6D SCFTs and Group Theory

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Based On

- 1502.05405/hep-th
  - with Jonathan Heckman, David Morrison, and Cumrun Vafa
- 1506.06753/hep-th
  - with Jonathan Heckman
- 1601.04078/hep-th
  - with Jonathan Heckman, Alessandro Tomasiello
- 1605.08045/hep-th
  - with David Morrison
- 1612.06399/hep-th
  - with Noppadol Mekareeya, Alessandro Tomasiello
- work in progress
  - with Fabio Apruzzi, Jonathan Heckman

Outline

I. Classification of 6D SCFTs
   i. Tensor Branches/Strings
   ii. Gauge Algebras/Particles

II. 6D SCFTs and Homomorphisms
   i. $\mathfrak{su}(2) \to \mathfrak{g}_{\mathrm{ADE}}$
   ii. $\Gamma_{\mathrm{ADE}} \to E_8$

III. 6D SCFTs and Automorphism Groups
   i. Automorphism Groups
   ii. Geometric Phases

The Big Picture

What is a 6D SCFT?

- S=supersymmetric (8 or 16 supercharges)
- C=conformal symmetry
- FT=Field theory in 5+1 dimensions

Why Study 6D SCFTs?

- Nahm: Maximal SCFT dimension is *six*
- Degrees of freedom $\neq$ particles (but it's a QFT!)
- QFT of M5-branes is a 6D SCFT
- Compactification $\Rightarrow$ 5D/4D/3D/2D Theories

Focus: $(1,0)$ SCFTs

Conformal Symmetry: $\mathfrak{so}(6,2)$

Supersymmetry: 8 Q's and 8 S's

R-symmetry: $\mathfrak{su}(2)_{\mathcal{R}}$

Studied since the 1990's

Many groups:

Witten '95; Strominger '95; Ganor and Hanany '96;

Seiberg and Witten '96; Bershadsky and Johansen '96;

Brunner and Karch '96; Blum and Intriligator '97;

Intriligator '97; Hanany Zaffaroni '97;

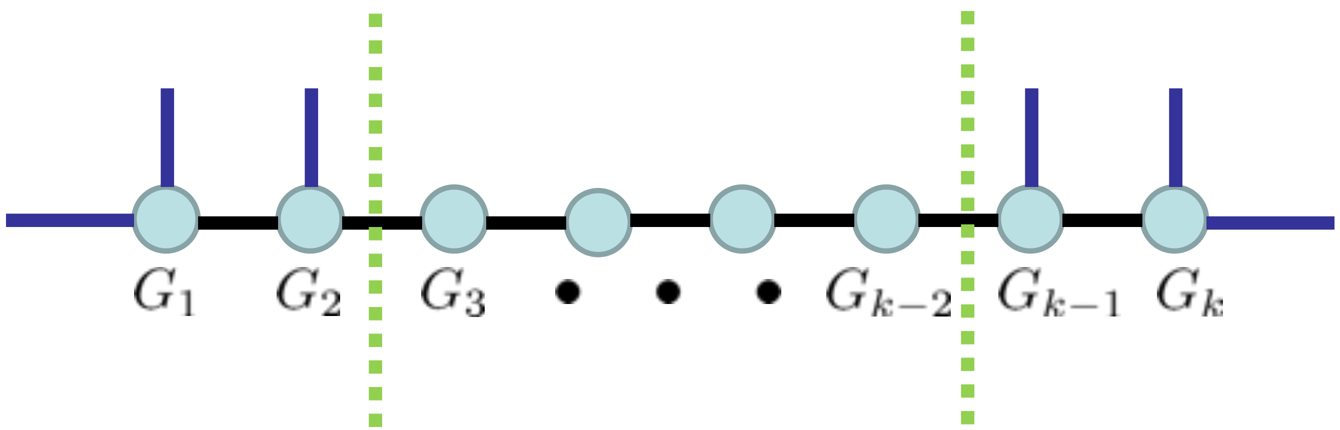
+ ....

But: Even now, still viewed as "mysterious"...

Classification of 6D SCFTs

Classification of 6D SCFTs

- 6D SCFTs can be classified via F-theory
- Nearly all F-theory conditions can be phrased in field theory terms
- 6D SCFTs = Generalized Quivers

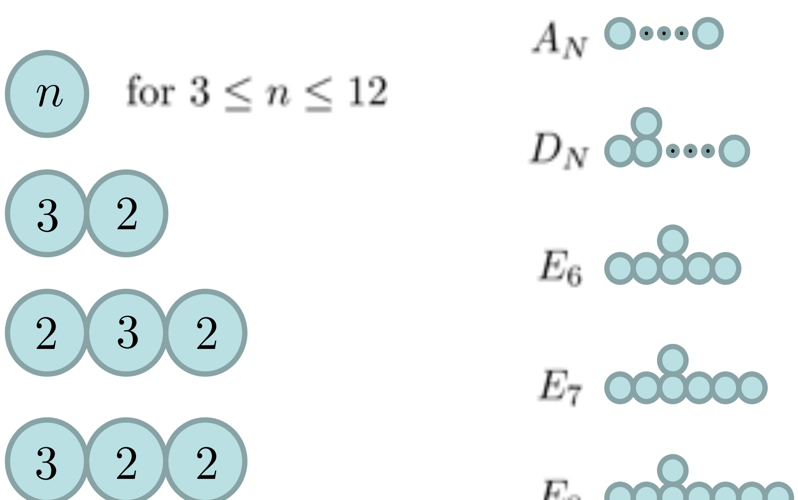


Classification of 6D SCFTs

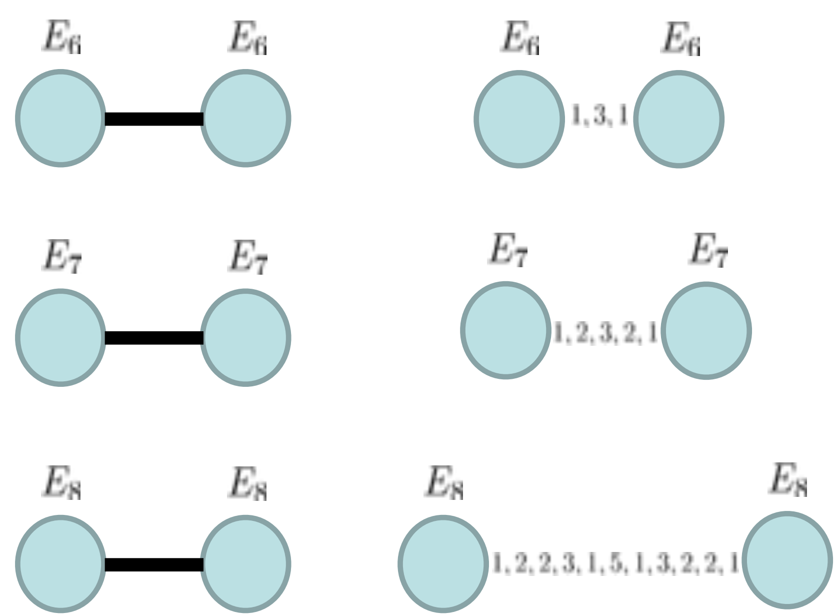
- Looks like chemistry

"Atoms"

c.f. Morrison and Taylor '12



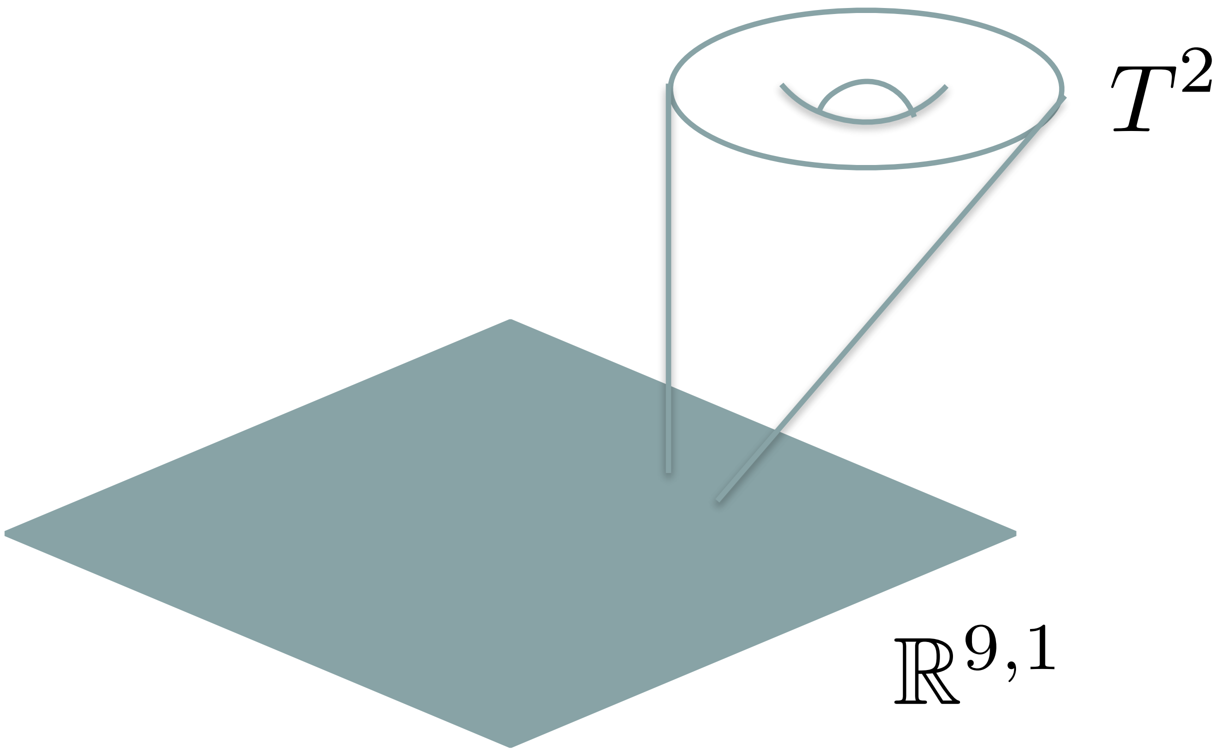
"Radicals"



What is F-theory?

Vafa '96

IIB: $\mathbb{R}^{9,1}$ with position-dependent coupling $\tau = C_0 + ie^{-\Phi}$



6D Theories and F-theory

Vafa '96, Vafa Morrison, I/II '96

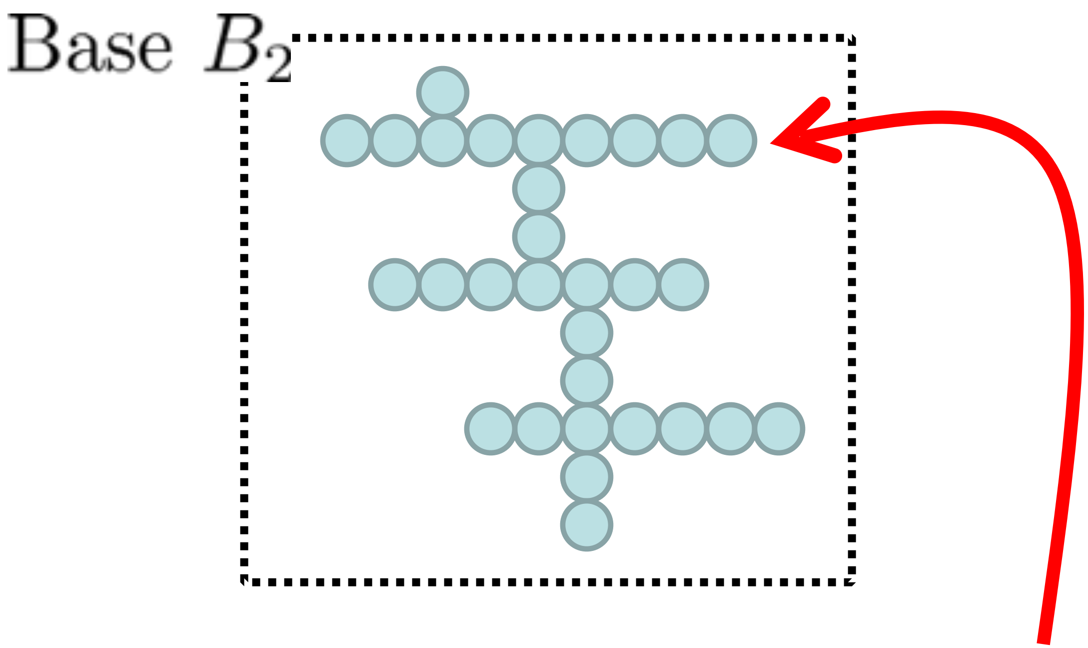
All known 6D theories have F-theory avatar*

IIB: $\mathbb{R}^{5,1} \times B_2$ with pos. dep. coupling $\tau(z_B)$

F-theory on $\mathbb{R}^{5,1} \times CY_3$

$$\begin{array}{c} T^2 \rightarrow CY_3 \\ \downarrow \\ B_2 \end{array}$$

Geometric Picture



Singularities in base $\Rightarrow$ strings (D3 / $\mathbb{P}^1$)

Singularities in fiber $\Rightarrow$ particles (7-brane on $\mathbb{P}^1$)

Tensionless Strings in F-theory

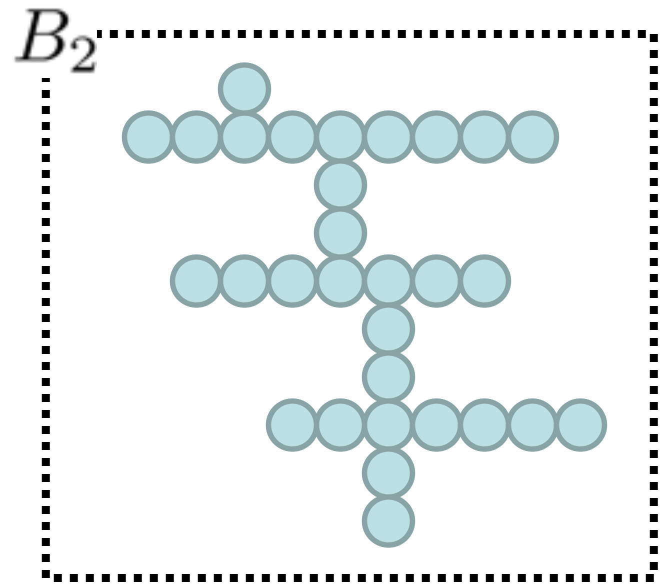
- Realized by D3-brane on collapsing $\mathbb{CP}^1$

  $$\text{Tension} = \text{Vol}(\mathbb{CP}^1) \to 0$$

	$\mathbb{R}^{5,1}$ (0–5)						$\mathbb{CP}^1$ (6–7)			
	0	1	2	3	4	5	6	7	8	9
D3	×	×	•	•	•	•	×	×	•	•

SCFT Limit

Start: A smooth base B_2

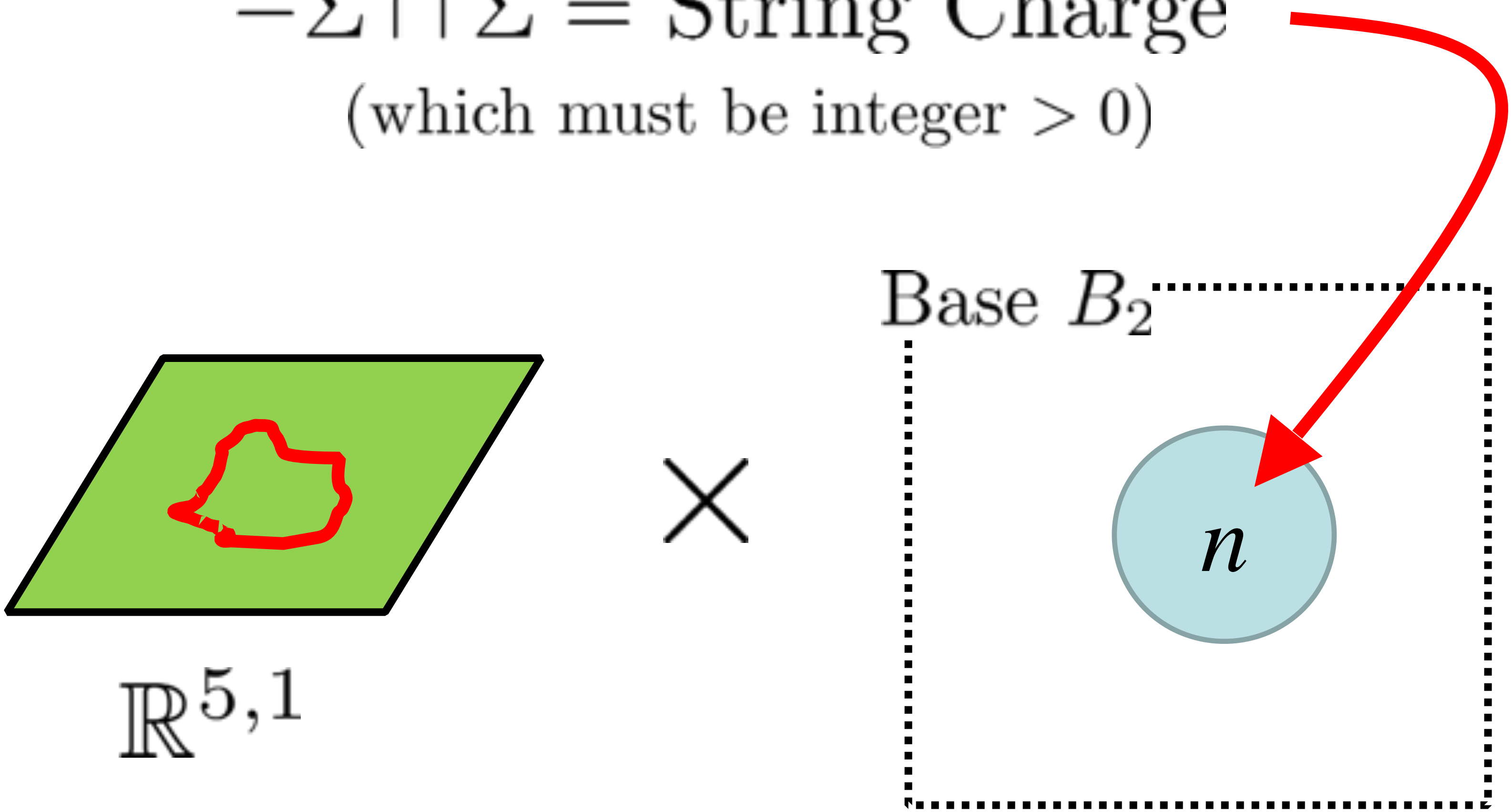


End: To get a CFT, sim. contract curves of B_2

Strings from D3 on a $\mathbb{P}^1$

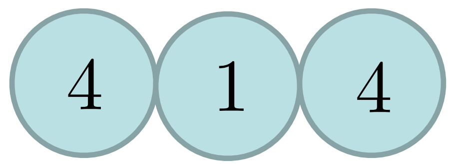
$$-\Sigma \cap \Sigma = \text{String Charge}$$

(which must be integer > 0)



Dirac Pairing for String Charge Lattice

Intersection Matrix $\longleftrightarrow$ Dirac Pairing



$$\Omega_{IJ} = \begin{pmatrix} -4 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -4 \end{pmatrix}$$

Ω_{IJ} negative definite $\Leftrightarrow$ Curves contractible

Example: All $(2,0)$ Theories

Witten '95, Strominger '95

Type IIB on $\mathbb{C}^2/\Gamma_{ADE}$

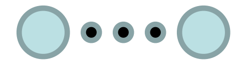
Resolution Involves:

Bouquet of $\mathbb{CP}^1$'s

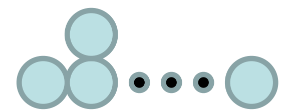
$\mathbb{CP}^1_i \cap \mathbb{CP}^1_j = -\text{Dynkin}_{ij}$

Note: $\mathbb{CP}^1_i \cap \mathbb{CP}^1_i = -2$

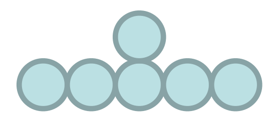
A_N



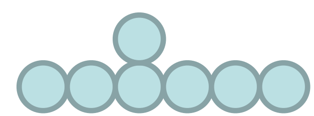
D_N



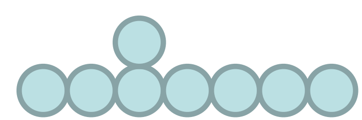
E_6



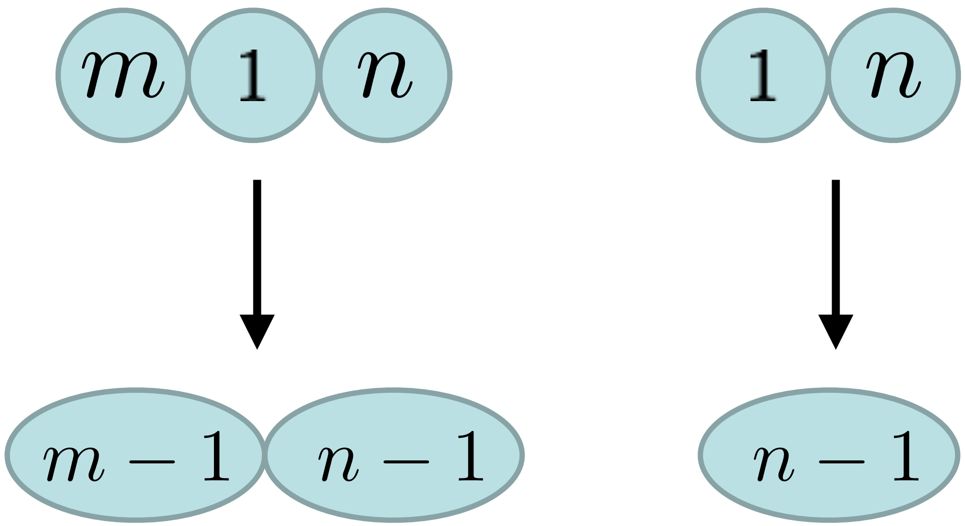
E_7



E_8



Blow-down Operations



Coarse Classification of Bases*

Heckman, Morrison, Vafa '13

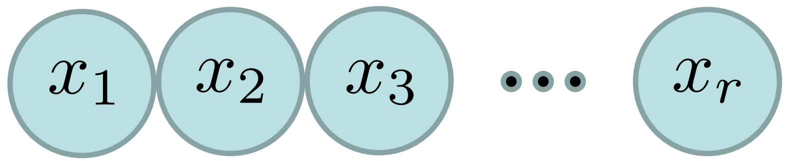
$$(2,0) \text{ SCFT} \Leftrightarrow \Gamma \subset SU(2)$$

$$(1,0) \text{ SCFT} \Rightarrow \Gamma \subset U(2)$$

*Bases related by blow-downs/blow-ups have same $\Gamma \subset U(2)$

Coarse Classification of Bases

Heckman, Morrison, Vafa '13

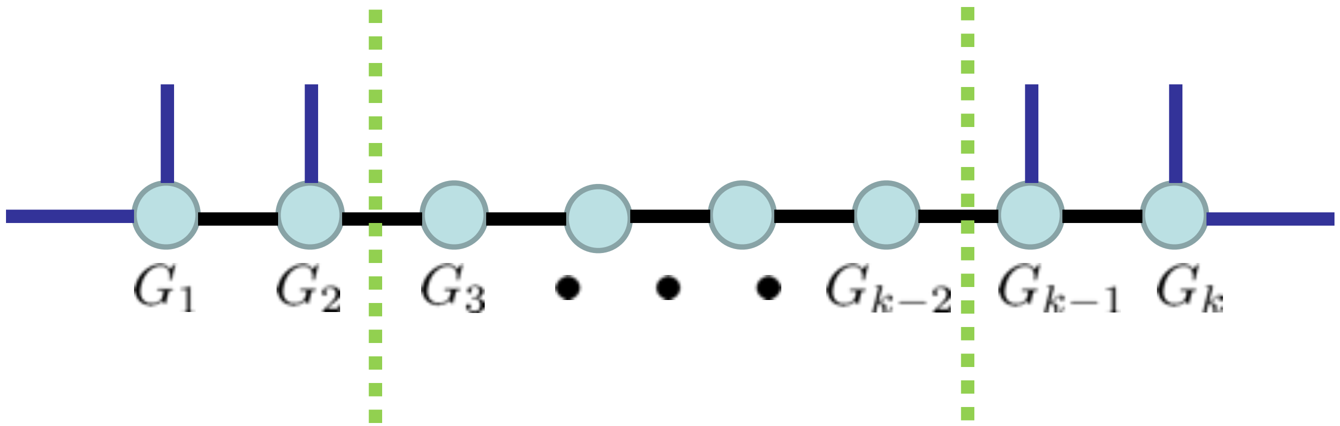


$$\frac{p}{q} = x_1 - \frac{1}{x_2 - \ldots \frac{1}{x_r}}$$

$$\Gamma : (z_1, z_2) \mapsto (e^{2\pi i/p} z_1, e^{2\pi i q/p} z_2) \qquad B_2 = \mathbb{C}^2/\Gamma$$

Complete Classification of Bases

The Base Quivers have a *very* simple structure!

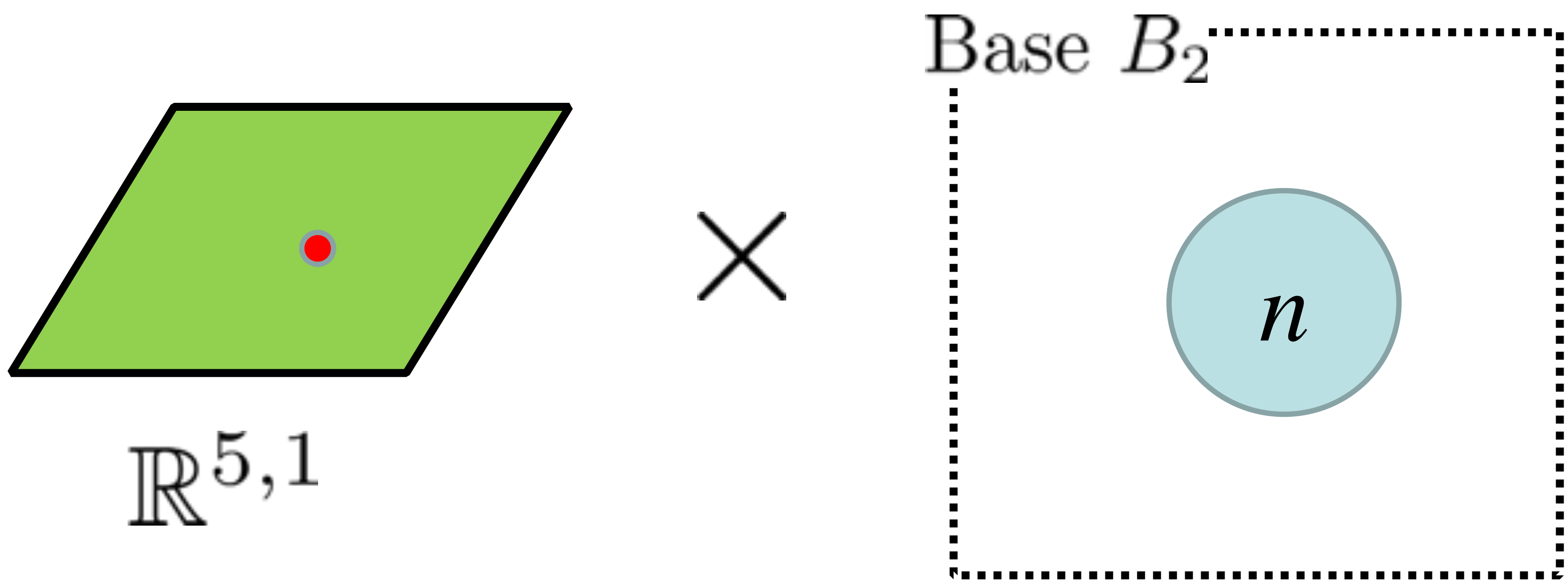


$$G_1 \subseteq G_2 \subseteq \cdots \subseteq G_m \supseteq \cdots \supseteq G_{k-1} \supseteq G_k$$

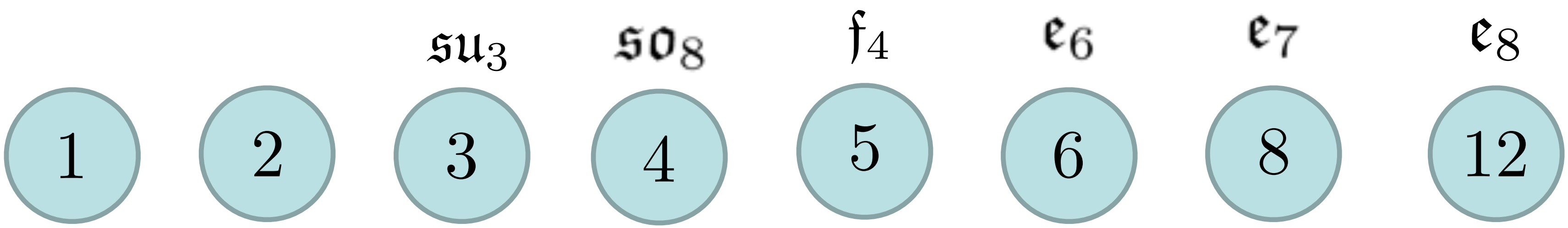
Particles from D7's on a $\mathbb{P}^1$

$3 \leq n \leq 12 \Rightarrow$ always have gauge fields

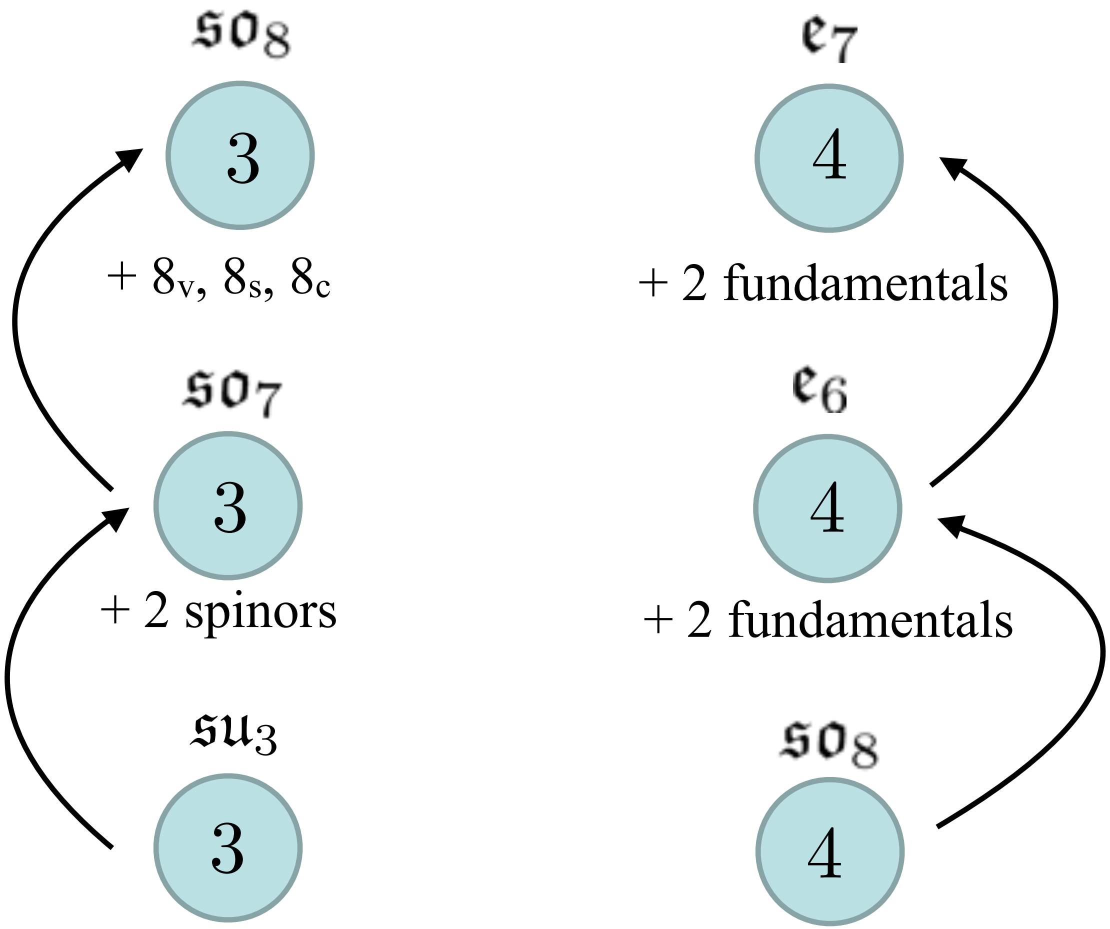
(elliptic fiber is singular: Morrison Taylor '12)



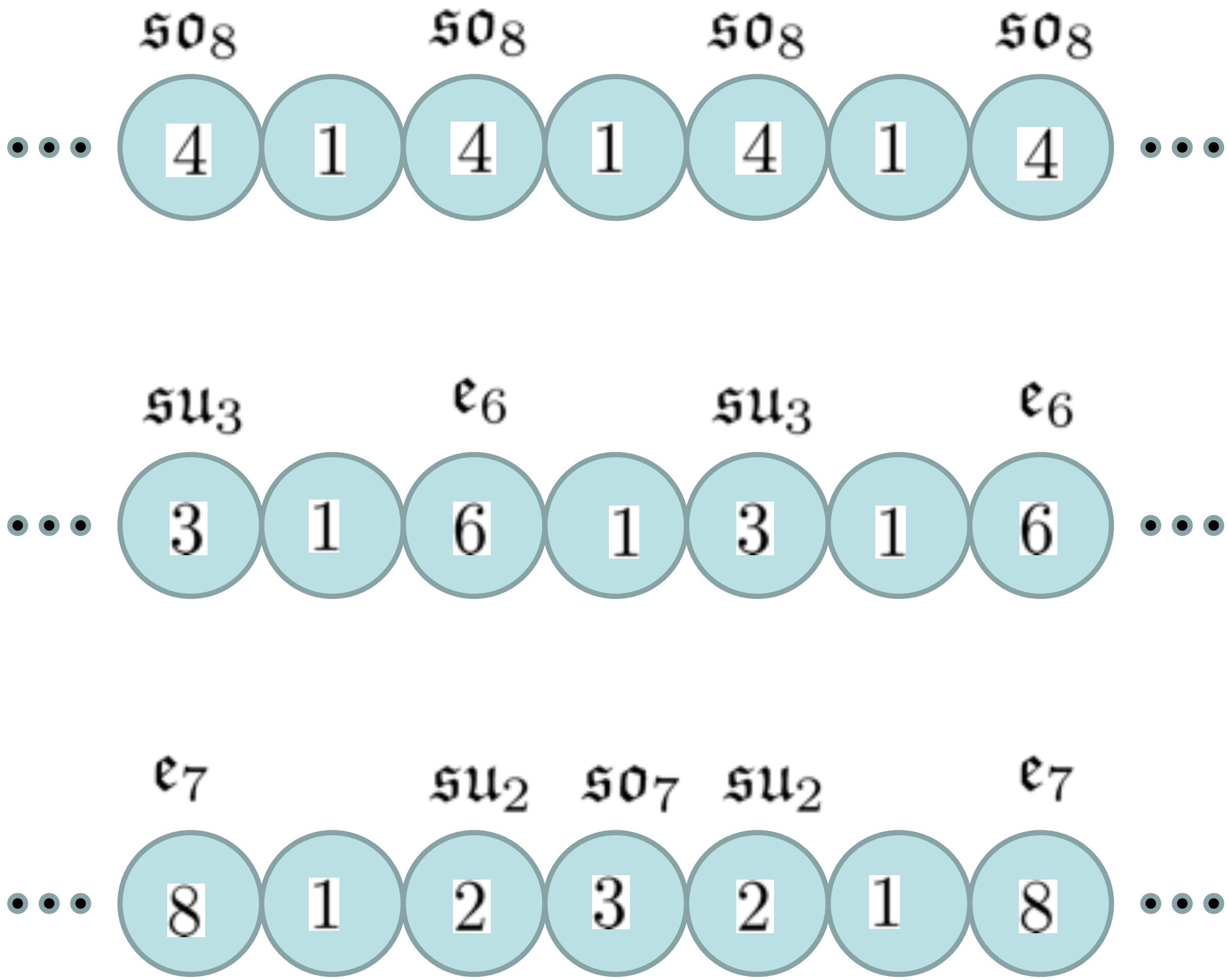
Minimal Gauge Algebras



Fiber Enhancements



Examples

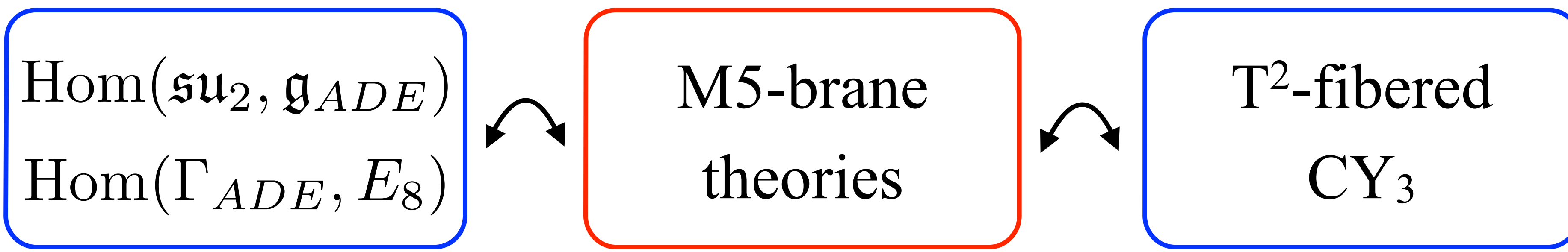


$\mathfrak{so}_8$ $\mathfrak{so}_8$ $\mathfrak{so}_8$ $\mathfrak{so}_8$
4 1 4 1 4 1 4
$\mathfrak{su}_3$ $\mathfrak{e}_6$ $\mathfrak{su}_3$ $\mathfrak{e}_6$
3 1 6 1 3 1 6
$\mathfrak{e}_7$ $\mathfrak{su}_2$ $\mathfrak{so}_7$ $\mathfrak{su}_2$ $\mathfrak{e}_7$
8 1 2 3 2 1 8

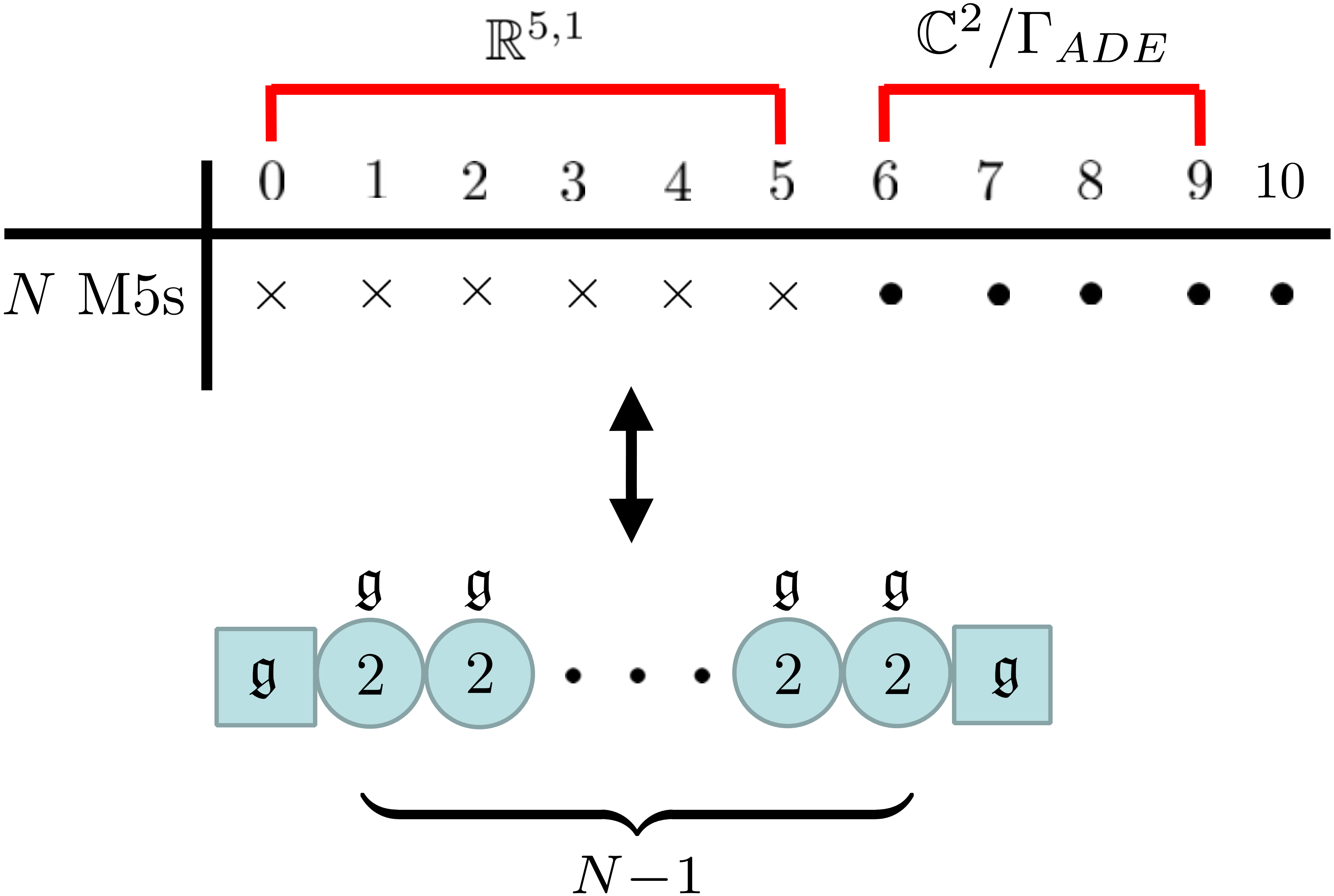
6D SCFTs and Homomorphisms

6D SCFTs and Homomorphisms

- Large classes of 6D SCFTs have connections to structures in group theory
- The correspondence has been verified explicitly

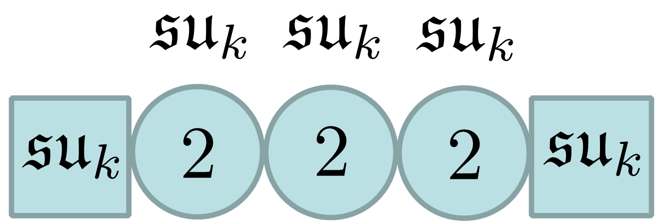


M5-Branes Probing $\mathbb{C}^2/\Gamma_{ADE}$

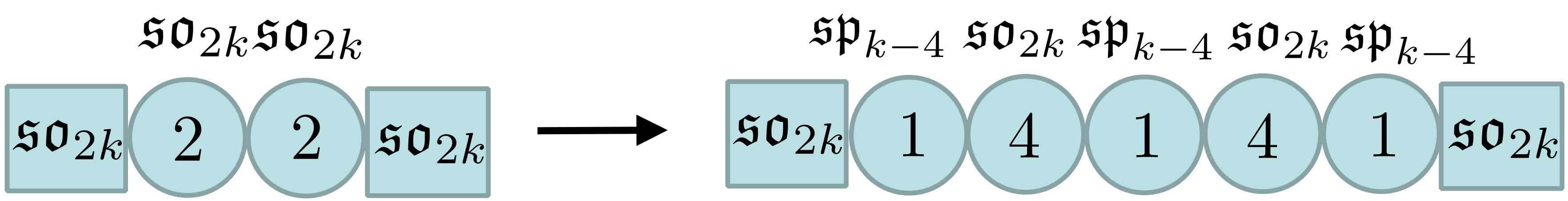


M5-Branes Probing $\mathbb{C}^2/\Gamma_{ADE}$

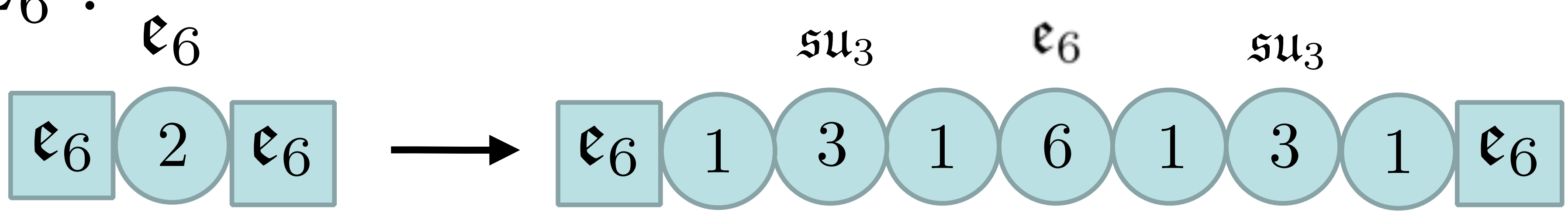
A_{k-1} :



D_k :



E_6 :



Nilpotent Deformations

- Matrix of normal deformations Φ characterizes positions of 7-branes
- View intersection points of $\mathbb{CP}^1$ in base as marked points
- Let adjoint field Φ have singular behavior at marked points $\Rightarrow$ Hitchin system coupled to defects:

$$\partial_A \Phi = \sum_p \mu_{\mathbb{C}}^{(p)} \delta_{(p)} \qquad F + [\Phi, \Phi^\dagger] = \sum_p \mu_{\mathbb{R}}^{(p)} \delta_{(p)}$$

Nilpotent Deformations

- Split $\mu_{\mathbb{C}} = \mu_s + \mu_n$, consider nilpotent part μ_n, get $\mathfrak{su}_2$ algebra:

$$J_+ = \mu_{\mathbb{C}} \qquad J_- = \mu_{\mathbb{C}}^\dagger \qquad J_3 = \mu_{\mathbb{R}}$$

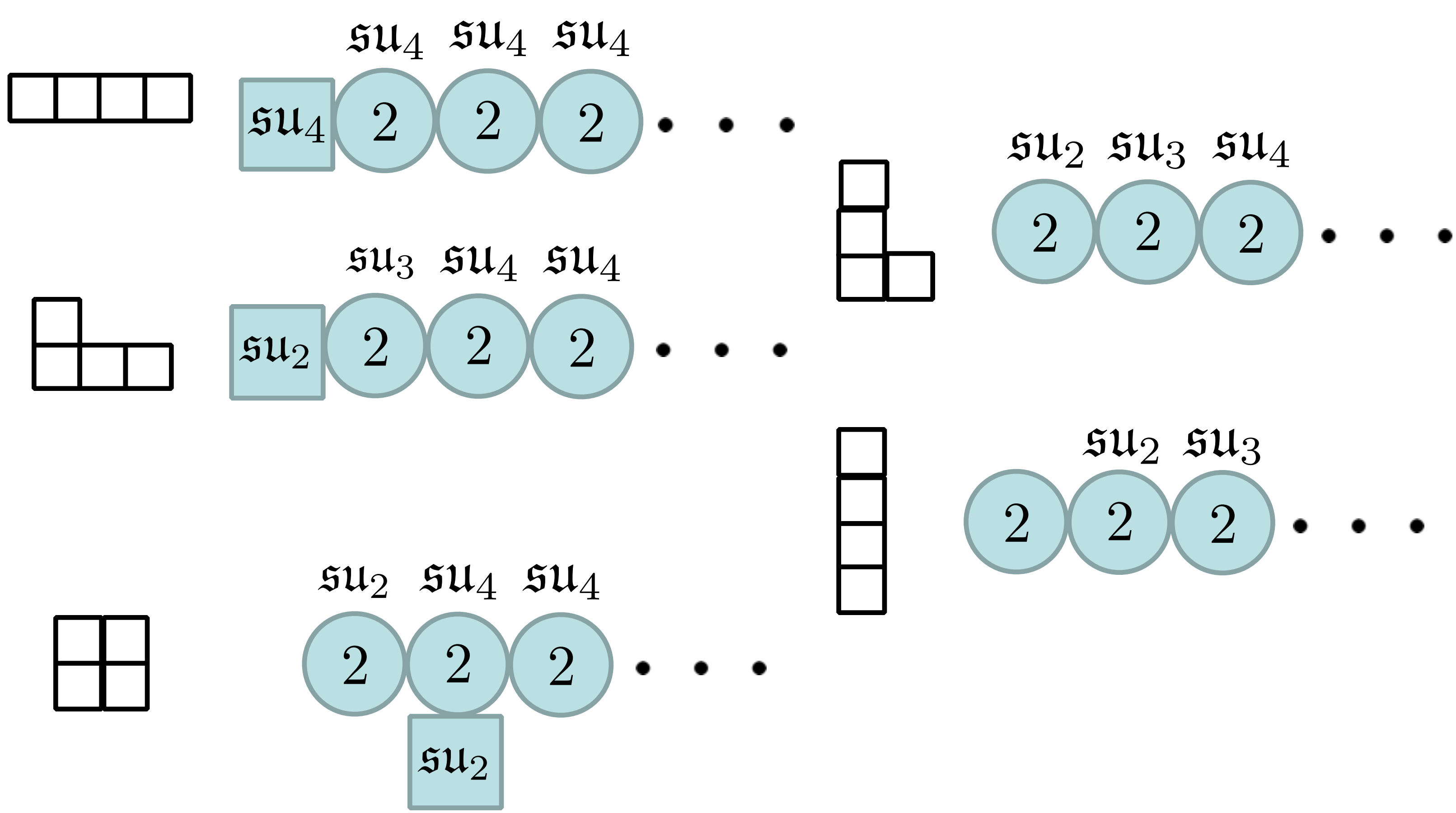
- Adjoint vevs $\Phi \sim \mu_{\mathbb{C}} \frac{dz}{z}$

$$\Rightarrow \text{ Classified by } \mathrm{Hom}(\mathfrak{su}(2), \mathfrak{g})$$

$$\text{(equivalently, by nilpotent orbits } J_+)$$

6D SCFTs and Hom($\mathfrak{su}(2), A_{k-1}$)

Hom($\mathfrak{su}(2), A_{k-1}$) labeled by partitions of k:

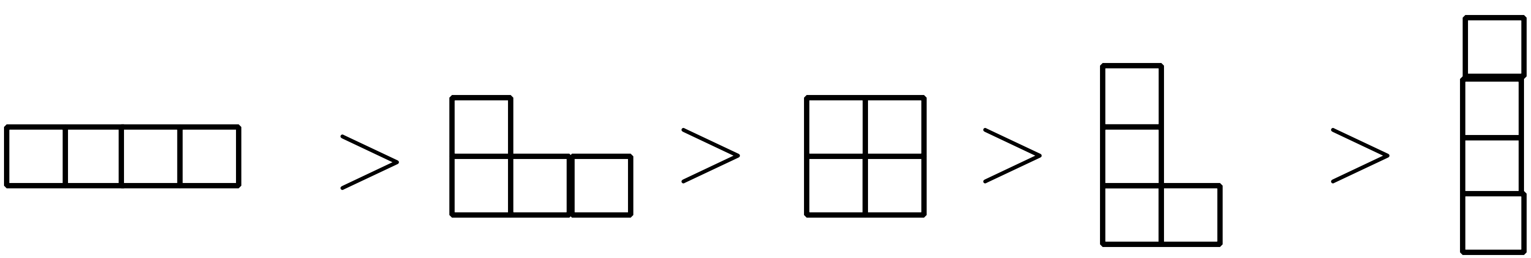


Partial Ordering of Nilpotent Orbits

$$\mathcal{O}_\mu \geq \mathcal{O}_\nu \Leftrightarrow \bar{\mathcal{O}}_\mu \supset \mathcal{O}_\nu$$

$$\Leftrightarrow \mu \geq \nu$$

$$\Leftrightarrow \sum_{i=1}^{m} \mu_i^T \geq \sum_{i=1}^{m} \nu_i^T \ \forall m$$



Renormalization Group Flows

High Energy Short Distance

$$\mathcal{T}_{UV}$$

$$\downarrow$$

$$\mathcal{T}_{IR}$$

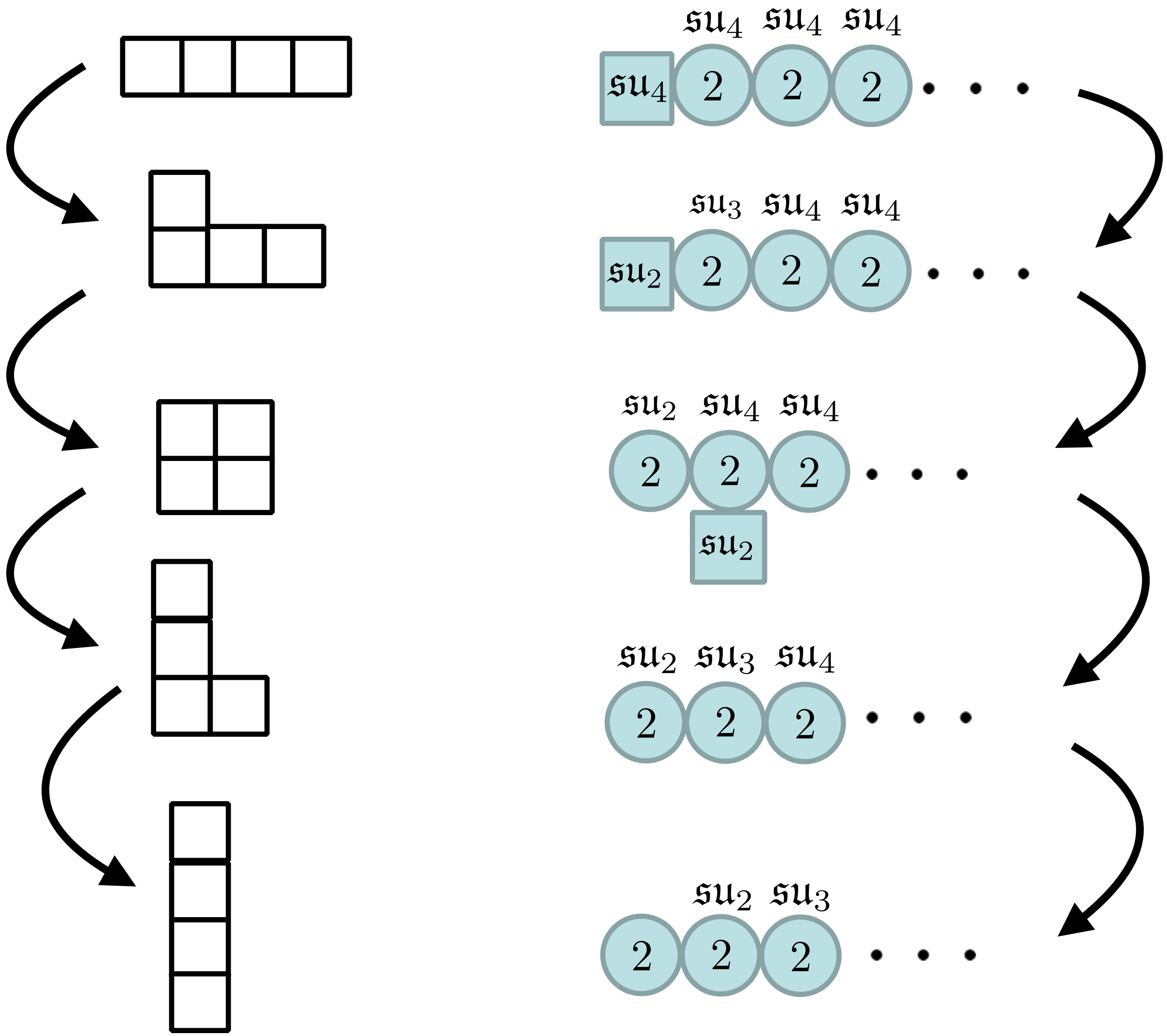
Low Energy Long Distance

Partial Ordering of Theories

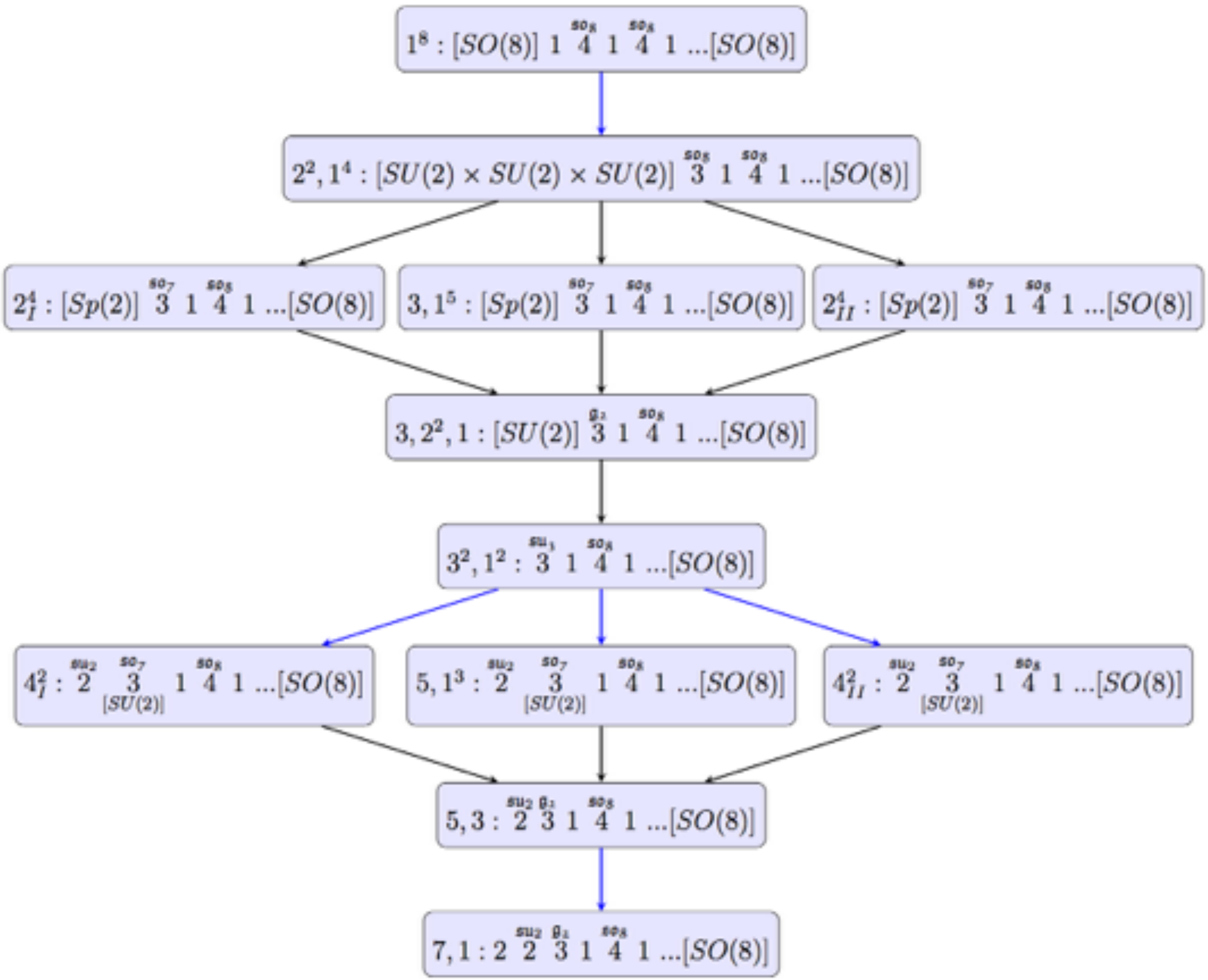
- Can define a partial ordering on theories using RG flows:

$$\mathcal{T}_1 \geq \mathcal{T}_2 \quad \Leftrightarrow \quad \exists \text{ flow } \begin{array}{c} \mathcal{T}_1 \\ \big\downarrow \\ \mathcal{T}_2 \end{array}$$

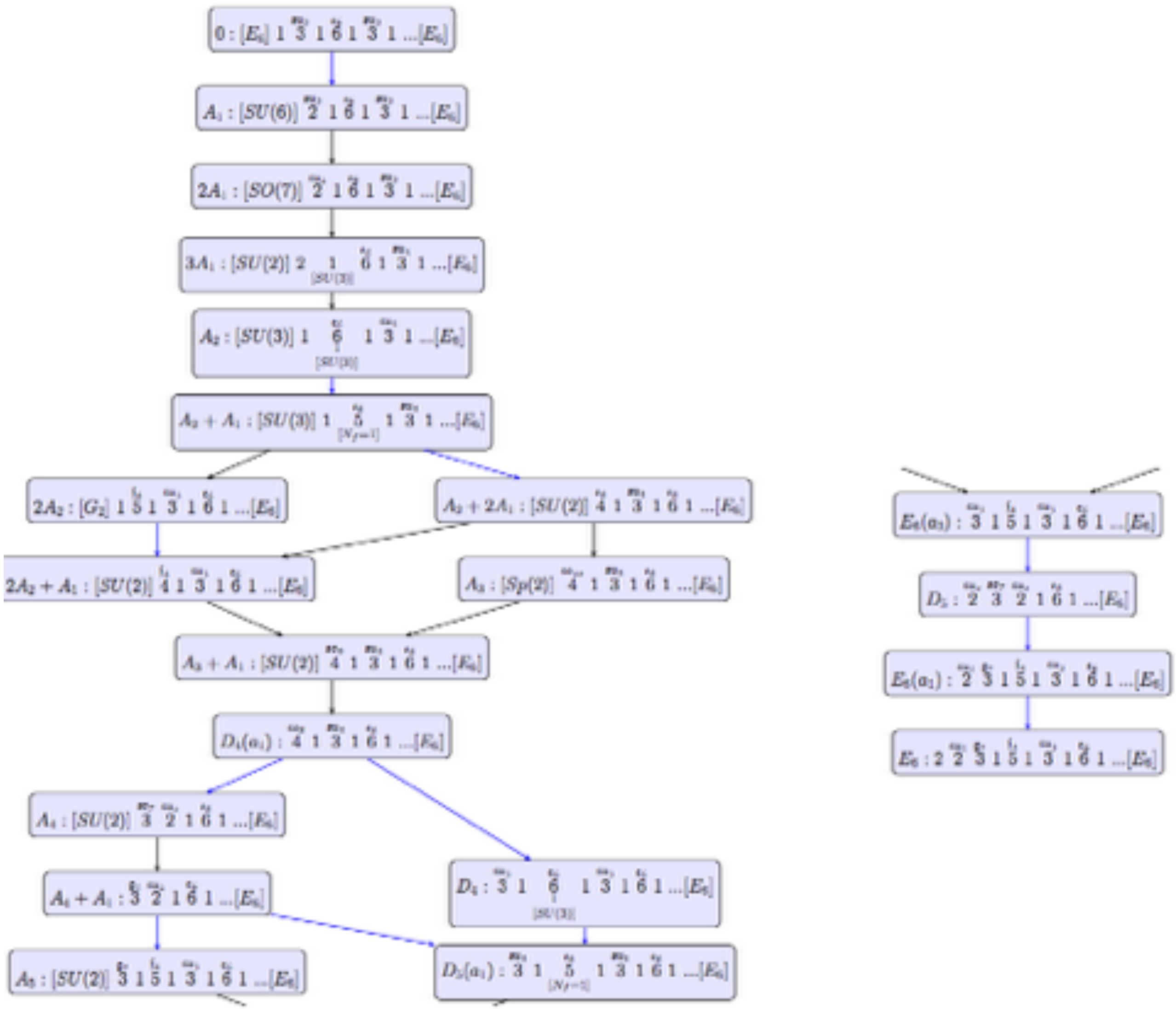
Nilpotent Orbit Ordering Matches RG Ordering!



6D SCFTs and Hom($\mathfrak{su}(2), D_k$)

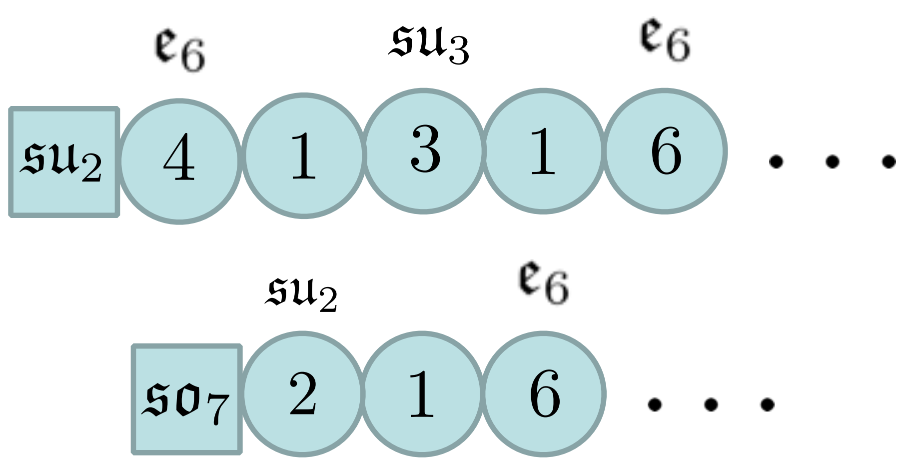


6D SCFTs and Hom($\mathfrak{su}(2), E_6$)



Nilpotent Orbits and Global Symmetries

- Consider nilpotent orbit $\mathcal{O}_\mu \in \mathfrak{g}$
- Let $F(\mu)$ be subgroup of G commuting with nilpotent element
- Claim: $F(\mu)$ is the global symmetry of the 6D SCFT associated with μ
- E.g.



6D SCFTs and Hom(Γ_{ADE},E_8)

- Consider M5-branes probing Horava-Witten wall and $\mathbb{C}^2/\Gamma_{ADE}$ singularity

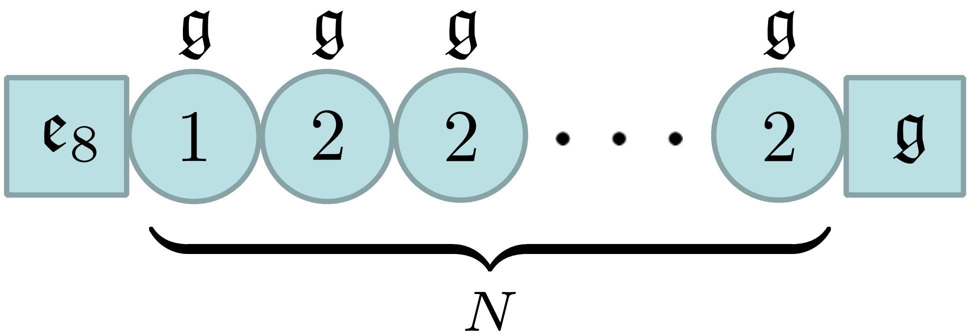
$\mathbb{R}^{5,1}$ (directions 0–5) $\quad$ $\mathbb{C}^2/\Gamma_{ADE}$ (directions 6–9)

	0	1	2	3	4	5	6	7	8	9	10
N M5s	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
E_8 Wall	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\times$

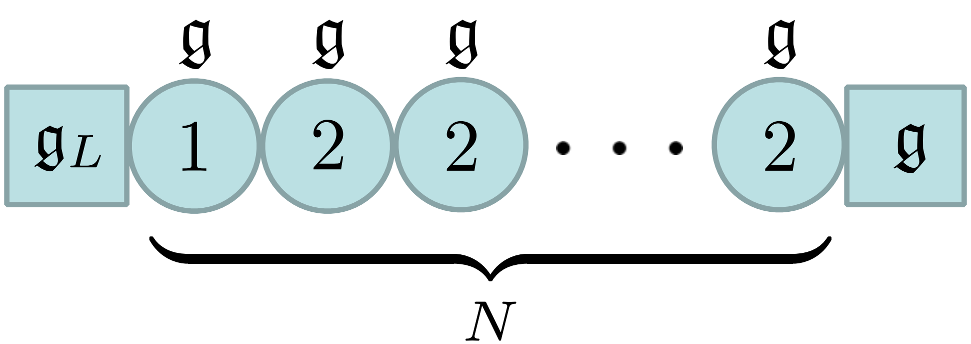
Boundary data $\simeq$ flat E_8 connections on $S^3/\Gamma_{\mathrm{ADE}}$

6D SCFTs and $\mathrm{Hom}(\Gamma_{\mathrm{ADE}}, E_8)$

- For trivial boundary data, get 6D SCFT:



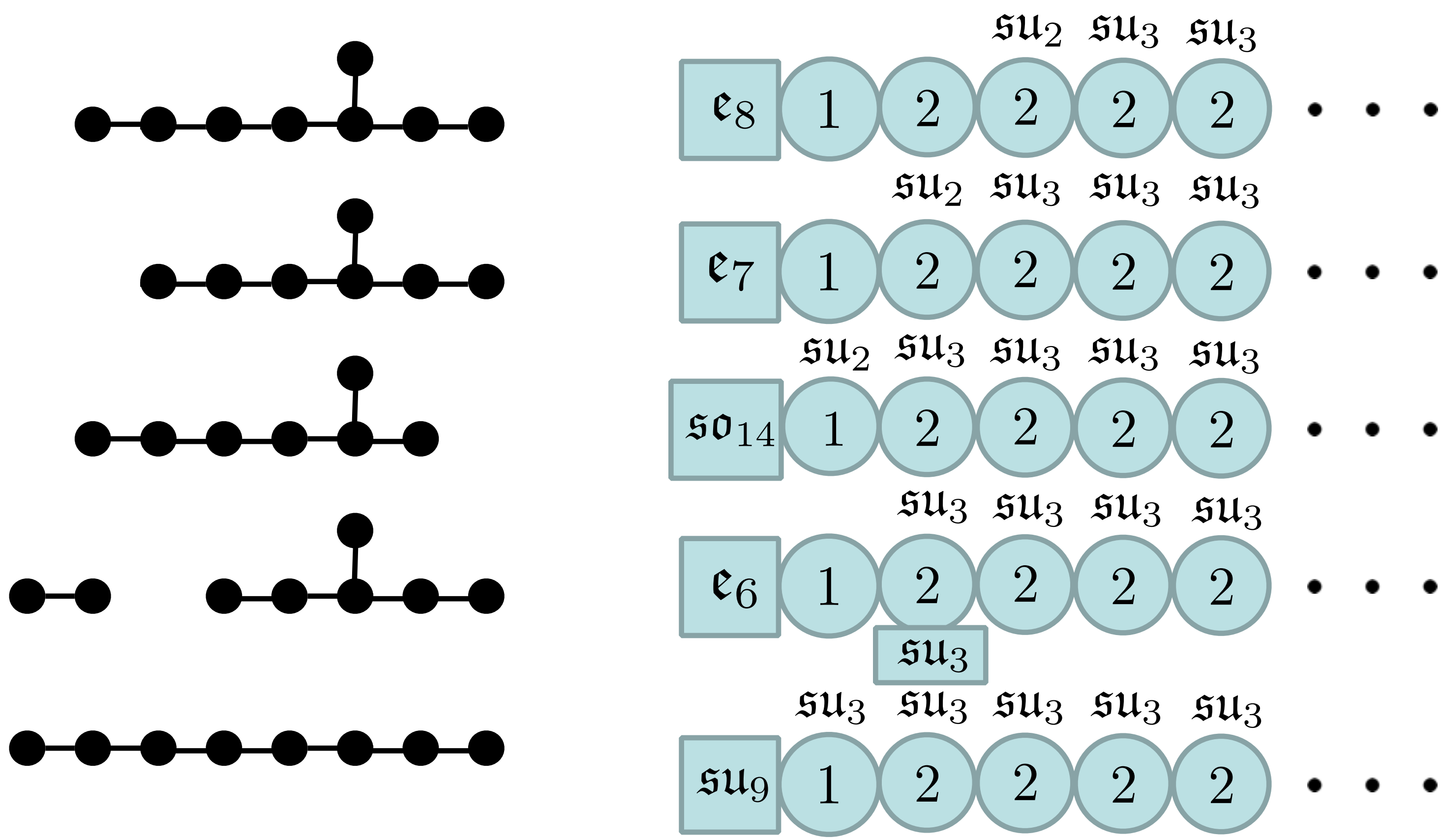
- For non-trivial boundary data, global symmetry is broken to a subgroup



6D SCFTs and $\mathrm{Hom}(\Gamma_{\mathrm{ADE}}, E_8)$

Flat E_8 connections on $S^3/\Gamma_{\mathrm{ADE}} \Leftrightarrow \mathrm{Hom}(\Gamma_{\mathrm{ADE}}, E_8)$

E.g. Γ_{A_2}, $\mathrm{Hom}(\mathbb{Z}_3, E_8)$:



6D SCFTs and Homomorphisms

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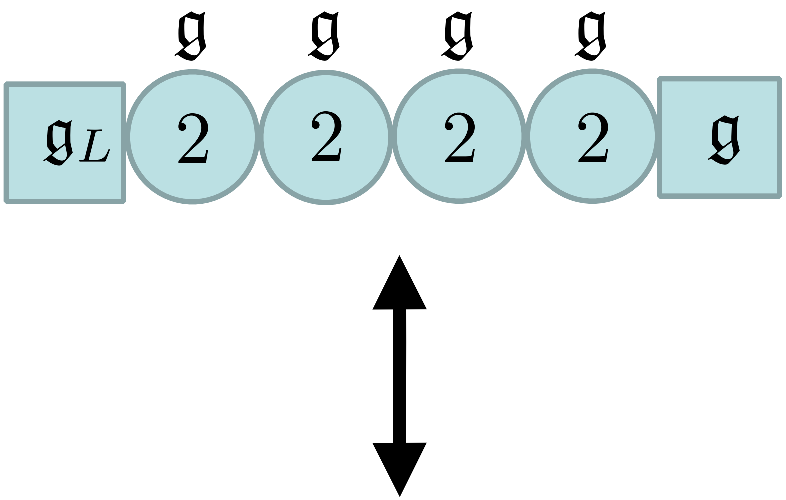
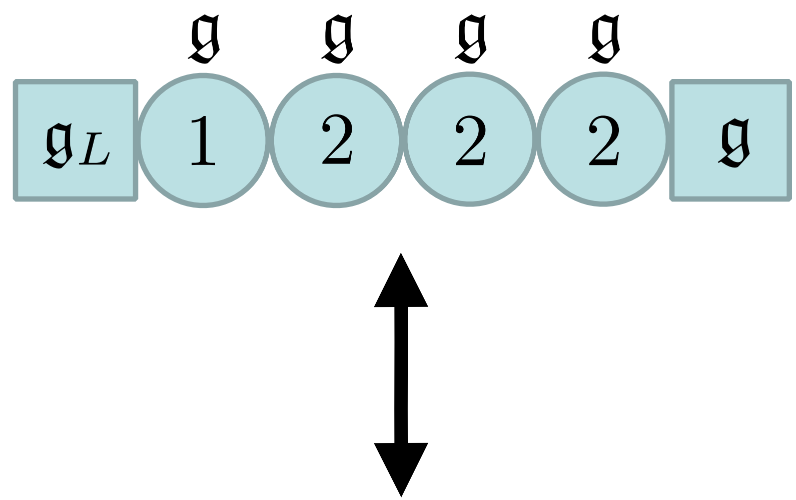


$$\mathrm{Hom}(\mathfrak{su}(2), \mathfrak{g})$$

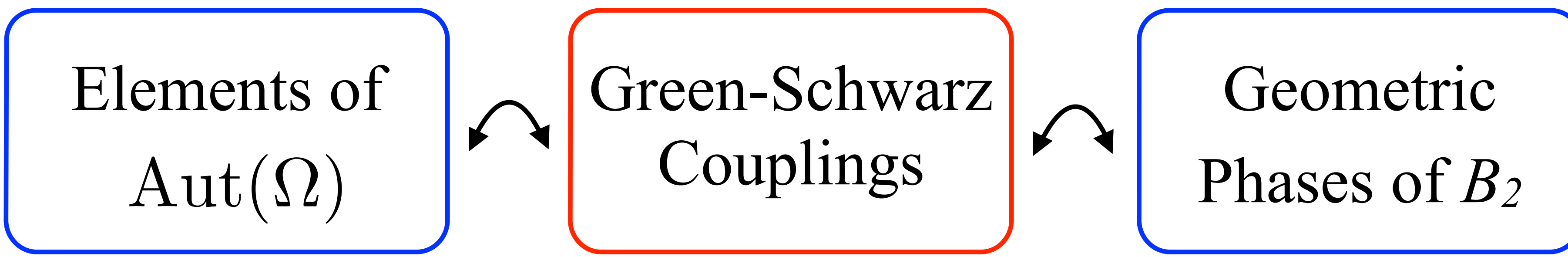


$$\mathrm{Hom}(\Gamma_{\mathfrak{g}}, E_8)$$

6D SCFTs and Automorphism Groups

6D SCFTs and Automorphism Groups

The Dirac pairing Ω of a 6D SCFT has an associated automorphism group $\mathrm{Aut}(\Omega)$, which is calculable



Automorphism Groups

Given $\Omega \in GL(n, \mathbb{Z})$, define $\mathrm{Aut}(\Omega)$ by

$$\mathrm{Aut}(\Omega) = \{\mu \in GL(n, \mathbb{Z}) | \mu^T \Omega \mu = \Omega\}.$$

Automorphism Groups of 6D SCFTs

For 6D SCFT, Dirac pairing Ω,

$$\mathrm{Aut}(\Omega) = \mathrm{Aut}(\Omega_{end}) \times \mathrm{Aut}(\mathbb{I}_k)$$

Dirac pairing after blowing down all -1 curves

Dirac pairing associated with *k* blow-downs

Automorphism Groups of 6D SCFTs

E.g.

$$\mathrm{Aut}(\textcircled{4}\textcircled{1}\textcircled{4}) = \mathrm{Aut}(\textcircled{3}\textcircled{3}) \times \mathrm{Aut}(\mathbb{I}_1)$$

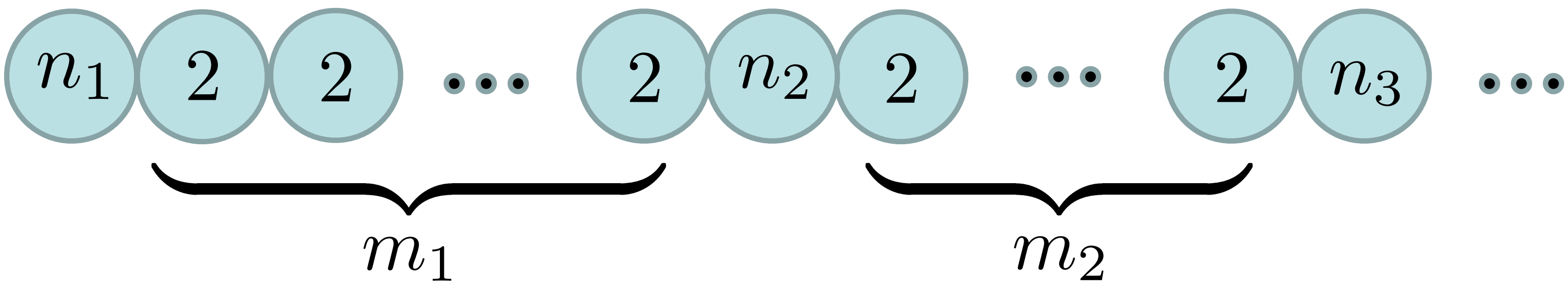
$$\mathrm{Aut}(\textcircled{1}\textcircled{2}\textcircled{2}) = \mathrm{Aut}(\textcircled{1}\textcircled{3}\textcircled{1})$$

$$= \mathrm{Aut}(\mathbb{I}_3)$$

Automorphism Groups of 6D SCFTs

In general,

Aut(



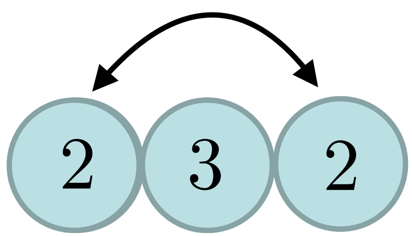
)

$$= \mathbb{Z}_2 \ltimes S_{m_1+1} \times S_{m_2+1} \times \ldots$$

$$\mathrm{Aut}(\mathbb{I}_k) = S_k \ltimes \mathbb{Z}_2^k$$

Outer Automorphisms

For a symmetric endpoint, $\mathrm{Aut}(\Omega)$ contains an additional factor associated with the quiver symmetry:

$\mathrm{Aut}($  $) = \mathbb{Z}_2 \rtimes (\mathbb{Z}_2 \rtimes (S_2 \times S_2))$

Symmetry of quiver

from left -2 curve

from right -2 curve

Green-Schwarz Couplings

- Group elements label distinct choices for Green-Schwarz coupling

$$\mathcal{L}_6 \supset \int \mu_{IJ} B_I \wedge \mathrm{Tr}(F_J \wedge F_J)$$

$$I_{GS} \supset \mathrm{Tr}(F_I \wedge F_I)\mu_{JI}\Omega^{-1}_{JK}\mu_{KL}\mathrm{Tr}(F_L \wedge F_L)$$

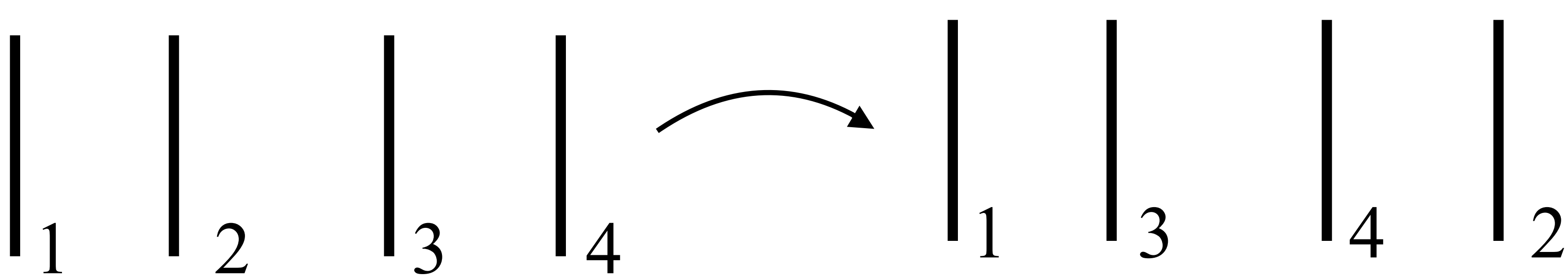
$$\mu_{IJ} \in \mathrm{Aut}(\Omega) \Leftrightarrow \text{Dirac Quantization}$$

(2,0) Automorphism Groups

For a (2,0) SCFT,

$$\mathrm{Aut}(\Omega_{\mathfrak{g}}) = \mathrm{Aut}(\mathfrak{g})$$

Group elements $\longleftrightarrow$ Permutations of M5-branes



Phases of 6D SCFTs

- For a general (1,0) SCFT, group elements label tensor branch phases:

$$\sum_I \mu^T_{IJ} \phi_I > 0$$

- These in turn correspond to geometric phases of the base B_2.

Summary

- So far…
  - 6D SCFTs have been classified
  - There are remarkable relationships between 6D SCFTs and two classes of homomorphisms
  - Phases of 6D SCFTs are labeled by automorphism groups of their Dirac pairing

Further Research

- In the future…
  - Can we classify full set of 6D RG Flows in terms of group theory data?
  - Can we understand compactifications to lower dimensions?
  - Can we understand these algebraic/geometric correspondences from a purely mathematical perspective?