Exploring QFT, phases and RG flows, via SUSY

Ken Intriligator (UCSD)
UNC + Duke, February16, 2017

Thank you for the invitation to visit.
I would also like to thank my….

Spectacular Collaborators

Clay Córdova

Thomas Dumitrescu

1506.03807: 6d conformal anomaly a from 't Hooft anomalies. 6d a-thm. for N=(1,0) susy theories.

1602.01217: Classify susy-preserving deformations for d>2 SCFTs.

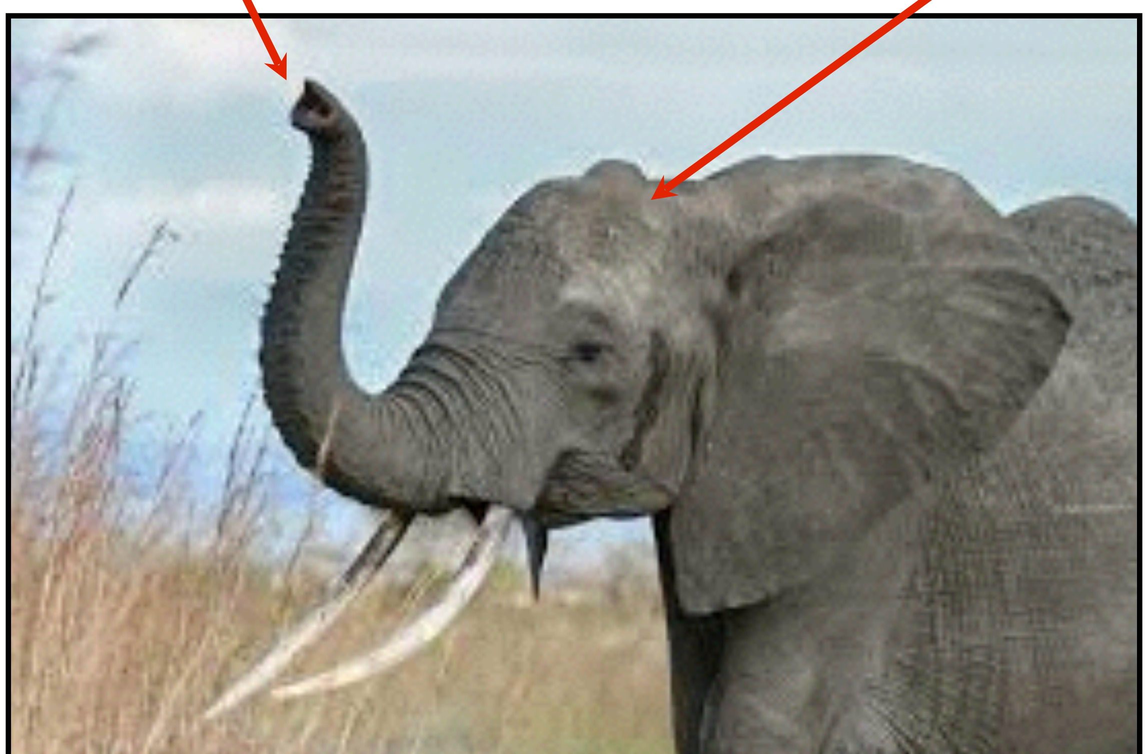
1612.00809: Multiplets of d>2 SCFTs (164 pages; we tried to keep it short).

+ to appear and in progress..

“What is QFT?”

Perturbation theory around free field Lagrangian theories

5d & 6d SCFTs, + deformations, compactifications



5d & 6d SCFTs, etc: new RG starting points. Also in 4d, QFTs that are not via free field+ ints.

(?unexplored…something crucial for the future?)

CFTs + perturbations

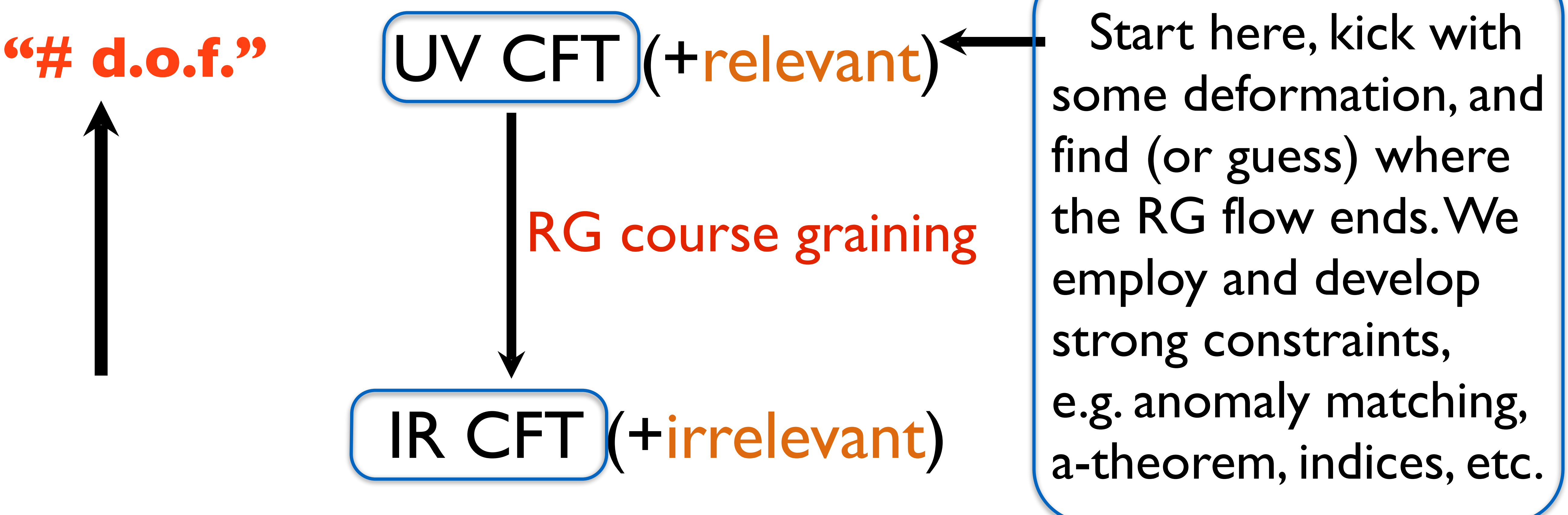
(Above 4d, starting from free theory, added interactions all look IR free. Quoting Duck Soup: “That’s irrelevant!”)

“That’s the answer! There’s a whole lot of relevants in the circus!”

RG flows, universality

In extreme UV or IR, masses become unimportant or decoupled. Enhanced, conformal symmetry in these limits. E.g. QCD: UV-free quarks and gluons in UV, and IR-free pions or mass gap in IR. Now many examples of non-trivial, interacting CFTs and especially with SUSY. Can deform them to find new QFTs.

RG flow cartoon:



RG flows

"# d.o.f."

UV CFT (+relevant)

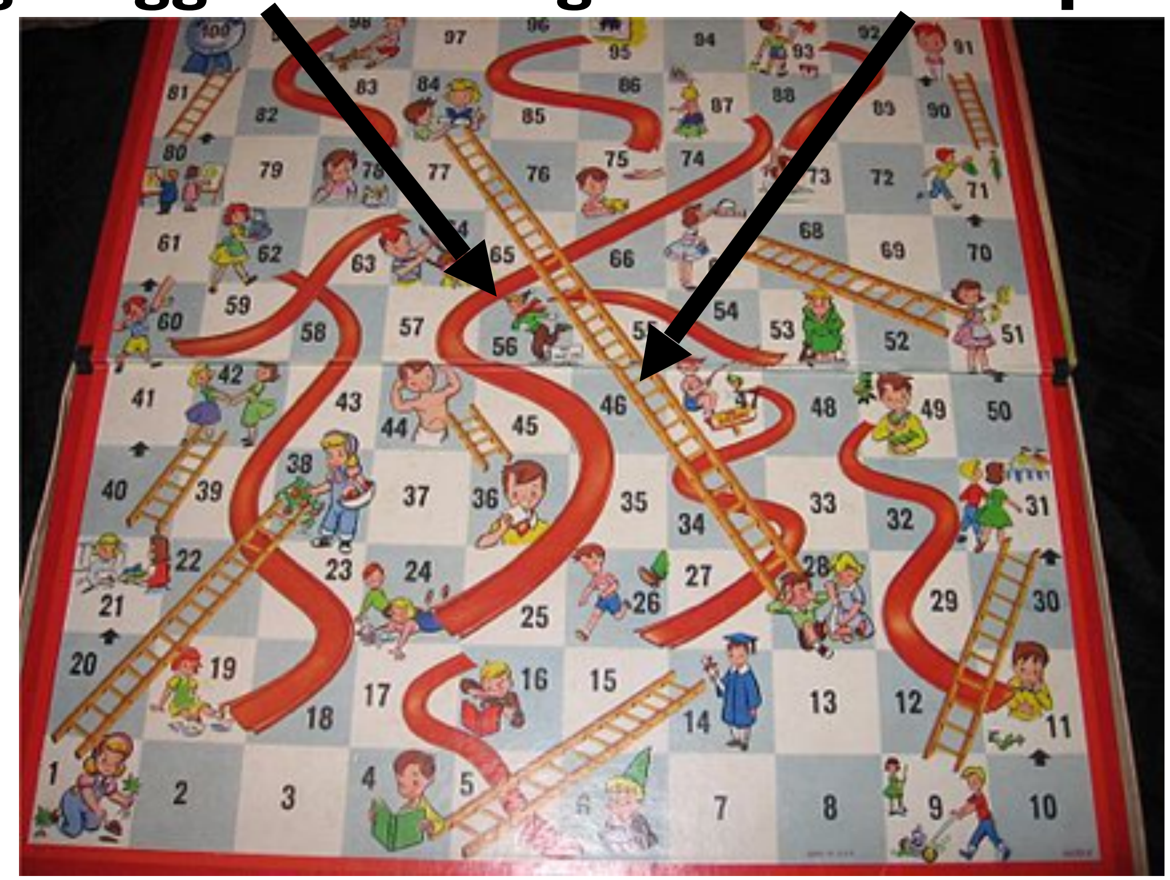
"chutes"

course graining

IR CFT (+irrelevant)

"ladders"

E.g. Higgs mass **E.g. dim 6 BSM ops**



RG flows

"# d.o.f."

UV CFT (+relevant)

RG course graining

IR CFT (+irrelevant)

The "**deformations**" examples:

- $$\text{“}\delta\mathcal{L}\text{”} = \sum_i g_i \mathcal{O}_i$$ (OK even if SCFT is non-Lagrangian)
- Move on the moduli space of (susy) vacua.
- Gauge a (e.g. UV or IR free) global symmetry.
- We focus on RG flows that preserve supersymmetry.

RG flow constraints

- d=even: 't Hooft anomaly matching for all global symmetries (including NGBs + WZW terms for spont. broken ones + Green-Schwarz contributions for reducible ones). Weaker d=odd analogs, e.g. parity anomaly matching in 3d.

- Reducing # of d.o.f. intuition. For d=2,4 (& d=6?) : a-theorem

$$a_{\rm UV} \geq a_{\rm IR} \qquad a \geq 0$$

For any unitary theory

d=even: $\langle T^\mu_\mu \rangle \sim a E_d + \sum_i c_i I_i$

(d=odd: conjectured analogs, from sphere partition function / entanglement entropy.)

- Additional power from supersymmetry.

6d a-theorem?

For spontaneous conf'l symm breaking: dilaton has derivative interactions to give Δa anom matching Schwimmer, Theisen; Komargodski, Schwimmer

6d case: $\mathcal{L}_{\text{dilaton}} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6}$ (schematic)

Maxfield, Sethi; Elvang, Freedman, Hung, Kiermaier, Myers, Theisen.

Can show that b>0 (b=0 iff free) but b's physical interpretation was unclear; no conclusive restriction on sign of Δa

Elvang et. al. also observed that, for case of (2,0) on Coulomb branch,

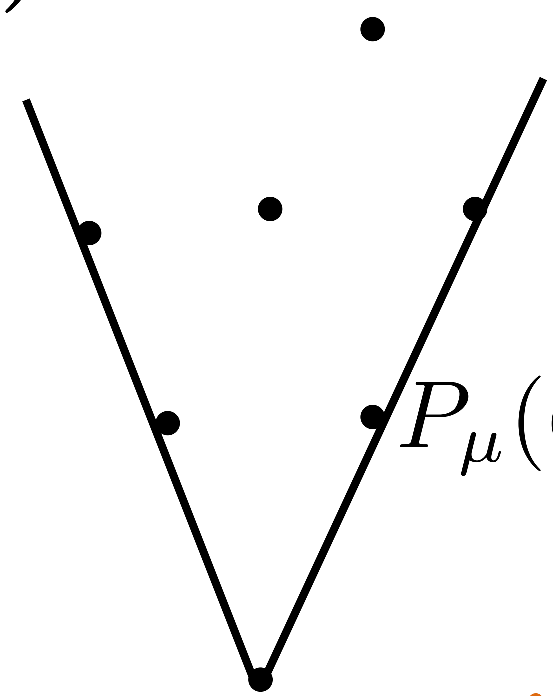
$$\Delta a \sim b^2 \quad >0.$$

Cordova, Dumitrescu, KI: this is a general req't of N=(1,0) susy, and b is related to an 't Hooft anomaly matching term.

CFTs, first w/o susy

$SO(d,2)$ Operators form representations

$P_\mu \uparrow$ $K_\mu \downarrow$



$P_\mu(\mathcal{O}_\mathcal{R})$ descendants= total derivatives, such deformations are **trivial**.

$\mathcal{O}_\mathcal{R}$ **primary** $K_\mu(\mathcal{O}_\mathcal{R}) = 0$

Unitarity: primary + all descendants must have + norm, e.g.

$$\left|P_\mu|\mathcal{O}\rangle\right|^2 \sim \langle\mathcal{O}|[K_\mu, P_\mu]|\mathcal{O}\rangle \geq 0$$

$$[P_\mu, K_\nu] \sim \eta_{\mu\nu} D + M_{\mu\nu}$$

Zero norm, null states if unitarity bounds saturated. E.g. conserved currents, or free fields. “Short” reps.

Classification of SCFT algebras= super-algebras:

Nahm '78

$d > 6$	no SCFTs can exist	
$d = 6$	$OSp(6,2\|\mathcal{N}) \supset SO(6,2) \times Sp(\mathcal{N})_R$	$(\mathcal{N},0)$ $8\mathcal{N}Qs$
$d = 5$	$F(4) \supset SO(5,2) \times Sp(1)_R$	8Qs
$d = 4$	$Su(2,2\|\mathcal{N} \neq 4) \supset SO(4,2) \times SU(\mathcal{N})_R \times U(1)_R$	$4\mathcal{N}Qs$
$d = 4$	$PSU(2,2\|\mathcal{N} = 4) \supset SO(4,2) \times SU(4)_R$	
$d = 3$	$OSp(4\|\mathcal{N}) \supset SO(3,2) \times SO(\mathcal{N})_R$	$2\mathcal{N}Qs$
$d = 2$	$OSp(2\|\mathcal{N}_L) \times OSp(2\|\mathcal{N}_R)$	$\mathcal{N}_L Qs + \mathcal{N}_R \bar{Q}s$

Unitary SCFTs: operators in unitary reps of the s-algs

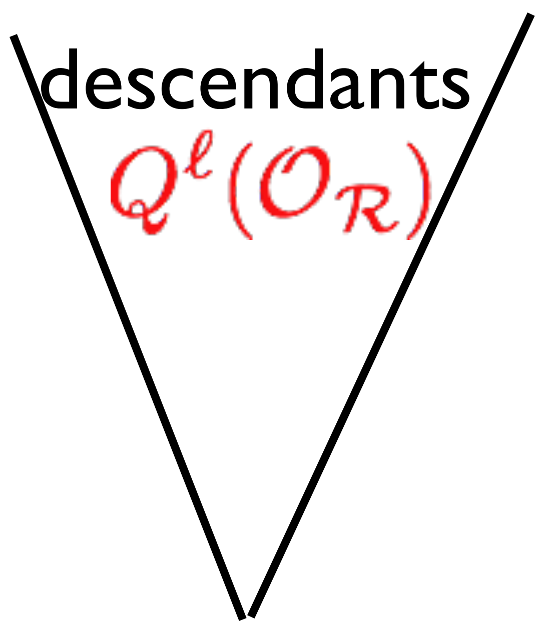
Dobrev and Petkova PLB '85 for 4d case. Shiraz Minwalla '97 for all d=3,4,5,6.

$P_\mu \uparrow \quad K_\mu \downarrow$

Q $\uparrow$ S $\downarrow$

$\{Q,Q\} = 2P_\mu$

$\{S,S\} = 2K_\mu$



$\left| P_{\mu_1} \ldots P_{\mu_\ell} |\mathcal{O}_\mathcal{R}\rangle \right|^2 \overset{!}{\geq} 0$

$\left| Q_1 \ldots Q_\ell |\mathcal{O}_\mathcal{R}\rangle \right|^2 \overset{!}{\geq} 0$

give CFT and SCFT unitarity bounds.

Bounds saturated for "short" multiplets. Have null ops, set to 0.

$\mathcal{O}_\mathcal{R}$ super-primary $S(\mathcal{O}_\mathcal{R}) = 0$

$\{Q,Q\} \sim P_\mu \sim 0$ modulo conf'l descendants.

Grassmann algebra.

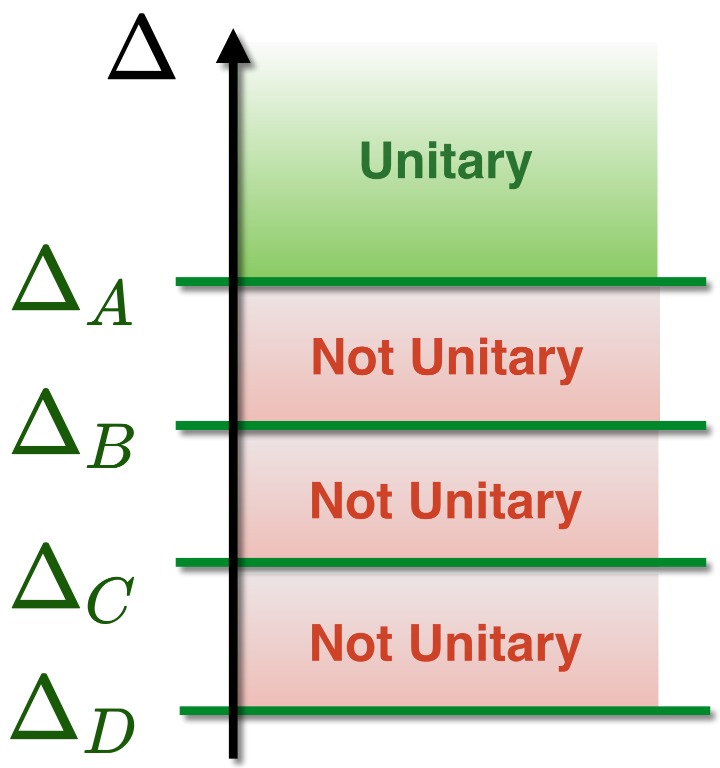
Level $Q^{\wedge\ell}(\mathcal{O}_\mathcal{R}) \quad \ell = 0 \ldots \ell_{max} \leq N_Q$

Multiplet is "long" iff

$\ell_{max} = N_Q$

otherwise, it's "short"

Unitarity constraints:



long

A-type, short at threshold.

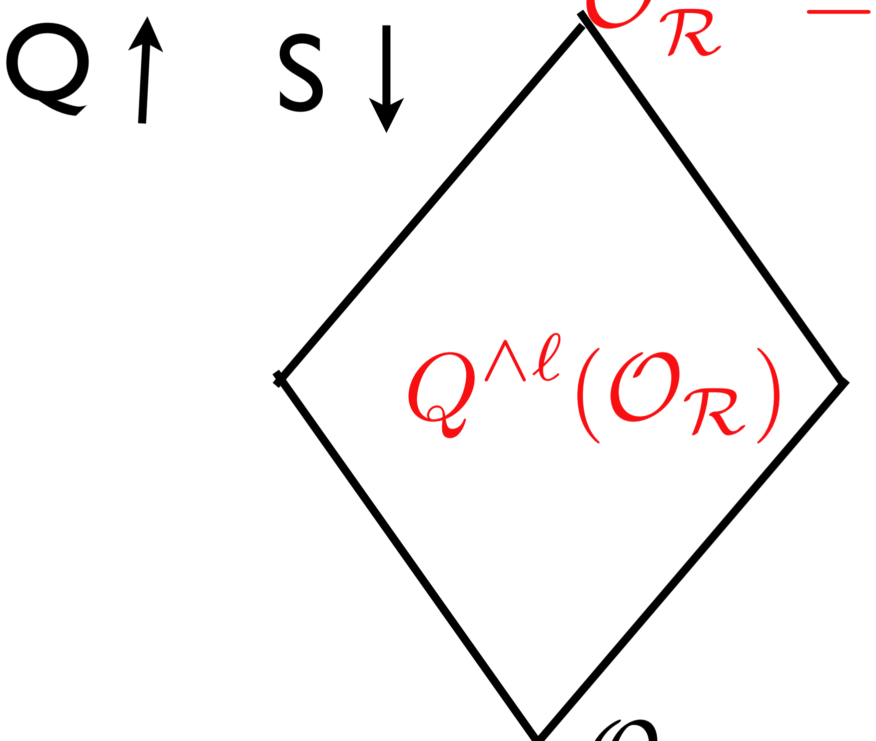
B-type, separated by gap.

$$\Delta_{A,B,C,D} = f(L_{\mathcal{V}}) + g(R_{\mathcal{V}}) + \delta_{A,B,C,D}$$

E.g. in d=6: $f(L) = \frac{1}{2}(j_1 + 2j_2 + 3j_3)$ $\quad g(R) = 2R$

$$\delta_{A,B,C,D} = 6, 4, 2, 0$$

Long generic multiplets:



$Q(\mathcal{O}_{\mathcal{R}}^{\rm top}) \sim 0$ *

modulo descendants

$\binom{N_Q}{\ell} d_{\mathcal{O}_{\mathcal{R}}}$ conformal primary ops at level *l*, $2^{N_Q} d_{\mathcal{O}_{\mathcal{R}}}$ total

$\mathcal{O}_{\mathcal{R}}$ super-primary $S(\mathcal{O}_{\mathcal{R}}) = 0$

Can generate multiplet from bottom up, via Q,or from top down, via S. Reflection symmetry. Unique op at bottom, so unique op at the top. Operator at top = susy preserving deformation (irrelevant for all d and N except for 3d, N=1) if Lorentz scalar. D-terms. Easy case.

Classify SCFT multiplets and all susy deformations

$Q(\mathcal{O}_{\mathcal{R}}^{\text{top}}) \sim 0$

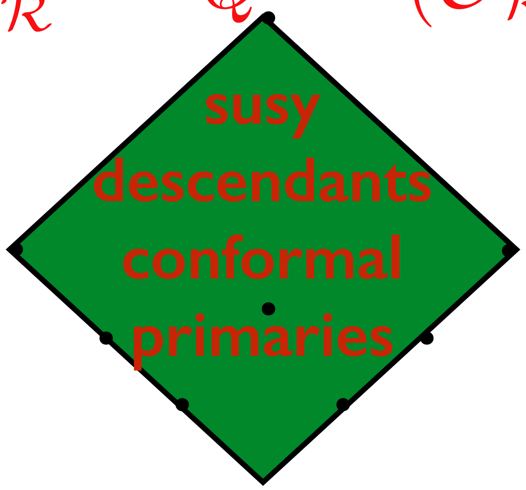
Cordova, Dumitrescu, KI

$\{Q, Q\} \sim P \sim 0$

D: $\mathcal{O}_{\mathcal{R}}^{\text{top}} = Q^{\wedge N_Q}(\mathcal{O}_{\mathcal{R}})$

S ↓

Q ↑

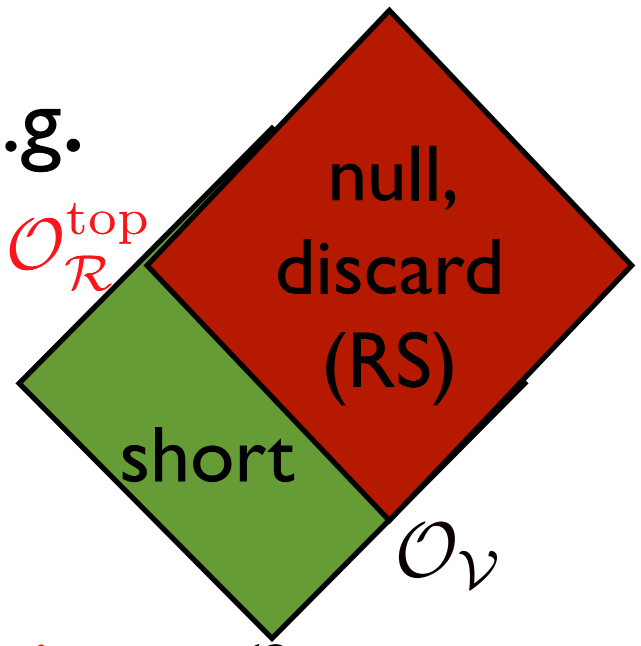


primary: $\mathcal{O}_{\mathcal{R}}$

Generic long =
"straightforward"

E.g.

F: $\mathcal{O}_{\mathcal{R}}^{\text{top}}$



$\mathcal{O}_{\mathcal{V}}$

primary: $\mathcal{O}_{\mathcal{R}}$

Generic short =
"proceed with caution"

Non-Generic Short
(small R-symm quant #s)

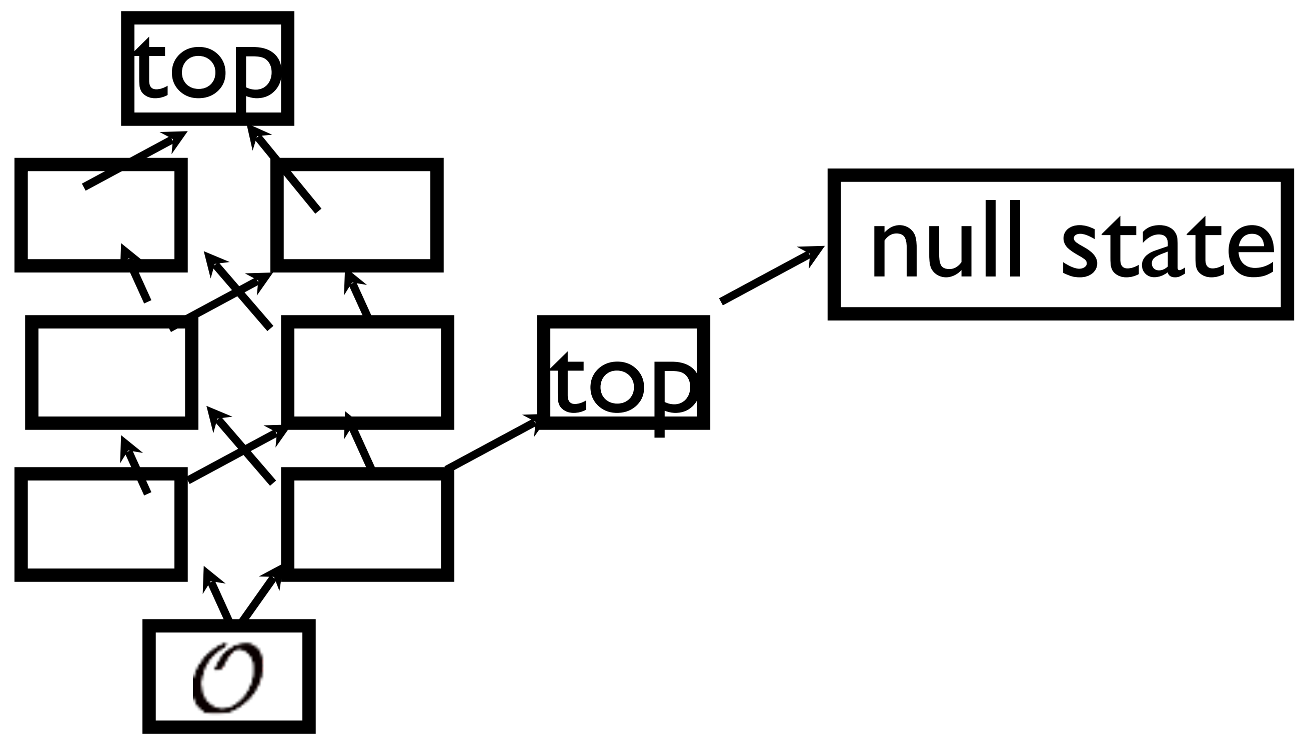
= a **ZOO** of sporadic cases.
E.g. Dolan + Osborn for some 4d N=2,4 cases.
We analyzed algorithms to eliminate only nulls; many problems. Non-trivial. We conjecture and test a general algorithm.

We then find the op. dim. constraints on the top components.
As we increase d or N, fewer or none relevant deformations.

Exotic zoo: e.g. cases (d=3) with mid-level susy top

(Find two, and multi-headed animals in the multiplet zoo)

Q ↑   S ↓



E.g. 3d $\mathcal{N} \geq 4$ $T_{\mu\nu}$ multiplet: the stress-tensor is at top, at level 4.

Another top, at level 2, Lorentz scalar. Gives susy-preserving "universal mass term" relevant deformations. First found in 3d N=8 (KI '98, Bena & Warner '04; Lin & Maldacena '05). Special to 3d. Indeed, they give a deformed susy algebra that is special to 3d (non-central extension).

Algorithm for mults.

Operators in reps of the algebra: $\mathfrak{G}(d,\mathcal{N}) \supset \mathfrak{so}(d,2) \oplus \mathfrak{R} \supset \mathfrak{so}(d) \oplus \mathfrak{R}$

We label the multiplets as: $\mathcal{M} = X_\ell[L_\mathcal{V}]^{(R_\mathcal{V})}_{\Delta_\mathcal{V}}$ $X \in \{L, A, B, C, D\}$

Group theory of the Lorentz and R-symmetry reps of the ops in the multiplet: $\wedge^\ell \mathcal{R}_Q \otimes \mathcal{V}$. Bypass full Clebsch-Gordon decomposition via the Racah Speiser algorithm. Important technical simplification, but also leads to some complications, esp. for operators with low R-symmetry reps. in properly eliminating the null multiplet, without e.g. over-subtracting. Our algorithm is inspired by some in prior literature, esp that of Dolan and Osborn for 4d N=2 and N=4. We find the previous algorithms fail in various exotic cases. Ours is conjectural but highly tested, and applicable for all d and N, as far as we know.

(Racah Speiser)

$$\lambda^{(1)} \otimes \lambda^{(2)} = \oplus_{a=1}^{\dim \lambda^{(2)}} (\lambda^{(1)} + \mu_a^{(2)})|_{RS} = \oplus_{a=1}^{\dim \lambda^{(2)}} (\lambda^{(1)} + \mu_a^{(1)})|_{RS}$$

$$(\lambda^{(1)} + \mu_a^{(2)})|_{RS} = \chi \tilde{\lambda} \qquad \chi = \pm 1, 0$$

Weyl reflect weight to fundamental Weyl chamber.

$$[\lambda_1, \ldots, \lambda_r]^{\sigma_i} = -[\lambda_1, \ldots \lambda_r] = \sigma_i([\lambda_1, \ldots, \lambda_r] + \rho) - \rho$$

E.g. SU(2): $[-\lambda_1] = -[\lambda_1 - 2]$ so $[-1] = 0$ and $[-2] = -[0]$

E.g. $[0] \otimes [2] = [2] \oplus [0] \oplus [-2] = [2]$

Apply to both Lorentz and R-symmetry. But obscures the Q action and subtractions have subtle cases, including leftover negative states, we propose how to handle them.

We give a complete classification for d=3,4,5,6

Detailed tour of the zoo of all multiplets, including a full picture of the various possible exotic short multiplets.

The complete classification of all susy-preserving deformations. They can be the start (if relevant) or end (if irrelevant) of susy RG flows between SCFTs, analyzed near the UV or IR SCFT fixed points. We also classify absolutely protected multiplets and all multiplets with conserved currents (incl higher spin) and free fields. Some CFT possibilities cannot appear in SCFTs, e.g. in 6d, no conserved 2-form current $j_{\mu\nu}$ (!)

E.g. d=4, N=3 SCFTs (all irrelevant)

Primary $\mathcal{O}$	Deformation $\delta\mathscr{L}$	Comments
$B_1\overline{B}_1\left\{\begin{matrix}(R_1+4,0\,;\,2R_1+8)\\ \Delta_{\mathcal{O}}=4+R_1\end{matrix}\right\}$	$Q^4\overline{Q}^2\mathcal{O}\in\left\{\begin{matrix}(R_1,0\,;\,2R_1+6)\\ \Delta=7+R_1\end{matrix}\right\}$	F-Term $(*)$
$B_1\overline{B}_1\left\{\begin{matrix}(0,R_2+4\,;\,-2R_2-8)\\ \Delta_{\mathcal{O}}=4+R_2\end{matrix}\right\}$	$Q^2\overline{Q}^4\mathcal{O}\in\left\{\begin{matrix}(0,R_2\,;\,-2R_2-6)\\ \Delta=7+R_2\end{matrix}\right\}$	F-Term $(*)$
$B_1B_1\left\{\begin{matrix}(R_1+2,R_2+2\,;\,2(R_1-R_2))\\ \Delta_{\mathcal{O}}=4+R_1+R_2\end{matrix}\right\}$	$Q^4Q^4\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2\,;\,2(R_1-R_2))\\ \Delta=8+R_1+R_2\end{matrix}\right\}$	–
$L\overline{B}_1\left\{\begin{matrix}(0,0\,;\,r+6)\ ,\ r>0\\ \Delta_{\mathcal{O}}=1+\frac{1}{6}r\end{matrix}\right\}$	$Q^6\mathcal{O}\in\left\{\begin{matrix}(0,0\,;\,r)\ ,\ r>0\\ \Delta=4+\frac{1}{6}r>4\end{matrix}\right\}$	F-term $(\star)$
$B_1\overline{L}\left\{\begin{matrix}(0,0\,;\,r-6)\ ,\ r<0\\ \Delta_{\mathcal{O}}=1-\frac{1}{6}r\end{matrix}\right\}$	$\overline{Q}^6\mathcal{O}\in\left\{\begin{matrix}(0,0\,;\,r)\ ,\ r<0\\ \Delta=4-\frac{1}{6}r>4\end{matrix}\right\}$	F-Term $(\star)$
$L\overline{B}_1\left\{\begin{matrix}(R_1+2,0\,;\,r+4)\ ,\ r>2R_1+6\\ \Delta_{\mathcal{O}}=2+\frac{2}{3}R_1+\frac{1}{6}r\end{matrix}\right\}$	$Q^6\overline{Q}^2\mathcal{O}\in\left\{\begin{matrix}(R_1,0\,;\,r)\ ,\ r>2R_1+6\\ \Delta=6+\frac{2}{3}R_1+\frac{1}{6}r>7+R_1\end{matrix}\right\}$	$(\dagger)$
$B_1\overline{L}\left\{\begin{matrix}(0,R_2+2\,;\,r-4)\ ,\ r<-2R_2-6\\ \Delta_{\mathcal{O}}=2+\frac{2}{3}R_2-\frac{1}{6}r\end{matrix}\right\}$	$Q^2\overline{Q}^6\mathcal{O}\in\left\{\begin{matrix}(0,R_2\,;\,r)\ ,\ r<-2R_2-6\\ \Delta=6+\frac{2}{3}R_2-\frac{1}{6}r>7+R_2\end{matrix}\right\}$	$(\dagger)$
$L\overline{B}_1\left\{\begin{matrix}(R_1,R_2+2\,;\,r+2)\ ,\ r>2(R_1-R_2)\\ \Delta_{\mathcal{O}}=3+\frac{2}{3}(R_1+2R_2)+\frac{1}{6}r\end{matrix}\right\}$	$Q^6\overline{Q}^4\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2\,;\,r)\ ,\ r>2(R_1-R_2)\\ \Delta=8+\frac{2}{3}(R_1+2R_2)+\frac{1}{6}r>8+R_1+R_2\end{matrix}\right\}$	$(\ddagger)$
$B_1\overline{L}\left\{\begin{matrix}(R_1+2,R_2\,;\,r-2)\ ,\ r<2(R_1-R_2)\\ \Delta_{\mathcal{O}}=3+\frac{2}{3}(2R_1+R_2)-\frac{1}{6}r\end{matrix}\right\}$	$Q^4\overline{Q}^6\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2\,;\,r)\ ,\ r<2(R_1-R_2)\\ \Delta=8+\frac{2}{3}(2R_1+R_2)-\frac{1}{6}r>8+R_1+R_2\end{matrix}\right\}$	$(\ddagger)$
$L\overline{L}\left\{\begin{matrix}(R_1,R_2\,;\,r)\\ \Delta_{\mathcal{O}}>2+\max\left\{\begin{matrix}\frac{2}{3}(2R_1+R_2)-\frac{1}{6}r\\ \frac{2}{3}(R_1+2R_2)+\frac{1}{6}r\end{matrix}\right\}\end{matrix}\right\}$	$Q^6\overline{Q}^6\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2\,;\,r)\\ \Delta>8+\max\left\{\begin{matrix}\frac{2}{3}(2R_1+R_2)-\frac{1}{6}r\\ \frac{2}{3}(R_1+2R_2)+\frac{1}{6}r\end{matrix}\right\}\end{matrix}\right\}$	D-Term

Table 25: Deformations of four-dimensional $\mathcal{N}=3$ SCFTs. The $\mathfrak{su}(3)_R$ Dynkin labels $R_1, R_2\in\mathbb{Z}_{\geq 0}$ and the $\mathfrak{u}(1)_R$ charge $r\in\mathbb{R}$ denote the R-symmetry representation of the deformation.

40

Maximal susy

In d=6,4,3,(+2), superconformal algebras exist for any N. Free-field methods (particle spectrum) show that there are higher-spin particles if more than 16 supercharges. Question: Can this be evaded with interacting SCFTs? We show that the answer is no. For d=4 and d=6, the algebra for more than 16 Qs has a short multiplet with a conserved stress-tensor, but it is not Q closed (mod P). Also higher spin currents. Free theory with wrong algebra. For d=3, stress-tensor is a mid-level "top" operator for all N, and higher spin currents. So d=3 has free field realizations for any N, no upper bound.

6d SCFTs and QFTs

- We show: **no** susy relevant operator deformations of 6d SCFTs. Only "flow" via going out along the moduli space. Theory flows to new, low-energy SCFT +(irrelevant ops).
- Spontaneous conformal breaking: low-energy theory contains the massless dilaton w/ irrelevant interactions.
- Global symmetries have 6d analog of 't Hooft anomaly matching conditions. The 't Hooft anomalies can often be exactly computed, e.g. by inflow or other methods.
- Anomaly matching for broken symmetries (via NG bosons) requires certain interactions in the low energy thy, like the WZW term but totally different in the details for 6d vs 4d.
- 6d a-theorem? Unclear w/o susy. We proved it for susy flows on the Coulomb branch. To appear: Higgs branch.

Anomaly polynomial

Alvarez-Gaume, Witten; Alvarez-Gaume+Ginsparg.

$$\mathcal{I}_{d+2} = \mathcal{I}_{d+2}^{\text{gauge}} + \mathcal{I}_{d+2}^{\text{gravity+global}} + \mathcal{I}_{d+2}^{\text{mixed}}$$

- $\mathcal{I}_{d+2}^{\text{gauge}}$: Must cancel, restricts G & matter.
- $\mathcal{I}_{d+2}^{\text{gravity+global}}$: ``'t Hooft anomalies" Const. on RG flows. Matching. Useful!
- $\mathcal{I}_{d+2}^{\text{mixed}}$: Mostly neglected. We argue must cancel for 6d SCFTs: since no $j_{\mu\nu}$

E.g. 6d: N=(2,0):

$$\mathcal{I}_g = r_g \mathcal{I}_{u(1)} + \frac{k_g}{24} p_2(F_{SO(5)_R})$$

- g: A,D,E group G
- $\mathcal{I}_{u(1)}$: Free (2,0) tensor mult
- $\frac{k_g}{24} p_2(F_{SO(5)_R})$: Interaction part

Duff, Liu, Minasian; Witten; Freed, Harvey, Minasian, Moore

N M5s+inflow: $k_{su(N)} = N^3 - N$ — Harvey, Minasian, Moore

Other methods: $k_g = h_g^\vee d_g$ — KI; Yi; Ohmori, Shimizu, Tachikawa, Yonekura. See also Ki-Myeong Lee et. al.

Exact info about mysterious SCFTs (and "L"STs).

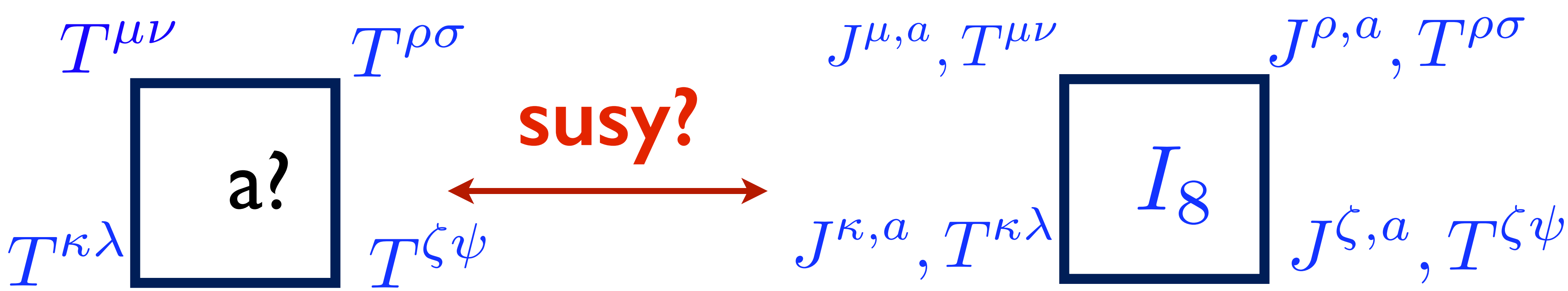
Longstanding hunch

e.g. Harvey Minasian, Moore '98

Supersymmetric multiplet of anomalies: should be able to relate conformal anomaly a to 't Hooft-type anomalies for the superconformal R-symmetry in 6d, as in 2d and 4d.

$T^{\mu\nu} \leftrightarrow J_R^{\mu,a}$ Stress-tensor supermultiplet

$g_{\mu\nu} \leftrightarrow A^a_{R,\mu}$ Source: bkgrd SUGRA supermultiplet



4-point fn with too many indices. Hard to get a (and to compute).

Easier to isolate anomaly term, and enjoys anomaly matching

On the moduli space

Need to supersymmetrize the dilaton LEEFT

Spontaneous conf'l symm breaking: dilaton has derivative interactions to give Δa anom matching Schwimmer, Theisen; Komargodski, Schwimmer

6d case: $$\mathcal{L}_{\text{dilaton}} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6}$$

(schematic; derivative order shown)

Maxfield, Sethi; Elvang, Freedman, Hung, Kiermaier, Myers, Theisen. b>0, but what is it good for? Interpretation? Clue: noticed that for N=(2,0) on Coulomb branch:

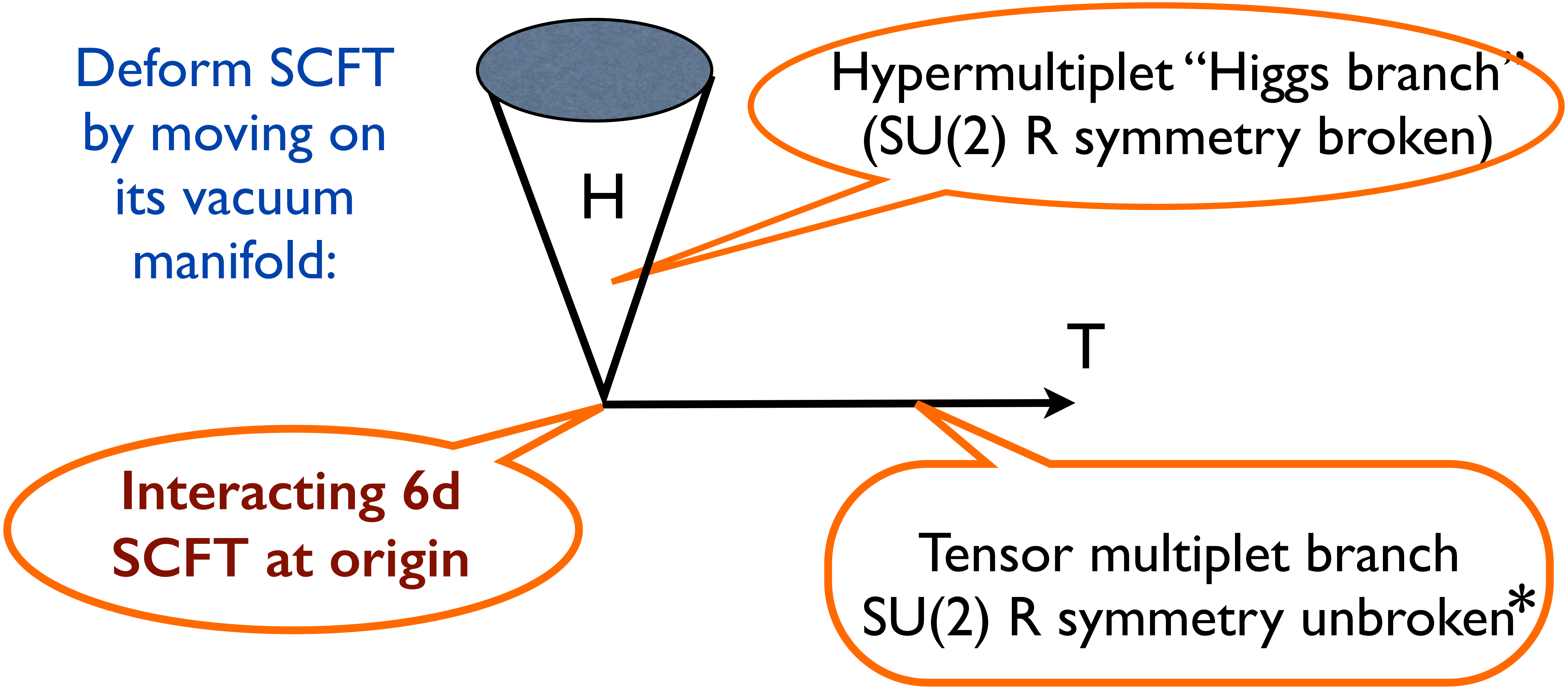
$$\Delta a \sim b^2$$

M&S: via (2,0) susy; EFHKMT: some scatt. amplitudes then, fits with AdS/CFT.

Cordova, Dumitrescu, Yin: proved it using (2,0) methods. Our parallel work proves for general (1,0) theories on Coulomb branch.

6d (1,0) susy moduli

Deform SCFT by moving on its vacuum manifold:



* Easier case. Just dilaton, no NG bosons. Dilaton = tensor multiplet.

(1,0) 't Hooft anomalies

$$\mathcal{I}_8^{\text{gravity+global}} \supset \alpha c_2(R)^2 + \beta c_2(R) p_1(T) + \gamma p_1^2(T) + \delta p_2(T)$$

Exactly computed for many (1,0) SCFTs

Ohmori, Shimizu, Tachikawa; & +Yonekura (OST, OSTY)

E.g. for theory of N small E8 instantons:

$$\mathcal{E}_N: \ (\alpha, \beta, \gamma, \delta) = (N(N^2+6N+3), -\frac{N}{2}(6N+5), \frac{7}{8}N, -\frac{N}{2})$$

N=(1,0) tensor branch anomaly matching:

KI; Ohmori, Shimizu, Tachikawa, Yonekura

$$\Delta \mathcal{I}_8 \equiv \mathcal{I}_8^{\text{origin}} - \mathcal{I}_8^{\text{tensor branch}} \sim X_4 \wedge X_4$$

must be a perfect square, match I_8 via X_4 sourcing B:

$$\mathcal{L}_{GSWS} = -iB \wedge X_4$$

$$X_4 \equiv 16\pi^2 (x c_2(R) + y p_1(T))$$

x, y = integer coefficients

6d (1,0) tensor LEEFT

Elvang et. al.

$$\mathcal{L}_{\text{dilaton}} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6}$$

Our deformation classification implies that b=D-term and

$$\Delta a = \frac{98304\pi^3}{7}b^2 > 0$$ **Proves the 6d a-theorem** for susy tensor branch flows.

b-term susy-completes to terms in $X_4 = \sqrt{\Delta I_8}$

b=(y-x)/2

$$X_4 \equiv 16\pi^2(xc_2(R) + yp_1(T))$$

By recycling a 6d SUGRA analysis from Bergshoeff, Salam, Sezgin '86 (!).

Upshot:

$$\boxed{a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta}$$

So exact 't Hooft anomaly coefficients give the exact conformal anomaly, useful! E.g. using this and **OST** for the anomalies:

$$a(\mathcal{E}_N) = \frac{64}{7}N^3 + \frac{144}{7}N^2 + \frac{99}{7}N$$

(N M5s @ M9 Horava-Witten wall.)

a-theorem, and sign, for theories with gauge fields

A free gauge field not conformal for d>4. It is unitary, but it can be regarded as a subsector of a non-unitary CFT.
El-Showk, Nakayama, Rychkov

Applying our formula to a free (1,0) vector multiplet gives

$$a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta \quad \rightarrow \quad a(\text{vector}) = -\frac{251}{210}$$

negative…same value later found for non-unitary, higher derivative (1,0) SCFT version by Beccaria & Tseytlin

We argue that unitary SCFTs satisfy the a-theorem and have a>0 even if they have vector multiplets (more, in an upcoming paper).

a, for 6d SCFTs with gauge flds:

E.g. SU(N) gauge group, 2N flavors, 1 tensor + anomaly cancellation for reducible gauge + mixed gauge + R-symmetry anomalies. Use $a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta$

V H T AC **:interactions**

$$a_{SCFT} = (N^2-1)(-\frac{251}{210}) + 2N^2(\frac{11}{210}) + \frac{199}{210} + \frac{96}{7}N^2 > 0.$$

Likewise verify that other generalizations have positive a.
Also, that Higgs branch flows satisfy the 6d a-theorem.

Conclude

- QFT is vast, expect still much to be found.
- susy QFTs and RG flows are rich, useful testing grounds for exploring QFT. Strongly constrained: unitarity, a-thm., etc. Can rule out some things. Exact results for others.
- Thank you !