Non-abelian D0-branes in curved backgrounds: from Matrix diffeomorphisms to a Geometric Myers Effect

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- Introduction
- D0-branes and diffeomorphisms
- Matrix diffeomorphisms from Open Strings
- A non-abelian DBI action
- Evidence for a geometric Myers effect
- Outlook & Conclusion

Stringy Geometry

- In String Theory, Geometry is defined by probes.

- "Geometry" is not universal
- Closed vs. Open Strings vs. D-branes
- Open with
$$B_{NS} \neq 0$$
yields noncommutative geometry
- Open with
$$F^{\perp}_{RR} \neq 0$$
gives rise to the Myers effect
- D-brane geometry, is well-defined. i.e. D$\emptyset$-metric is not renormalized.

Douglas

D-branes in Curved Space

- Question:

  What is the geometry of N coincident D-branes?

- Single D-brane effective action:
  Particle in Curved Space

$$S = \int e^{-\varphi} \sqrt{|G_{ij}\, \partial_a x^i \partial_b x^j + F_{ab}|}$$

- N coincident D-branes in flat space

$$x^i \in \mathbb{R} \sim U(1) \longrightarrow X^i \in U(N)$$

$$S = \int \frac{1}{2} \mathrm{Tr}\, \partial_a X^i \partial^a X_i + \frac{F^2}{4} - [X^i, X^j]^2$$

- N coincident D-branes in curved space

$$S = \int \mathrm{tr}\; G_{ij}(X)\, \partial X^i \partial X^j$$

to determine this!

Matrix valued diffeomorphisms

- What is the action of N coincident D0branes in curved space?

  - One expects

$$S = -m \int \mathrm{tr} \sqrt{G_{ij}(X)\dot{X}^i\dot{X}^j} + \dots$$

  - Ordering? Lore

$$[X^i, X^j] \neq 0$$

  - Additional interactions? YES (viz. potential)

- Characteristic of curved space: Invariance under diffeomorphisms

  - Recall

$$S_{part} = \int \sqrt{g_{ij}(x)\dot{x}^i\dot{x}^j}$$

$$\delta S_{part} = 0 \; ; \quad \delta x^i = \xi^i(x) \; ; \quad \delta g_{ij} = -D_{(i}\xi_{j)}$$

  - Graviton is gauge field of diff. Coupling is unique

- Search for a symmetry principle: $\boxed{\text{Matrix Diffeomorphisms}}$

Overview

- **What**: the action for D0 branes in curved space

- **How**: by requiring diffeomorphism invariance PLUS .... string thy

  - Other approaches
    - Susy and off-diagonal fluctuations and geodesics — Douglas Kato Ooguri
    - Susy and kappa symmetry — Bergshoeff, de Roo Sevrin
    - (Linear) couplings from Matrix theory — Taylor vRaamsdonk Okawa Ooguri
    - Coset-like approaches from dN^2-sigma models — Chamsedine
      ⋮

- **Why**:
  - Probes of Quantum Geometry (Myers)
  - Connection between D0-brane Mechanics and Matrix theory
  - Matrix Models:
    - O(N) vector models
    - LG models
    - Random Matrix Theory

Diff invariant version of N-particle QM.

Intermezzo: NC Geometry in String Theory

- STANDARD non-commutative Geometry

Connes
Seiberg Witten
Kontsevich
Cattaneo Felder
Connes Douglas Hull
⋮

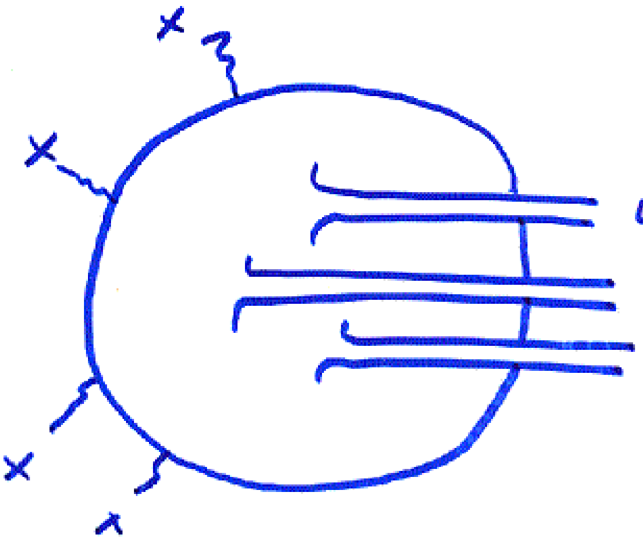
$$[x^i, x^j] = i\vartheta^{ij}$$

$$\partial_{[k}\vartheta_{ij]} = 0$$

Open strings

Associative

$\star$ product



$$\vartheta = B_{NS}^{-1}$$

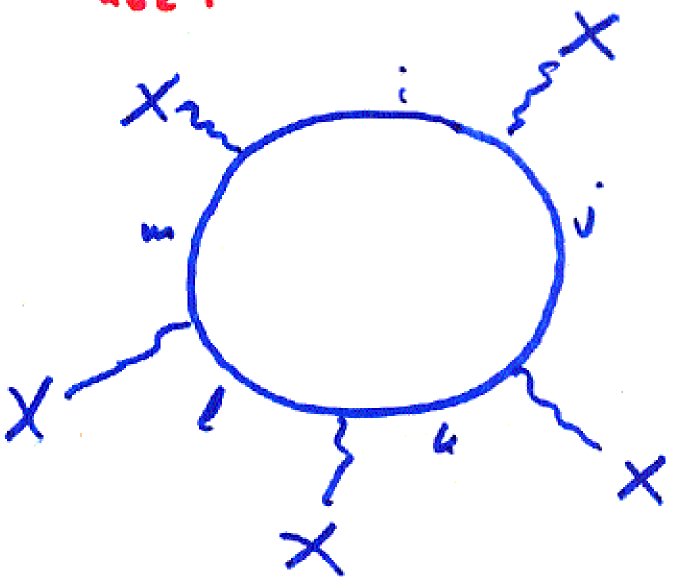
- D∅ - brane non-commutative Geometry

Witten
⋮

$$[X^\mu, X^\nu] = X^{\mu a} X^{\nu b} f_{abc} T^c$$

Open Strings

$\tilde{\star}$-product?



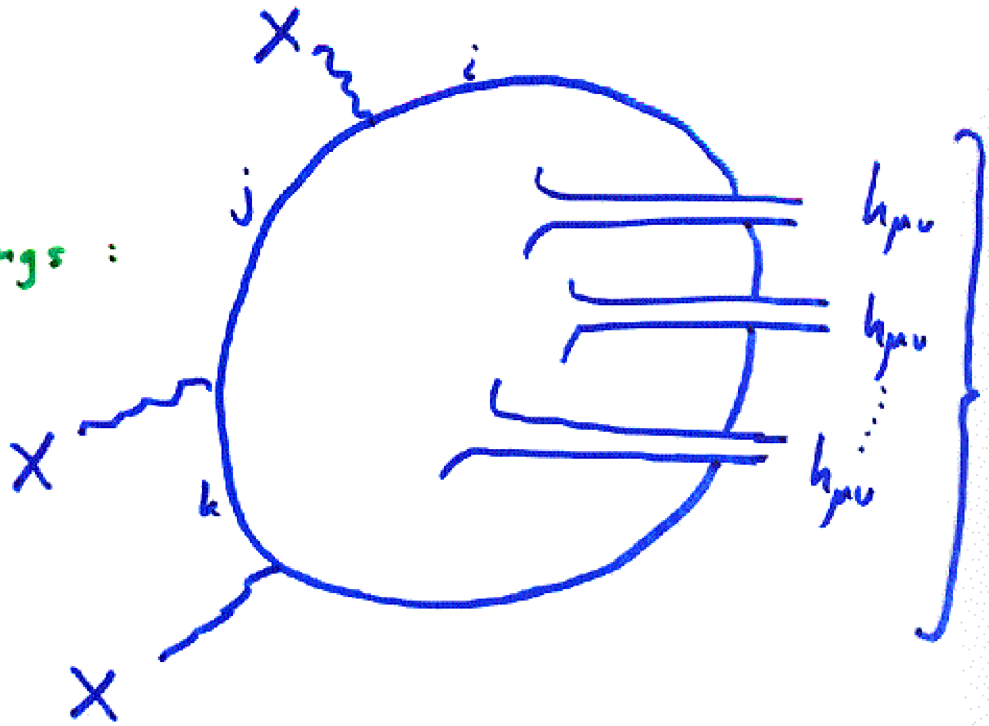
Contents

- Exact formulation of the problem
  - Matrix DIFFeomorphisms
  - Representations and linear couplings
- An action by way of open strings
  - Characteristics of amplitudes
  - Normal Coordinates and diffeomorphisms
  - Matrix Geometry
  - → An action
- Properties
  - Potential / T-duality
  - D-geometry constraints
- Consequences
  - A Geometric Myers effect ?
  - ⋮

• D∅-branes coupled to gravity

Open strings :

action exists



Exact formulation

- Given the action

$$S[g,X] = \int \mathrm{tr}\; G_{ij}(X)\, \dot{X}^i \dot{X}^j$$
$$\equiv \int \mathrm{tr}\left(\dot{X}^i \dot{X}^j X^{k_1} \ldots X^{k_n}\right) \frac{\partial^n}{n!} g_{ij}(0)$$

infinite # couplings

find the precise orderings/couplings such that

D-geometry

- the action has a single trace — Tseytlin
- If $X^i = \mathrm{diag}(x^i_1, \ldots, x^i_N)$ the reduces to a sum of N particle actions

Potential

- Off-diagonal fluctuations have masses proportional to $d(x_1, x_N)$ — Douglas
- Classical Moduli space is the symmetric product $\mathcal{M}^N / S_N$

- the action is invariant under $X \xrightarrow{?} X'(X)$ for $g \to g'$ — NEW
- Linear couplings agree with known results (Taylor vR ; Ozaka Ooguri) — NEW

Linearized Coupling and Linearized diffeomorphisms

- Single D-brane (particle)

$$\boxed{\begin{aligned} S &= \int d\tau \; g_{ij}(x)\,\dot{x}^i \dot{x}^j = \int d\tau \; \dot{x}^i \dot{x}^j \left(\eta_{ij} + h_{ij}(x)\right) \\ &\equiv \int d\tau \; \dot{x}^i \dot{x}_i + T^{ij}(x)\, h_{ij}(x) \end{aligned}}$$

$$T^{ij k_1 \dots k_n}_{(x)} \quad \underbrace{\frac{\partial^n}{n!} h_{ij}\Big|_{x=0}}_{\text{couplings}}$$

- Current (stress-tensor) $T^{ij}(x)$ conserved (on shell)

"follows" from invariance under infinitesimal diffs:

$$\boxed{\delta x^i = \xi^i(x) \quad ; \quad \delta h_{ij} = -D_i(h)\xi_j - D_j(h)\xi_i}$$

Linearized diffs II:

- To first order in ε, the action

$$\boxed{S = \int d\tau\, \dot{x}^i \dot{x}_i + T^{ij}(x)\, h_{ij}(x)}$$

is also invariant under linear diffs.

$$h_{ij} \sim \varepsilon \qquad ; \qquad \xi^i \sim \varepsilon$$

coupling constant

$$\delta x^i = \xi^i(x)$$

$$= x^{k_1} \ldots x^{k_n} \frac{\partial^n}{n!} \xi^i \Big|_{x=0}$$

$$\delta h_{ij} = -\partial_i \xi_j - \partial_j \xi_i + \mathcal{O}(\varepsilon)$$

$$\delta \partial^n h_{ij} = -\partial^n \partial_{(i} \xi_{j)} + \mathcal{O}(\varepsilon)$$

- "Reconstruction": to first order the gauge field coupling to a conserved current yields an invariant action iff

$$\delta A_i = -\partial_i \varepsilon$$

Linearized Coupling and Linearized diff. III

- Linear coupling to graviton (stress-tensor); known from Matrix theory and string amplitudes

(Taylor v R; Ooguri Ooguri)

$$S = \int d\tau \; \dot{X}^i \dot{X}_i + T^{ij(k_1 \dots k_n)} \frac{\partial^{(n)}}{n!} h_{ij} + \dots$$

$$T^{ij(k_1 \dots k_n)} = \mathrm{Str}(\dot{X}^i \dot{X}^j X^{k_1} \dots X^{k_n})$$

Tseytlin

DIFFERENT FOR BOSONIC STRING

- Stress tensor is conserved (on shell)

$\Longleftrightarrow$

Invariant under linearized diffeomorphisms

$$\delta h_{ij} = -\partial_i \xi_j - \partial_j \xi_i + \dots$$

$$\delta X^i = \xi^i(X) \equiv \mathrm{Sym}(X^{k_1} \dots X^{k_n}) \frac{\partial^n}{n!} \xi(0)$$

- "Reconstruction" of higher order terms ....

Structure of Matrix diffeomorphisms

- .... derive by Noether method

- Very difficult!
- Need to guess 2^{nd} order

$$\delta^{(2)}X \sim \partial h \, \xi \, [[X,X],X]$$
$$S^{(2)} \sim h^2 \, [X,\dot{X}]^2$$

- "Normally" need either $\delta^{(2)}X$ or $S^{(2)}$
- Important to note that

$$\delta X = \xi [g, X]$$

gauge field

- Group DIFF of matrix-valued diffs.

- Lift diff to DIFF

Group Structure

diff $\quad x''(x'(x)) = x''(x)$

¿ DIFF $\quad X''(X'(X)) = U X''(X) U^{+}$?

- HAS NO SOLUTION
- $\{g, X\}$ form a representation

Intermezzo

- To implement Matrix-valued diffeomorphisms:

  - Seek an action that obeys

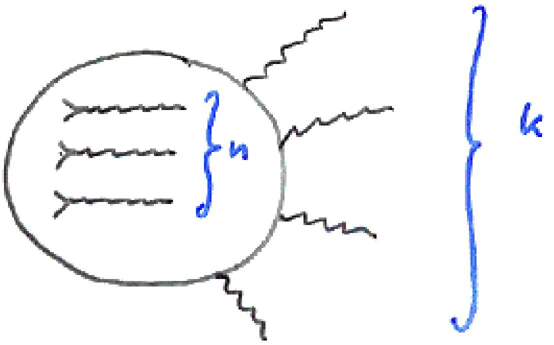
  $$S[X, g] = S[X'(g, X), g'(g)]$$

  - Extremely non linear problem
  - Brute force (Noether) "fails"
  - Can we find some extra information?

D-brane actions from open strings

- D-brane effective action = effective action reconstructed from open string amplitudes.

- Disc amplitudes with k open-string vertices and n graviton vertices



- Diffeomorphism invariance of string theory $\Rightarrow$ diff invariance of the effective action
- $n=1$ graviton amplitudes yield linearized result
- Rather than compute amplitudes, try to glean information from the general structure

String Effective Actions
Intermezzo

- Why not use β-functions?

  - β-functions work for massless fields (marginal operators) in a BG-field method
  - the matrix X corresponds to the open-string field V with vertex

$$V_i(k)\, \partial_n Y^i e^{ikY}$$

  - Hence we would need "to "turn-on" off-diagonal parts of X, but these correspond to massive operators

- Does the LEEA in fact exist?

  - Does the matrix structure survive $\alpha' \to 0$
  - Physical expectation YES
  - Provided, masses $\langle X \rangle \ll \sqrt{\alpha'}$
    curvature $R \ll \sqrt{\alpha'}$
  - Inspection of two-graviton amplitudes suggests a consistent $\alpha' \to 0$ limit exists

Lessons from open strings

- The action from amplitudes

Dirichlet

$Y(0)=Y(\pi)$

- Vertices = fluctuations around x, η_{ij}

$$V_i(k)\,\partial_n Y^i e^{ikY} \quad ; \quad h_{ij}(k)\,\partial Y^i \bar\partial Y^j e^{ikY}$$

- Action

$$S = S[h_{ij}(x), \tilde{X}^i]_{\eta, x}$$

$$\tilde{X}^i = (\eta + h)^{ij} V_j$$

- Can we relate this to $S[g, x]$?

- Vertex $V_i(k=0) \Rightarrow$ shift in x !

- V_i transform as vectors

matrices

caution : contact terms

$\tilde{X}^i \sim \Delta x^i$

$\Rightarrow$ Can we preserve the vector transformation properties?

Normal Coordinates

• Particle

- Final action depends on

$$X = \bar{x} + \tilde{x} + \mathcal{O}(\tilde{x}^2)$$

- X should transform as a vector
  $\Leftrightarrow$ expansion is known : covariant non-linear BG-field expansion

Alvarez G
Freedman
Mukhi

$$x = \bar{x} + \tilde{x}^i - \sum \Gamma^i{}_{k_1 \dots k_n} \tilde{x}^{k_1} \dots \tilde{x}^{k_n}$$

- In that case, all other quantities will be covariant

$$S[h(x(\bar{x}, \tilde{x})), x(\bar{x}, \tilde{x})] = S[g(\bar{x})\, D \dots DR, \tilde{x}]$$

- Special Coordinate Choice where non-linear terms are absent

contact terms

$$\Gamma^i{}_{(k_1 \dots k_n)}(\bar{x}) = 0$$

$$\Leftrightarrow$$

$X^i(\tau) = \tau\, Y^i$ is a solution to the geodesic equation : <u>the field equation of the action.</u>

Matrix normal Coordinates

- Matrices

- The action we seek to solve

$$S \sim \int d\tau \, G_{ij}(X)_{\delta\alpha} \, \dot{X}^{i\alpha\beta} \, \delta_{\beta\gamma} \, \dot{X}^{j\gamma\delta}$$

is a special case of the dN^2-NLσM

$$S \sim \int d\tau \; G_{i\alpha\beta, j\gamma\delta}(X) \, \dot{X}^{i\alpha\beta} \, \dot{X}^{j\gamma\delta}$$

if we view the triple $\{i\alpha\beta\} \sim I$. Note manifest diffeomorphism invariance.

- Matrix normal coordinates defined by

$$X^{i\alpha\beta}(t) = t \, Y^{i\alpha\beta}$$

is a solution to the field equation

- This fixes regular plus "new" diffs

$$\delta X \sim [X, X]$$

will come back to these later

- Action will contain dN^2-dim covariant quantities

$$S = S[G_{IJ}, D_{K_1} \ldots D_{K_N} R, \tilde{X}^I]$$

which are functionals of d-dim covariant quantities g_{ij} & $D_{k_1} \ldots D_{k_n} R$!!

Normal Coordinates & Diffeomorphisms

- Every action appears DIFF invariant ?!

– as the action, for <u>any</u> ordering

$$S[\, g_{ij}(x),\ D_{k_1} \dots D_{k_n} R(x),\ \tilde{X} \,]$$

only depends on covariant quantities

- Normal coordinates are based on a special point x ; diff invariance means independence of the base point x

Transformation to new normal coordinates must leave action S invariant !!

– diff translates in the symmetry

$$\delta S = 0 \qquad ; \qquad \text{"} \delta x = \varepsilon \text{"}$$

$$\delta\, D \dots D R \;\hat{=}\; \varepsilon^i D_i D \dots D R$$

$$\delta \tilde{X} = \varepsilon - \Gamma^{(n)}(\tilde{X})\, \varepsilon$$

solution to geodesic equation = field equation of S !!

– imposes that the action only depends on $X = \varepsilon + \tilde{X}$

Shift Symmetry in BG field formalism

- BG-field formalism in YM theory

  - BG-Quantum Split

$$\delta(A_\mu + Q_\mu) = D_\mu(A+Q)\,\Lambda$$

$$\Rightarrow \quad \begin{aligned} \delta A_\mu &= D_\mu(A)\,\Lambda \\ \delta Q_\mu &= [Q_\mu, \Lambda] \end{aligned}$$

  - Shift Identity / Symmetry

$$\delta A_\mu = \varepsilon_\mu \quad ; \quad \delta Q_\mu = -\varepsilon_\mu$$

- (Covariant) BG-field formalism in NLσM

  - <u>NONLINEAR</u> BG-Quantum Split

$$\delta(x + \xi - \dots) = \varepsilon(x + \xi - \dots)$$

$$\Rightarrow \quad \begin{aligned} \delta x^\mu &= \varepsilon^\mu(x) \\ \delta \xi^\mu &= -\varepsilon\cdot\partial\,\xi^\mu + \xi^\nu \partial^\mu \varepsilon_\nu \end{aligned}$$

  - NONLINEAR Shift symmetry (combined w change to new RNC)

$$\delta(\overset{0}{x} + \xi) = \tilde{x} + \chi - {}'' \Gamma^{(n)} {}'' \chi^n$$

$$\Rightarrow \delta \xi = \varepsilon + \chi + \varepsilon\, \nabla^n R\, \chi^{n+2}$$

An Algorithm for Diffs

- Diffs as maps Normal Coord$_1$ $\leftrightarrow$ Normal Coord$_2$ have advantage, that invariance can be imposed algorithmically.

- Write action S with all possible terms
- Impose Normal Coord$_1$ by requiring

$$X(\tau) = \tau Y$$

is a solution to field eq $\Rightarrow$ constraints$_1$

- Change to Normal Coord$_2$ around ε by solving field eq order by order; substitute in action S and require that

$$S_{NC2}[g, D..DR, \tilde{X}, \varepsilon] = S_{NC1}[g, \varepsilon^i D_i D..R, \tilde{X}]$$

$\Rightarrow$ constraints$_2$

- Solve system of constraints 1 and 2

Normal coordinates & Matrix Geometry

- Expand the action (dN^2-sigma model)

$$\boxed{S = G_{IJ}\,\dot{X}^I \dot{X}^J \equiv g_{i\alpha\beta\, j\gamma\delta}\,\dot{X}^{i\alpha\beta}\dot{X}^{j\gamma\delta}}$$

in normal coordinates

- dN^2 Curvature tensors $R_{IJKL}(g, R_{ijkl})$ are functionals of d-curvature tensors.

- These functionals must obey the usual props.

- Impose on R_{IJKL}, $\nabla_I R_{JKLM}$, etc

a) $R_{IJKL} = -R_{JIKL} = R_{KLIJ}$
b) $R_{I[JKL]} = 0$
c) $\nabla_{[I} R_{JK]LM} = 0$
d) $[\nabla_I, \nabla_J] R_{KLMN} = R_{IJK}{}^{T} R_{TLMN} + \ldots$

etc.

e) $U(N)$ indices contracted in a single trace
f) Basept invariance

$$\delta^{\alpha\beta}\,\nabla_{i\alpha\beta} = \nabla_i$$

g) To first order, symmetrized result
h) Correct $U(1)$ limit

Solution

- Constraint e) implies that

$$\boxed{\begin{aligned} R_{IJKL} &= R_{ijkl}\,\Delta_{\alpha\beta\gamma\delta} + \text{orderings.} \\ \Delta_{\alpha\beta\gamma\delta} &= \delta_{\alpha_2\beta_2}\,\delta_{\beta_2\gamma_1}\,\delta_{\gamma_2\delta_1}\,\delta_{\delta_2\alpha_1} \end{aligned}}$$

- At order 4, R_{IJKL}, and 5, $\nabla_I R_{JKLM}$ symmetrized result. Order 6 requires new structures due to $[\nabla_I, \nabla_J] R_{KLMN}$ condition.

- Answer : 120 possible terms

$\downarrow$

32-dim space of solutions

$$\begin{aligned} &Tr\left(X^n X^m X^i \dot{X}^j \dot{X}^k X^l\right) T^1_{nmiklj} \\ + \;& Tr\left(X^n X^m X^i \dot{X}^k X^l \dot{X}^j\right) T^2_{nmiklj} \\ + \;& Tr\left(X^n X^i \dot{X}^k X^m X^l \dot{X}^j\right) T^3_{nmiklj} \end{aligned}$$

$$T^1_{nmiklj} = \left(-\frac{7}{120} - 2\beta\right) R_{mklp} R_{nij}{}^{p} + 23 \text{ terms}$$

$$T^2_{nmiklj} = \frac{1}{180} R_{mjkp} R_{nilp} + 31 \text{ terms}$$

$$T^3_{nmiklj} = \left(-\frac{47}{120} - \frac{\alpha}{5} - \frac{6\beta}{5}\right) R_{mikp} R_{njl}{}^{p} + 15 \text{ terms}$$

- No obvious structure emerges

Properties of the Solution

- There are **new** vertices cp. with $U(1)$
  - $U(1)$ only $(R\dot{x})(R\dot{x})$
  - $U(N)$ e.g. from T^2 : $R_{mjkp}\, \dot{X}^j \dot{X}^k$
  - appears to hold for all values of the 32 - parameters.
- The construction of the solution obscures that some of the 32 parameters will be fixed at higher order
  - e.g. $\Delta S \sim \underline{Tr\left(Asym\left(\dot{X}^j X^k X^l\right) ASym\left(\dot{X}^m X^n X^p\right)\right)}$ $\underline{R_{ijkl}}\ \underline{R'_{mnp}}$ is left unfixed at this order.
  - Unlikely all of them will be fixed at higher order
    - Bosonic string & superstring are different reps of Matrix DIFF, expect more.

The Potential, fermions, other contributions

- Need Potential for remaining two constraints of D-Geometry.

- Introduce vielbeins

$$G_{IJ} = E_I^{At} E_J^{Bt} \qquad E_I^{At} \equiv E^A_{i\alpha\beta}$$

[$U(N)$-beins $\quad G_{i\alpha\beta\, j\gamma\delta} = E_{i\alpha\delta}(X)\, E_{j\beta\gamma}(X)$
compatible with single trace]

- Diff $\longrightarrow$ $SO(dN^2)$ transformation of $E_{i\beta\alpha}\dot{X}^i$

i.e. $Tr\, E_i^A \dot{X}^i$ transforms nicely

- Note that $[X, \ldots]$ acts as $d/d\tau$ implies that $Tr([E_i^A, X^i]V)$ transforms nicely

- Some Guesswork leads to the potential

$$V = \frac{1}{2} Tr[X^i, X^j]^2 \rightarrow$$

$$= \frac{1}{4} Tr\, [E_i^A, X^i][E_j^B, X^j][E_k^A, X^k][E_\ell^B, X^\ell]$$

- <u>Also</u> yields correct linearized answer

Spectrum of Quadratic Fluctuations

- Remaining constraints of D-Geometry

$$\boxed{\begin{array}{l}\bullet\ \text{Quadratic fluctuations } \lambda \text{ have masses} \\ \qquad m_\lambda \sim d(x_i, x_j) \\ \bullet\ \text{Classical Moduli space is } \mathcal{M}^N/S_N\end{array}}$$

- Quadratic fluctuations

$$\boxed{\begin{array}{l}\bullet\ \text{diff invariance + normal coordinates imply that we only need to check} \\ V\big|_{\lambda^2} \sim d(0,\lambda) = g_{ij}(0)\,\lambda^i\lambda^j \\ \bullet\ \text{CHECKS OUT !!}\end{array}}$$

- Classical Moduli space $\mathcal{M}^N/S_N$ also follows.

Fermions and Susy

- Need Vielbeins for susy / fermions

  - Appearance of vielbeins in potential is encouraging

- Susy constraints ?!

  - Susy on the worldvolume should follow from kappa symmetry

  - For kappa symmetry

$$\delta S^{kin} = - \delta S^{WZ}$$

  need WZ-term.

  - ~~Do~~ additional constraints arise? for non-abelian kappa-symmetry

  only closes on-shell

Gravitational Myers Effect I

- The Myers effect

Myers

- In the presence of RR 4-form flux

$$V(X) = -\frac{1}{4}\,\mathrm{Tr}\,[X^i, X^j]^2 - F_{0ijk}\,\mathrm{Tr}\,X^0 X^i X^j X^k$$

- Two solutions

$$\frac{\partial V}{\partial X} = 0 \quad \begin{cases} X^i = \mathrm{diag}\, x^i_\lambda \\ [X^i, X^k] = i F_{0ikj} X^j \end{cases}$$

least energetic

- The non-trivial solution corresponds to a polarized D2 - brane.

- Signal for Myers effect is the presence of a tachyonic mode in the spectrum of quadratic fluctuations around the trivial solution.

Jatkar et. al.

Gravitational Myers Effect II

- Question : Is there a Gravitational Myers effect ?

- Is there a non-trivial critical point for the action

$$S(X) = \frac{1}{2} \underline{\underline{Tr}}\, E_i^A \dot{X}^i \underline{\underline{Tr}}\, E_j^A \dot{X}^j - \frac{1}{4} Tr\, [E, X]^4$$

- Highly non linear
- Difficult to find solutions

Sahakian

- Signal of Gravitional Myers effect : spectrum of <u>quadratic</u> <u>fluctuations</u>

¿ CONFLICT ?

- D-Geometry : off-diagonal fluctuations have $m^2 \simeq d^2(x,y) \geq 0$

Gravitational Myers effect III

- "Trivial" Solution is N freely falling non-interacting particles.

- $X^{bg} = \mathrm{diag}(x^i_\lambda)$

  with $\ddot{x}^i_\lambda + \Gamma^i{}_{jk}\dot{x}^j_\lambda \dot{x}^k_\lambda = 0$

  for each entry x^i_λ.

- In D-Geometry, ultra-trivial background

  $\dot{x}^i_\lambda = 0$

- Myers effect possible iff $\dot{x}^i_\lambda \neq 0$

  Connection w. Giant Graviton?

- Quadratic fluctuations ?!

- Single Particle;

  $\delta S^{kin} = \delta x^i M_{ij} \delta x^j$

  $M_{ij} = R_{imnj}\dot{x}^m \dot{x}^n + \nabla_\tau \nabla_\tau$

- M_{ij} is the kernel of the geodesic deviation operator; measures tidal forces!

Gravitational Myers effect $\underline{\text{IV}}$

- Spectrum of Quadratic off-diagonal fluctuations (around N- freely falling particles)

$$\delta^{(2)} S = \delta^{(2)} S^{kin} + \delta^{(2)} S^{pot}$$

$$\Rightarrow \quad \delta^{(2)} S^{pot} = -\frac{1}{4} [x^i, y^j][x^i, y^j]$$

$$\delta^{(2)} S^{kin} = \mathrm{Tr}\; y^i M_{ij} y^j$$

$$M_{ij} = R_{imnj}\, \dot{x}^m \dot{x}^n + \nabla_z \nabla_z g_{ij}$$

- Myers effect occurs when

source of stress-energy

balance

tidal forces $\leftarrow$ kinetic term

internal forces $\leftarrow$ potential term

- Does this happen? $\boxed{\text{Yes !}}$

When ambient space is <u>negatively</u> curved locally

<u>and</u> if $g_s^{2/3} \ll \dfrac{\omega^2 \ell_s^2}{L^2}$

Non-commutative Geometry

- Standard non-com geometry

  - Open strings with $B_{NS} \neq 0$

  $$f \cdot g \to f * g \equiv e^{iB\partial_x\partial_y} f(x)\, g(y)\Big|_{y=x}$$

- Close relations

  Liu Michelson
  Ozuka Ooguri

  - Amplitude calculations both exhibit "Wilson lines"; linear action can be written as

  $$S = \int_0^1 d\lambda\, h_{ij}(k)\, \mathrm{Tr}\left(e^{i\lambda k X}\, \dot{X}^i e^{i(1-\lambda) k X}\, \dot{X}^j\right)$$

  - Deformation quantization; diffs preserving B_{NS}
    $\Rightarrow$ non-com gauge transformations

  - 1 - 1 Map $\mathcal{F} = B + F \xleftrightarrow{T} [X, X]$

  $$X^i X^j + X^j X^i = 2x^i x^j$$
  $$[X^i, X^j] = B^{ij}$$
  $$[X^k, [X^i, X^j]] = B^{kl}\partial_l B^{ij}$$

  Is there a true Seiberg-Witten like map?

Outlook and Conclusions

- Have succeeded in finding a matrix-diffeomorphism invariant action
  - Constructed "Matrix-Geometry"
  - Presented a solution to order 6

- Have glimpsed at the intricate geometrical structure
  - ... obvious patterns still lacking
  - Connection with non-commutative geometry

- Open questions and further directions
  - WZ term; kappa & supersymmetry
  - Explicit Computational Myers effect
  - Applications; Schwarzschild
  - Other Closed String backgrounds
  - Fundamental Symmetry structure
    → Matrix theory ?
  - Non-Commutative YM ? v Raamsdonk