

Non-abelian  $D\bar{0}$  - branes in  
curved backgrounds : from  
Matrix diffeomorphisms to a  
Geometric Myers Effect

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- Introduction
- $D\bar{0}$  - branes and diffeomorphisms
- Matrix diffeomorphisms from Open Strings
- A non-abelian DBI action
- Evidence for a geometric Myers effect
- Outlook & Conclusion

# Stringy Geometry

- In String Theory, Geometry is defined by probes.

- "Geometry" is not universal
- Closed vs. Open Strings vs. D-branes
- Open with  $B_{NS} \neq 0$   
yields noncommutative geometry
- Open with  $F_{RR} \neq 0$   
gives rise to the Myers effect
- D-brane geometry, is well-defined.  
i.e.  $D\phi$ -metric is not renormalized.

Douglas

## D-branes in Curved Space

- Question:

What is the geometry of  $N$  coincident D-branes?

- Single D-brane effective action:  
Particle in Curved Space

$$S = \int e^{-\varphi} \sqrt{|G_{ij} \partial_a x^i \partial_b x^j + F_{ab}|}$$

-  $N$  coincident D-branes in flat space

$$x^i \in \mathbb{R} \sim U(1) \rightarrow X^i \in U(N)$$

$$S = \int \frac{1}{2} \text{Tr} \partial_a X^i \partial^a X_i + \frac{F^2}{4} - [X^i, X^j]^2$$

-  $N$  coincident D-branes in curved space

$$S = \int \text{tr} G_{ij}(X) \partial X^i \partial X^j$$

to determine this!

## Matrix valued diffeomorphisms

- What is the action of  $N$  coincident D0branes in curved space?

- One expects

$$S = -m \int \text{tr} \sqrt{g_{ij}(X) \dot{X}^i \dot{X}^j} + \dots$$

- Ordering?

Lore

$$[X^i, X^j] \neq 0$$

- Additional interactions?  
(viz. potential)

YES

- Characteristic of curved space:  
Invariance under diffeomorphisms

- Recall

$$S_{\text{part}} = \int \sqrt{g_{ij}(x) \dot{x}^i \dot{x}^j}$$

$$\delta S_{\text{part}} = 0 \quad ; \quad \delta x^i = \xi^i(x) \quad ; \quad \delta g_{ij} = -D_i \xi_j$$

- Graviton is gauge field of diff.  
Coupling is unique

- Search for a symmetry principle:  
Matrix Diffeomorphisms

## Overview

- What: the action for D0-branes in curved space
- How: by requiring diffeomorphism invariance PLUS ..... string theory

### - Other approaches

- Susy and off-diagonal fluctuations and geodesics Douglas Kato Ooguri
- Susy and kappa symmetry Bergshoeff, de Roo Serrin
- (Linear) couplings from Matrix theory Taylor v Raamsdonk Okawa Ooguri
- Coset-like approaches from  $dN^2$ -sigma models Chamsedine  
⋮

- Why:
    - Probes of Quantum Geometry (Myers)
    - Connection between D0-brane Mechanics and Matrix theory
    - Matrix Models:
      - $O(N)$  vector models
      - LG models
      - Random Matrix Theory
- Diff invariant version of  $N$ -particle QM.

# Intermezzo: NC Geometry in String Theory

- STANDARD non-commutative Geometry

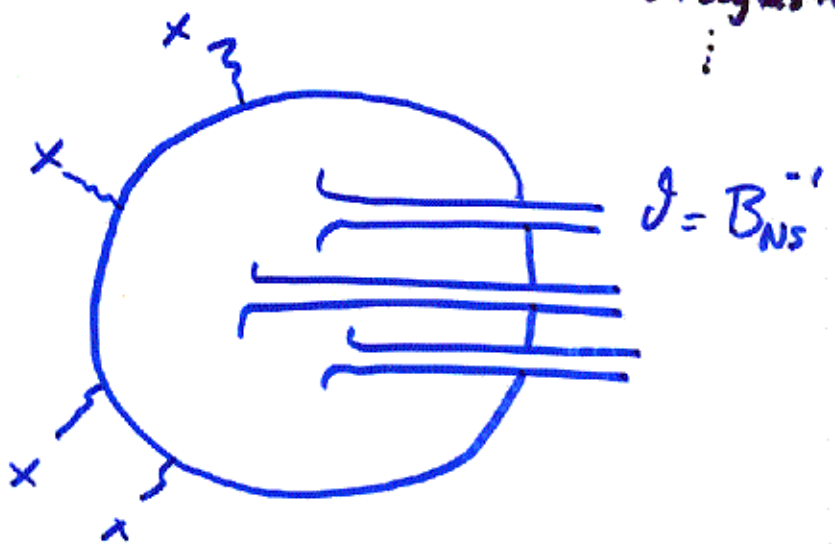
Connes  
Seiberg Witten  
Kontsevich  
Cattaneo Felder  
Connes Douglas Hull  
...

$$[x^i, x^j] = i\theta^{ij}$$

$$\partial_k \theta_{ij} = 0$$



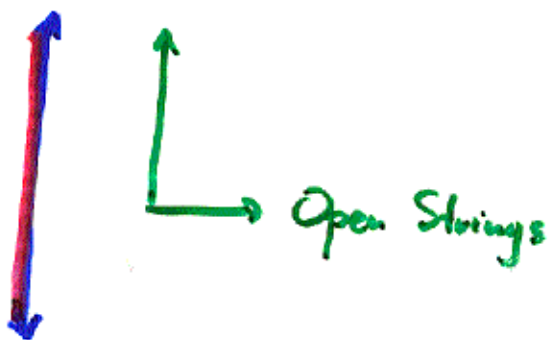
Associative  
\* product



- D0-brane non-commutative Geometry

Witten  
...

$$[X^a, X^b] = X^m X^c f_{abc} T^c$$



$\tilde{*}$ -product?

