} \; (4, 1\,2) \qquad M_{\alpha}^{A} \; (4\,2\,1)$

collective coordinates of D-instanton

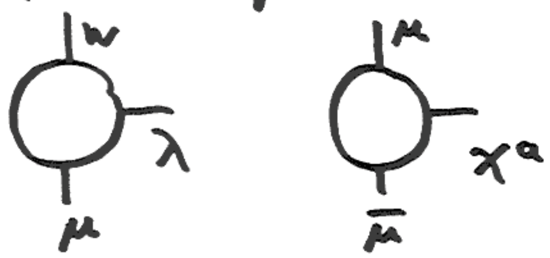
ND $\quad w_\alpha \, w_\alpha \; (1, 1\,2) \quad \mu^A, \mu^A \; (4\,1\,1)$

Vertex operator contain twist fields which change N to D b.c

NN $\quad A_n \; (1\,2\,2) \qquad \phi_a \; (6, 1\,1)$

$\bar{\Lambda}^{\alpha}{}_{A} \; (4, 1\,2 \qquad \Lambda^{\alpha A} \; (4\,2\,1)$

Simplest diagrams



etc

'Matrix' action which weights the integration over the collective coordinates

$$S = -\frac{g_0^2}{2}(w^{\alpha} w_{\dot\alpha})^2 \quad \chi_a \chi^a w^{\alpha} w_{\dot\alpha} - \pi \mu^A \mu^B \Sigma_{AB}^{a} \chi_a$$

$$+ \pi(\mu^A \lambda_A^{\alpha} w_{\alpha} + \bar\mu^A \lambda_A^{\alpha} w_{\dot\alpha})$$

BPS D-p brane breaks ½ of the Supersymmetry

D3 D-1 breaks 3/4 alltogether

II B 32 Supersymmetries 2 Majorana Weyl spinors

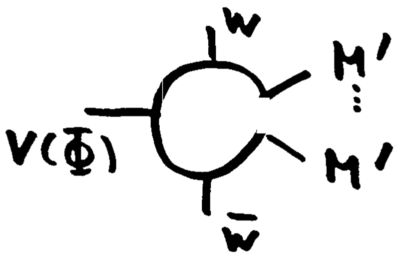
	$\xi_{\dot\alpha A}$	$\xi_\alpha{}^A$	$\eta_{\dot\alpha A}$	$\eta_\alpha{}^A$
D3	u	b	b	u
D-1	u	u	b	b

Note fermionic zeromodes λ_A^{α} and M_α^A are like Goldstino modes for $\eta^{\alpha}{}_A$ and $\eta_\alpha{}^A$

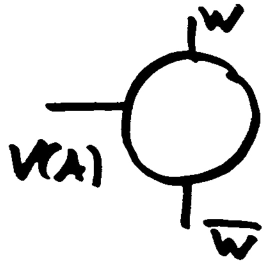
Integration over fermionic collective coordinates $\lambda, M' \mu$ yields vanishing result unless enough fermionic zero modes are inserted. λ, μ can be saturated by S'

$$\int d^8\lambda \, d^4\mu \, d^4\bar\mu \, (\mu^A \lambda_A^{\alpha} w_{\alpha})^4 (\bar\mu^A \lambda_A w_{\alpha}) = (\bar w^{\alpha} w_{\dot\alpha})^4$$

What about M' They have to be absorbed by disk diagrams with a 3 brane vertex inserted



simplest example

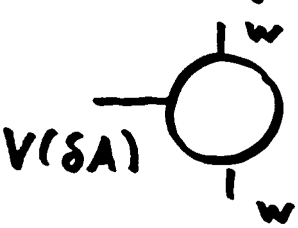


$$\langle F^- \rangle = \langle cV(A_m)\, c\, V(w)\ \ cV(\bar{w}) \rangle = F^-_{mn}\, \bar{w}^{\dot{\alpha}}\, \sigma^{mn}_{\dot{\alpha}\dot{\beta}}\, w^{\dot{\beta}}$$

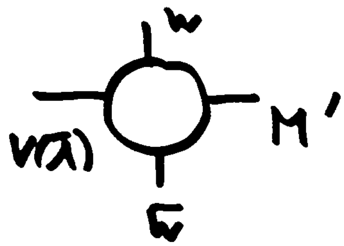
Note that M' is associated with the broken D-1 susy which is unbroken by the D3 brane hence one can use

$$V(\delta_\eta A_m) = [\eta Q, V(\bar{\Lambda})]$$

where $\delta_\eta A_m = -i\, \eta_a{}^A\, \sigma^{m\, a\dot{a}}\, \bar{\Lambda}_{\dot{a} A}$ is a $N=4$ SYM susy transformation. Using the fact that $\eta Q = \oint V_{M'}(\eta)$ one can relate



to



$$\langle \delta_\eta F^- \rangle = -i\, \bar{w}^{\dot{\alpha}}\, \sigma^{mn}_{\dot{\alpha}\dot{\beta}}\, w^{\dot{\beta}}\, \bar{\eta}^A\, \sigma_{[m}\partial_{n]}\bar{\Lambda}_A$$

Further application of $N=4$ Susy can relate the 3 pt function to amplitudes with up to 4 M'

$$\delta\varphi^{AB} = \tfrac{1}{2}\left(\Lambda^{\alpha[A}\eta_\alpha{}^{B]}\right) + \frac{1}{2}\,\varepsilon^{ABCD}\,\bar{\xi}_{\dot\alpha C}\bar{\Lambda}_D{}^{\dot\alpha}$$

$$\delta\Lambda_\alpha{}^A = -\frac{1}{2}F^-_{mn}\,\sigma^{mn}{}_\alpha{}^\beta\,\eta_\beta{}^A + 4i\,D_{\alpha\dot\alpha}\varphi^{AB}\,\bar{\xi}^{\dot\alpha}{}_B$$

$$\delta A_m = -i\,\Lambda^{\alpha A}\sigma^m{}_{\alpha\dot\alpha}\,\bar{\xi}^{\dot\alpha}{}_A - i\,\eta^{\alpha A}\,\sigma^m{}_{\alpha\dot\alpha}\,\bar{\Lambda}^{\dot\alpha}{}_A$$

produces

$$\langle\delta_\eta^2 F^-\rangle = \bar{w}\sigma^{mn}w\;\eta^B\sigma_{mp}\eta^A\,\partial_n\partial_p\varphi_{AB}$$

$$\langle\delta_\eta^3 F^-\rangle = \bar{w}\sigma^{mn}w\;\varepsilon_{ABCD}\,\eta^B\sigma_{pm}\eta^A\eta^C\,\partial_n\partial_p\Lambda^D$$

$$\langle\delta^4 F^-\rangle = \bar{w}\sigma^{mn}w\;\varepsilon_{ABCD}\,\eta^B\sigma_{pm}\eta^A\eta^C\sigma_{kl}\eta^D\,\partial_n\partial_p F_{kl}$$

Integration over collective coordinates induces higher derivative interactions for example using.

$$\int d^4x\int d^4w\int d^8\eta\;\langle\delta_\eta^2F^-\rangle\langle\delta_\eta^2F^-\rangle\langle\delta_\eta^2F^-\rangle\langle\delta_\eta^2F^-\rangle$$

will produce exactly a term of the form $(\partial^2\phi)^4$ where the kinematic structure is exactly the same as at tree level.

$\int d^8\eta$

and similarly for $(\partial F)^4$ and $(\partial F)^2(\partial^2\phi)^4$ etc

heterotic / type I' duality & 7branes

Consider heterotic SO(32) string compactified a T^2

$$T = B_{12}^{NSNS} + i R_1 R_2 \; , \quad U = i \frac{R_2}{R_1}$$

Introduce Wilson Lines on T^2 breaking SO(32) $SO(8)^4$

$$Y^{(1)} = (0^4, 0^4, 1/2^{\,4}, 1/2^{\,4})$$
$$Y^{(2)} = (0^4, 1/2^{\,4}, 0^4, 1/^{\,4})$$

A hetrotic one loop calculation of $\mathrm{tr}_{SO(8)} F^4$ is one loop exact. The result is given by

$$T_2 \int d^8x \sqrt{g}\; \mathrm{tr} F^4 \; \frac{1}{2} \left\{ \ln | T_2 \eta(4T)^4 | - \ln | T_2 \eta(2T)^4 | \right.$$

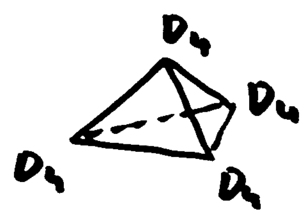
Use hetrotic / type I duality

$$T \to B_{12}^{RR} + i \frac{R_1 R_2}{\lambda} \; , \quad U \to i \frac{R_2}{R_1}$$

Under Two T-dualities type I is mapped a typ I' or IIB orientifold.

$$T \to \tau = C_0 + i\frac{1}{g} \qquad U \to U = i\frac{R_1}{R_2}$$

- This type II orientifold given by $T^2/\Omega(-1)^F I$ was first described by Sen and is the simplest example of an F-theory compactification.



4 orientifold sevenplanes with 4 D7 branes on top of each.

- Note that world sheet instantons on the heterotic side are weighted by $e^{2\pi i N T}$ on the type II side this gets mapped into D-instanton contributions weighted by $e^{2\pi i N \tau}$
- the SO(8) gauge symmetry comes from 4 D7 branes on top of the orientifold plane hence the heterotic calculation of thresholds of the form $\mathrm{tr}F^4$, $(\mathrm{tr}F^2)^2$ will produce localized D-instanton effects on the seven brane.
- Its also possible to calculate $\mathrm{tr}R^4$ $(\mathrm{tr}R^2)^2$ curvature terms localized on the D7 branes.

The heterotic one loop amplitude of $\mathrm{tr}F^4$ and $(\mathrm{tr}F^2)^2$ are related by Susy to anomaly cancelling term and the 'elliptic genus', hence they are one loop exact.

Lerche Schellekens Warner
Lerche

The one loop amplitude for het $SO(32)$ on T^2 is given by:

$$I_Q = \int \frac{d^2\tau}{\tau_2} \sum_A \frac{T_2}{\tau_2}\, e^{\left(2\pi i T \det A - \frac{\pi T_2}{\tau_2 U_2}\left|(1\,U)\,A\binom{\tau}{1}\right|^2\right)} \; Q\,C(\tau, A)$$

here A parametrizes the windings & momenta on T^2

$$A = \begin{pmatrix} m_1 & n_1 \\ m_2 & n_2 \end{pmatrix}$$

The partition function for the heterotic string with Wilson lines breaking $SO(32)$ to $SO(8)^4$ is given by

$$C(\tau, A) = \sum_{ab} \Theta^4\begin{bmatrix} a \\ b \end{bmatrix}(0|\tau)\, \Theta^4\begin{bmatrix} a+m_2 \\ b+n_2 \end{bmatrix} \Theta^4\begin{bmatrix} a+m_1 \\ b+n_1 \end{bmatrix} \Theta^4\begin{bmatrix} a+m_1+m_2 \\ b+n_1+n_2 \end{bmatrix}$$
$$\cdot\ 1/\eta^{24}$$

Jatkar, Kiritsis

The threshold is calculated by a charge insertion operator Ellis which depends on the amplitude & the spin structure $\begin{bmatrix} a \\ b \end{bmatrix}$

$$Q_{F^4}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = -\frac{1}{2^8 3}\, \theta_3^4\, \theta_4^4\,, \quad Q_{F^4}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \frac{1}{2^8 3}\, \theta_2^4\, \theta_4^4$$

$$Q_{F^4}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = -\frac{1}{2^8 3}\, \theta_2^4\, \theta_3^4$$

Integral over fundamental domain $\mathcal{F}$ of T can be evaluated 'unfolding' a $SL(2,\mathbb{Z})$ transformation of T can be undone by a $SL(2,\mathbb{Z})$ transformation on A. For nondegenerate orbits $\det A \neq 0$ the fundamental domain unfolds on the upper half plane (where integrals can be done) Dixon Kaplunovsky & Louis

$$A = \pm \begin{pmatrix} k & j \\ 0 & p \end{pmatrix} \qquad k>0 \quad 0 \le j \le k \quad p \in \mathbb{Z}$$

splits into four sectors

$$= \begin{pmatrix} 2k & 2j \\ 0 & 2p \end{pmatrix} \qquad A^2 = \begin{pmatrix} 2k+1 & 2j \\ 0 & 2p \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 2k+1 & 2j+1 \\ 0 & 2p \end{pmatrix} \qquad A^4 = \begin{pmatrix} 2k & 2j \\ 0 & 2p+1 \end{pmatrix}, \begin{pmatrix} 2\hat{k} & 2j+1 \\ 0 & 2p+1 \end{pmatrix}$$

$$\begin{pmatrix} 2k & 2j+1 \\ 0 & 2p \end{pmatrix}$$

For the $\operatorname{tr} F_1^4$ threshold $QC(A^{(i)})$ simplify

$$QC(A^1) = \frac{1}{2^8 3} \frac{1}{\eta^{24}} \left(-\theta_2^{16}\theta_3^4\theta_4^4 + \theta_3^{16}\theta_2^4\theta_4^4 - \theta_4^{16}\theta_2^4\theta_3^4 \right) = 1$$

$$Q(A^2) = -1/3 \qquad Q(A^3) = -1/3 \qquad Q(A^4) = -1/3$$

Using the (trivial) fact $QC(A^1) = QC(A^2) + QC(A^3) + QC(A^4) + 2$

one can rearrange the summation ranges

$I_{(F^4)}$ becomes

$$I_{F^4} = 2 \int_{H} \frac{d^2T}{T_2^2}\, T_2 \sum_{\substack{k>0 \\ 0\le j<k \\ p\in Z}} \exp\left(2\pi i\, 4kpT - \frac{\pi\, 4T_2}{T_2 U_2} |kT+j+pU|^2\right)$$

$$\int_{H'} \frac{d^2T}{T_2^2}\, T_2 \sum_{\substack{k>0 \\ 0\le j<k \\ p\in Z}} \exp\left(2\pi i\, 2kpT - \frac{\pi\, 2T_2}{T_2 U_2} |kT+j+pU|^2\right)$$

$$= \sum_N \frac{1}{2}\left(\sum_{N|m} \frac{1}{m} e^{-2\pi N 4T} - \frac{1}{2} \sum_{N|m} \frac{1}{m} e^{-2\pi N 2T}\right)$$

The calculation of $I_{(F^2)^2}$ is more complicated since $QC(A^i)$ are not constants but an identity between the $QC(A^i)$ makes it possible to rearrange the summation in the sectors such that,

$$I_{(F^2)^2} = \sum_N \left(\frac{1}{4} \sum_{N|m} \frac{1}{m} e^{2\pi i\, 12NT} - \frac{1}{8} \sum_{N|m} \frac{1}{m} e^{2\pi i\, 4NT}\right)$$

Similarly one can calculate $\mathrm{tr} R^4$ & $(\mathrm{tr} R^2)^2$ terms and $\mathrm{tr} F_i^2\, \mathrm{tr} F_j^2$ terms. It is also possible to consider other Wilson line breakings using the same method

Lerche Stieberger & Warner

Orientifolds and $N=2$ $USp(2N)$ theories

Simplest F-theory example on K3 is Sen's orientifold:

$$\text{IIB on } T^2/(-1)^{F_L}\Omega I, \quad I: z \to -z \qquad \text{Sen}$$

corresponds to orientifold sevenplanes on the four fixed pts of I on T^2 and 4 D7 branes on top of each O-fold plane.

- charges cancel locally and dilaton is constant over the base
- There is an enhanced $SO(8)$ symmetry associated with each of the 4 D7 branes.

Introduction of N_c D3 brane ('probe') produces a gauge theory on the D3 brane:

$USp(2N_c)$ $N=2$ SYM — Banks, Douglas & Seiberg; Douglas, Lowe, Schwarz

4 hypermultiplets in fundamental of $USp(2N_c)$
1 " in AST

$\beta = 0$ for any N_c, hence it is possible to introduce a large number of D3 branes and study the near horizon geometry of D3 brans in the vicinity of O7/D7 a la Maldacena.

The near horizon geometry for large N_c D3 branes is given by $AdS_5 \times S_5/Z_2$ where the metric on the angular part is given by

Fayyazuddin, Spalinski
Aharony, F., Maldacena

$$ds^2_{\Omega_5} = d\theta^2 + \sin^2(\theta)\, d\phi^2 + \cos^2(\theta)\, d\Omega_3$$

where $0 \leq \theta \leq \pi/2$ and $\phi \in [0, 2\pi(1-\alpha/2)]$ with $\alpha = 1$ for the D_4 case. The S_5/Z_2 has a fixed point at which is an S_3 and the 7 branes are wrapped around the S_3 and fill AdS_5.

Fields in Supergravity / primaries of the CFT are classified by charges

$$SU(2)_R \times SU(2)_L \times U(1)_R \times SO(8) \times USp(2N_c)$$

2 types of fields / chiral primaries:

- Bulk supergravity fields, with changed periodicity and monodromy
- 'twisted' fields which are localized on the 7 branes and carry SO(8) quantum numbers.

8 dim vector field A_M for the 7brane decomposes into KK modes

a vector $A_\mu = \sum_k A_\mu{}^k Y^k$ in $(k,k,0,\text{adj},1)$

scalars $A_a = \sum_k A_k Y_a{}^k$ in $(k,k+2,0,\text{adj},1)$ $+ (k+2,k,0,\text{adj},1)$

scalar $Z = \sum_k Z_k Y^k$ in $(k,k,2,\text{adj},1)$

conformal dimension & quantum numbers lead to identification on the gauge theory side of the currents.

$$A_a^{k=1} \quad \Delta = 2 : \quad Z^{[IJ]} = q_A^I q_B^J C^{AB}$$

$$A_\mu^{k=1} \quad \Delta = 3 : \quad J_\mu^{[IJ]} = q^{[I} \partial_\mu q^{J]} + i \psi^{[I} \gamma_\mu \psi^{J]}$$

global $SO(8)$ current.

From a heterotic 1 loop calculation we get D-instanton induced terms in the 7 brane world volume.

$$I_7 = \int d^8x \, t_8 \, \mathrm{tr} F^4 \, Z_{(1)} + \int d^8x \, t_8 (\mathrm{tr} F^2)^2 \, Z_{(2)}$$

$$Z_1 = \sum_N \left(\frac{1}{2} \sum_{N|m} \frac{1}{m} e^{2\pi i \, 2N\tau} - \frac{1}{2} \sum_{N|m} \frac{1}{m} e^{2\pi i \, 4N\tau} \right) + \text{c.c.}$$

$$Z_2 = \sum_N \left(\frac{1}{4} \sum_{N|m} \frac{1}{m} e^{2\pi i \, 2N\tau} - \frac{1}{8} \sum_{N|m} \frac{1}{m} e^{2\pi i \, 4N\tau} \right) + \text{c.c.}$$

These terms induce a four point function for 4 vectors in AdS_5

$$I = \int \frac{d^4z\, dz_0}{z_0^5}\, z_0^8\, t_8^{mnpqerst} D_m A_n^A D_p A_q^B D_e A_r^C D_s A_t^D$$

$$\times \left\{ F_{ABCD}(T) + G_{ABCD}(T) \right\}$$

where F and G are determined by group theory & Z_1 Z_2:

$$F_{ABCD}(T) = \mathrm{tr}(t_A t_B t_C t_D) \times Z_{(1)}($$

$$G_{ABCD}(T) = \mathrm{tr}(t_A t_B)\mathrm{tr}(t_C t_D) \times Z_{(2)}(T)$$

Via the GKP/W prescription I can be related to instanton contributions to the correlator involving of $J_{\mu A}(x)$

$$\langle J_{\mu A}(x_1)\, J_{\nu B}(x_2)\, J_{\rho C}(x_3)\, J_{\lambda D}(x_4) \rangle$$

$$= t_8^{mnpqerst} \int \frac{d^4z\, dz_0}{z_0^5}\, z_0^{12}\, \frac{J_{0[m}(z-x_1)\, J_{n]\mu}(z-x_1)}{(z_0^2 - (z-x_1)^2)^3}$$

$$\times \frac{J_{0[p}(z-x_2)\, J_{q]\nu}(z-x_2)}{(z_0^2 + (z-x_2)^2)^3}\; \frac{J_{0[e}(z-x_3)\, J_{r]\rho}(z-x_3)}{(z_0^2 + (z-x_3)^2)^3}$$

$$\times \frac{J_{0[s}(z-x_4)\, J_{t]\lambda}(z-x_4)}{(z_0^2 + (z-x_4)^2)^3} \left\{ F_{ABCD}(T) + G_{ABCD}(T) \right\}$$

where $J_{m\mu}$ is related to the bulk to boundary propagator for a vector field

$$J_m = z_0^2+(z-x)^2\Big) \frac{\partial}{\partial z_m} \left(\frac{(z-x)_\mu}{z_0^2+(z-x)^2} \right)$$

This provides a prediction for large N_c correlators of conformal currents J_μ^A in $USp(2N_c)$ gauge theory coming from instantons.

- Only even instanton number contributes to the ($O(8)$ parity even) correlators.
- Form of $Z_{(1)}$ & $Z_{(2)}$ is simpler than the corresponding IIB instanton functions $f_{(p,q)}$: No perturbative ~~fluctu~~ corrections around the instanton (semi classical approximation is exact)
- The 'partition function' for a charge $2k$ instanton should be related to the matrix mechanics of D instantons in the presence of O7/D7:

$$= \mathrm{tr}\Big(\tfrac{1}{2}[X_i, X_j]^2 + [\phi_a, X_i]^2 + \tfrac{1}{2}[\phi_a,\phi_b]^2$$

$$+ ig[\theta\,(\phi_1+i\phi_2), \theta] + ig\,\lambda[\phi_1 - i\phi_2, \lambda]$$

$$+ ig\,\theta\,\Gamma_i[X^i,\lambda]\Big) + ig\,\chi_I(\phi_1-i\phi_2)\chi_I$$

where X^i, $i=1\ldots8$, θ^a $a=1\ldots8$ transform in symmetric second rank tensor representation of $SO(k)$ ϕ_a $a=1,2$ λ^α $a=1\ldots8$ transform as adjoint of $SO(k)$, χ_I transforms in the fundament?

- Check using semiclassical instanton techniques — Green, Naculich & Samadi, Hollowood

ADHM construction for $USp(2N_c)$ theories

Constructed as an orbifold of the $U(2N)$ theory

Saddle point in the large N_c limit for ADHM integrals

K instantons commute & are on top of each other

1 collective coordinates are $AdS_5 \times S_3$ contributes to

flavor current correlator Matrix integral remains of $O(k)$ instantons

2. collective coordinates are $AdS_5 \times S_5 / Z_2$

contributes to bulk correlators like R^4

Conclusions

- String theory in an D-instanton background is a useful way to analyze D-instanton effects
- Together with duality symmetries this can determine instanton corrections to D brane world volume theories
- D-brane fill AdS in the Maldacena limit and thus provide new instanton contributions the boundary CFT correlators