Oxidizing SuperYang-Mills from $(N = 4,\ d = 4)$ to $(N = 1,\ d = 10)$

Sudarshan Ananth
University of Florida

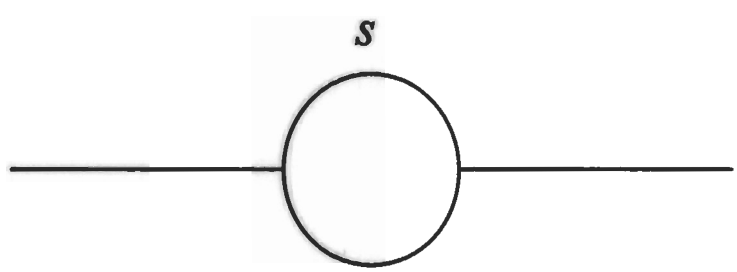
Divergences

$$\beta = -\frac{11}{3}\, C_a^{(2)} + \frac{2}{3}\, C_f^{(2)} + \frac{1}{3}\, C_s^{(2)}$$

where ,

$$Tr\,(\,T_r^A\,T_r^B\,) \;=\; C_r^{(2)}\,\delta^{AB}$$

- Richard Hughes, 1980



$$(-1)^{2s}\;\frac{1}{3}\left(1 - 12\,s^2\right)$$

Dynkin Indices

For a representation R, the pth Dynkin index is,

$$I^{(p)}[R] = \sum_{w \text{ in } R} (w \cdot w)^{\frac{p}{2}}$$

- Tom Curtright, 1981

The UV behavior of a reduced theory ($D \to d$),

is determined by the $SO(D-2)$ Dynkin indices,

$$(-1)^f \left\{ \frac{I^{(0)}}{d-1} - 4\frac{I^{(2)}}{r} \right\} (p^\mu p^\nu - \delta^{\mu\nu} p^2)\ \mathcal{G}$$

$\mathcal{N} = 4$ Yang-Mills

Beta function vanishes to all orders: Brink, Lindgren and Nilsson(1982), Mandelstam(1982).

- Obtained by reduction from ($\mathcal{N} = 1, d = 10$) Yang-Mills
- Maximally supersymmetric theory in four dimensions

UV behavior, dictated by the $d = 10$ little group, $SO(8)$.

$SO(8)$ triality produces highly non-trivial cancelations.

*

$d = 10$ SuperYang-Mills

The spectrum of ten-dimensional Yang-Mills consists of a single gauge boson and a Majorana-Weyl Gaugino.

$$S = \int d^{10}x \{ -\frac{1}{4} F^a_{MN} F^{MN\,a} + \frac{i}{2} \bar{\lambda}^a \Gamma^M D_M \bar{\lambda}^a \}$$

These fields are linked by $\mathcal{N} = 1$ supersymmetry, and are members of the adjoint representation of a Lie algebra.

Both massless fields, transform as eight-dimensional representations(one bosonic, one fermionic) of $SO(8)$.

Light-Cone gauge is imposed, by setting

$$A_- = 0$$

The A_+ component is then eliminated, using the equations of motion, leaving only the transverse(physical) degrees of freedom.

$d = 4$ SuperYang-Mills

When the $d = 10$ theory is dimensionally reduced to $d = 4$, the resulting spectrum consists of one complex bosonic field (the gauge field), four complex Grassmann fields and six scalars.

$$SO(8) \supset SO(2) \times SO(6)\ .$$

The eight vector in ten dimensions, reduces to

$$\mathbf{8}_v = \mathbf{1}_1 + \mathbf{1}_{-1} + \mathbf{6}_0\ ,$$

and the eight spinors to

$$\mathbf{8}_s = \mathbf{4}_{1/2} + \bar{\mathbf{4}}_{-1/2}\ .$$

The representations on the right-hand side belong to $SO(6) \sim SU(4)$, with subscripts denoting the helicity.

There are six scalar fields, two vector fields, four spinor fields and their conjugates.

Notation

With the metric $(-,+,+,\ldots,+)$, introduce

$$
\begin{aligned}
x^{\pm} &= \frac{1}{\sqrt{2}}(x^0 \pm x^3)\,; & \partial^{\pm} &= \frac{1}{\sqrt{2}}(-\partial_0 \pm \partial_3)\,;\\
x &= \frac{1}{\sqrt{2}}(x_1 + i\,x_2)\,; & \bar{\partial} &= \frac{1}{\sqrt{2}}(\partial_1 - i\partial_2)\,;\\
\bar{x} &= \frac{1}{\sqrt{2}}(x_1 - i\,x_2)\,; & \partial &= \frac{1}{\sqrt{2}}(\partial_1 + i\partial_2)\,,
\end{aligned}
$$

so that

$$
\partial^+ x^- = \partial^- x^+ = -1\,; \qquad \bar{\partial} x = \partial \bar{x} = +1\,.
$$

Introduce, anticommuting Grassmann variables θ^m and $\bar{\theta}_m$,

$$
\{\theta^m\,,\,\theta^n\} = \{\bar{\theta}_m\,,\,\bar{\theta}_n\} = \{\bar{\theta}_m\,,\,\theta^n\} = 0\,,
$$

which transform under $SU(4)$ $(m,n,p,q,\ldots = 1,2,3,4)$.

$$
\bar{\partial}_m \equiv \frac{\partial}{\partial\,\theta^m}\,; \qquad \partial^m \equiv \frac{\partial}{\partial\,\bar{\theta}_m}\,.
$$

The eight original gauge fields A_i , $i = 1, \ldots, 8$ appear as

$$A \;=\; \frac{1}{\sqrt{2}}\,(A_1 + i\,A_2)\ , \qquad \bar{A} \;=\; \frac{1}{\sqrt{2}}\,(A_1 - i\,A_2)\ ,$$

and as antisymmetric $SU(4)$ bi-spinors

$$C^{m\,4} \;=\; \frac{1}{\sqrt{2}}\,(A_{m+3} + i\,A_{m+6})\ , \qquad \overline{C}_{mn} \;=\; \frac{1}{2}\,\epsilon_{mnpq}\,C^{pq}\ .$$

All the physical degrees of freedom can be captured in one complex superfield

$$\begin{aligned}\phi\,(y) \;=\; & \frac{1}{\partial^+}\,A\,(y) + \frac{i}{\partial^+}\,\theta^m\,\bar{\chi}_m(y) + \frac{i}{\sqrt{2}}\,\theta^m\,\theta^n\,\overline{C}_{mn}\,(y)\\ & + \frac{\sqrt{2}}{6}\theta^m\,\theta^n\,\theta^p\,\epsilon_{mnpq}\,\chi^q(y) + \frac{1}{12}\,\theta^m\,\theta^n\,\theta^p\,\theta^q\,\epsilon_{mnpq}\,\partial^+\,\bar{A}\,(y)\end{aligned}$$

All fields have adjoint indices (not shown here), and are local in the “chiral” light-cone coordinates,

$$y \;=\; (x,\,\bar{x},\,x^+,\,y^- \equiv x^- - \frac{i}{\sqrt{2}}\,\theta^m\,\bar{\theta}_m)\ .$$

$(\mathcal{N}=4, d=4)$ Action

In terms of this superfield, the Yang-Mills action is simply,

$$\int d^4x \int d^4\theta\, d^4\bar{\theta}\, \mathcal{L}\,,$$

where $\mathcal{L}$ is,

$$-\,\bar{\phi}\,\frac{\Box}{\partial^{+2}}\,\phi$$

$$+\;\frac{4g}{3}\,f^{abc}\left\{\frac{1}{\partial^+}\,\bar{\phi}^a\,\phi^b\,\bar{\partial}\,\phi^c \;+\; \frac{1}{\partial^+}\,\phi^a\,\bar{\phi}^b\,\partial\,\bar{\phi}^c\right\}$$

$$-\;g^2 f^{abc}\,f^{ade}\left\{\frac{1}{\partial^+}(\phi^b\,\partial^+\phi^c)\frac{1}{\partial^+}\,(\bar{\phi}^d\,\partial^+\,\bar{\phi}^e)+\frac{1}{2}\,\phi^b\bar{\phi}^c\,\phi^d\,\bar{\phi}^e\right\}$$

Grassmann integration is normalized: $\int d^4\theta\,\theta^1\theta^2\theta^3\theta^4 = 1$.

Chiral Derivatives

Introduce the chiral derivatives,

$$d^m = -\partial^m - \frac{i}{\sqrt{2}}\,\theta^m\,\partial^+ \;; \qquad \bar{d}_n = \;\bar{\partial}_n + \frac{i}{\sqrt{2}}\,\bar{\theta}_n\,\partial^+ \;,$$

which satisfy the anticommutation relations

$$\{\,d^m\,,\,\bar{d}_n\,\} \;=\; -i\,\sqrt{2}\,\delta^m{}_n\,\partial^+ \;.$$

One verifies that ϕ and its complex conjugate $\bar{\phi}$ satisfy the chiral constraints

$$d^m\,\phi = 0\;; \qquad \bar{d}_m\,\bar{\phi} = 0\;,$$

as well as the "inside-out" constraints

$$\bar{d}_m\,\bar{d}_n\,\phi \;=\; \frac{1}{2}\,\epsilon_{mnpq}\,d^p\,d^q\,\bar{\phi}\;,$$

$$d^m\,d^n\,\bar{\phi} \;=\; \frac{1}{2}\,\epsilon^{mnpq}\,\bar{d}_p\,\bar{d}_q\,\phi\;.$$

SuperPoincaré Algebra

The SuperPoincaré algebra, splits up into kinematical and dynamical pieces. The kinematical generators(at $x^+ = 0$) are,

- the three momenta,

$$p^+ \;=\; -\,i\,\partial^+\;, \qquad p \;=\; -\,i\,\partial\;, \qquad \bar{p} \;=\; -\,i\,\bar{\partial}\;,$$

- the transverse space rotation,

$$j \;=\; x\,\bar{\partial} - \bar{x}\,\partial + S^{12}\;,$$

$$S^{12} \;=\; \frac{1}{2}\,(\,\theta^\alpha\,\bar{\partial}_\alpha \;-\; \bar{\theta}_\alpha\,\partial^\alpha\,) \;+\; \frac{i}{4\sqrt{2}\,\partial^+}\,(\,d^\alpha\,\bar{d}_\alpha - \bar{d}_\alpha\,d^\alpha\,)\;.$$

$$[\,j\,,\,d^\alpha\,] \;=\; [\,j\,,\,\bar{d}_\beta\,] \;=\; 0\;.$$

- and the "plus-rotations",

$$j^+ \;=\; i\,x\,\partial^+\;, \qquad \bar{j}^+ \;=\; i\,\bar{x}\,\partial^+\;.$$

$$j^{+-} \;=\; i\,x^-\,\partial^+ - \frac{i}{2}\,(\,\theta^\alpha\bar{\partial}_\alpha + \bar{\theta}_\alpha\,\partial^\alpha\,)\;,$$

SuperPoincaré Algebra

The dynamical generators are,

- the light-cone Hamiltonian,

$$p^- \;=\; -\,i\frac{\partial\bar\partial}{\partial^+}$$

- and the dynamical boosts,

$$j^- \;=\; i\,x\,\frac{\partial\bar\partial}{\partial^+} \;-\; i\,x^-\,\partial \;+\; i\left(\theta^\alpha\bar\partial_\alpha + \frac{i}{4\sqrt{2}\,\partial^+}\,(\,d^\alpha\,\bar d_\alpha - \bar d_\alpha\,d^\alpha\,)\right)\frac{\partial}{\partial^+}$$

$$\bar j^- \;=\; i\,\bar x\,\frac{\partial\bar\partial}{\partial^+} \;-\; i\,x^-\,\bar\partial \;+\; i\left(\bar\theta_\beta\partial^\beta + \frac{i}{4\sqrt{2}\,\partial^+}\,(\,d^\beta\,\bar d_\beta - \bar d_\beta\,d^\beta\,)\right)\frac{\bar\partial}{\partial^+}$$

These generators satisfy,

$$[\,j^-\,,\,\bar j^+\,] \;=\; -\,i\,j^{+-} - j\;, \qquad [\,j^-\,,\,j^{+-}\,] \;=\; i\,j^-\;.$$

Supersymmetries

In a similar fashion, the supersymmetries split into,

- kinematical supersymmetries,

$$q^{\alpha}_{+} = -\partial^{\alpha} + \frac{i}{\sqrt{2}}\,\theta^{\alpha}\,\partial^{+}\ ;\qquad \bar{q}_{+\,\beta} = \quad \bar{\partial}_{\beta} - \frac{i}{\sqrt{2}}\,\bar{\theta}_{\beta}\,\partial^{+}\ ,$$

satisfying

$$\{\,q^{\alpha}_{+}\,,\,\bar{q}_{+\beta}\,\} = i\,\sqrt{2}\,\delta^{\alpha}{}_{\beta}\,\partial^{+}\ ,$$

$$\{\,q^{\alpha}_{+}\,,\,\bar{d}_{\beta}\,\} = \{\,d^{\alpha}\,,\,\bar{q}_{+\beta}\,\} = 0\ .$$

- and dynamical supersymmetries,

$$q^{\alpha}_{-}\ \equiv\ i\,[\,\bar{j}^{-}\,,\,q^{\alpha}_{+}\,]\ =\ \frac{\bar{\partial}}{\partial^{+}}\,q^{\alpha}_{+}\ ,\qquad \bar{q}_{-\,\beta}\ \equiv\ i\,[\,j^{-}\,,\,\bar{q}_{+\,\beta}\,]\ =\ \frac{\partial}{\partial^{+}}\,\bar{q}_{+\,\beta}$$

They obey,

$$\{\,q^{\alpha}_{-}\,,\,\bar{q}_{-\,\beta}\,\}\ =\ i\,\sqrt{2}\,\delta^{\alpha}{}_{\beta}\,\frac{\partial\bar{\partial}}{\partial^{+}}\ .$$

Ten-Dimensional SuperYang-Mills in Light-Cone Superspace

$SO(6)$ Coordinates

Transverse light-cone variables need to be generalized to eight. We stick to previous notation, and introduce six extra coordinates and their derivatives as antisymmetric bi-spinors,

$$x^{m\,4} = \frac{1}{\sqrt{2}}(\,x_{m+3} + i\,x_{m+6}\,)\,, \qquad \partial^{m\,4} = \frac{1}{\sqrt{2}}(\,\partial_{m+3} + i\,\partial_{m+6}\,)$$

$$\bar{x}_{pq} = \frac{1}{2}\,\epsilon_{pqmn}\,x^{mn}\ ; \qquad \bar{\partial}_{pq} = \frac{1}{2}\,\epsilon_{pqmn}\,\partial^{mn}$$

The derivatives satisfy,

$$\bar{\partial}_{mn}\,x^{pq} \;=\; (\,\delta_m{}^p\,\delta_n{}^q \,-\, \delta_m{}^q\,\delta_n{}^p\,) \;=\; \delta^{pq}_{mn}\ ,$$

and

$$\partial^{mn}\,x^{pq} \;=\; \frac{1}{2}\,\epsilon^{pqrs}\,\partial^{mn}\,\bar{x}_{rs} \;=\; \epsilon^{mnpq}\ .$$

Oxidation

There are no modifications to the chiral superfield, except for the added dependence on the extra coordinates.

Simply generalize the transverse derivatives in the $d=4$ superspace action and we end up with the $d=10$ action

We propose the following operator

$$\bar{\nabla} \equiv \bar{\partial} + \frac{i}{4\sqrt{2}\,\partial^+}\,\bar{d}_p\,\bar{d}_q\,\partial^{pq}\ ,$$

The simple prescription,

$$\partial \to \nabla \qquad\qquad \bar{\partial} \to \bar{\nabla}$$

"oxidizes" the $d=4$ theory into its parent version. *

Procedure

- Construct the ten-dimensional SuperPoincaré algebra
- Define the new derivative, ∇ and verify its transformation properties
- Verify ten-dimensional SuperPoincaré invariance of the "oxidized" action

$$SO(8) \supset SO(2) \times SO(6) \ ,$$

$SO(8):$ 28 generators

$SO(6):$ 15 generators

$SO(2):$ 1 generator

The remaining 12 generators, belong to the coset,

$$SO(8) \, / \, SO(2) \times SO(6) \ .$$

$SO(8)$ Generators

The $SO(6) \sim SU(4)$ generators are given by

$$
\begin{aligned}
J^m{}_n \quad = \quad & \frac{1}{2}\,(\,x^{mp}\,\bar{\partial}_{pn} \;-\; \bar{x}_{pn}\,\partial^{mp}\,) \;-\; \theta^m\,\bar{\partial}_n \;+\; \bar{\theta}_n\,\partial^m \\
& +\frac{1}{4}\,(\,\theta^p\,\bar{\partial}_p \;-\; \bar{\theta}_p\,\partial^p\,)\,\delta^m{}_n \;+\; \frac{i}{2\sqrt{2}\,\partial^+}\,(\,d^m\,\bar{d}_n - \bar{d}_n\,d^m\,) \\
& +\frac{i}{8\sqrt{2}\,\partial^+}\,(\,d^p\,\bar{d}_p - \bar{d}_p\,d^p\,)\,\delta^m{}_n
\end{aligned}
$$

$SO(8)/(SO(2) \times SO(6))$ Coset generators read,

$$
\begin{aligned}
J^{pq} \quad = \quad & x\,\partial^{pq} - x^{pq}\,\partial + \frac{i}{\sqrt{2}}\,\partial^+\,\theta^p\,\theta^q \\
& -i\,\sqrt{2}\,\frac{1}{\partial^+}\,\partial^p\,\partial^q + \frac{i}{\sqrt{2}\,\partial^+}\,d^p\,d^q\ ,
\end{aligned}
$$

$$
\begin{aligned}
\bar{J}_{mn} \quad = \quad & \bar{x}\,\bar{\partial}_{mn} - \bar{x}_{mn}\,\bar{\partial} + \frac{i}{\sqrt{2}}\,\partial^+\,\bar{\theta}_m\,\bar{\theta}_n \\
& -i\,\sqrt{2}\,\frac{1}{\partial^+}\,\bar{\partial}_m\,\bar{\partial}_n + \frac{i}{\sqrt{2}\,\partial^+}\,\bar{d}_m\,\bar{d}_n\ .
\end{aligned}
$$

All $SO(8)$ transformations are specially constructed so as not to mix chiral and antichiral superfields.

$SO(8)$ Algebra

These generators satisfy the $SO(8)$ commutation relations,

$$\begin{aligned}
\Big[J\,,\,J^{mn}\Big] &= J^{mn} \\
\Big[J^m{}_n\,,\,J^{pq}\Big] &= \delta^q{}_n\,J^{mp} - \delta^p{}_n\,J^{mq} \\
\Big[J^{mn}\,,\,\bar{J}_{pq}\Big] &= \delta^m{}_pJ^n{}_q + \delta^n{}_qJ^m{}_p - \delta^n{}_pJ^m{}_q - \delta^m{}_qJ^n{}_p \\
&\quad - (\,\delta^m{}_p\,\delta^n{}_q - \delta^n{}_p\,\delta^m{}_q\,)\,J\;.
\end{aligned}$$

Other generators include,

$$\begin{aligned}
J^+ &= i\,x\,\partial^+\;; \qquad \bar{J}^+ = i\,\bar{x}\,\partial^+ \\
J^{+\,mn} &= i\,x^{mn}\,\partial^+\;; \qquad \bar{J}^+{}_{mn} = i\,\bar{x}_{mn}\,\partial^+\;.
\end{aligned}$$

The dynamical boosts are now,

$$\begin{aligned}
J^- = i\,x\,\frac{\partial\bar{\partial}}{\partial^+} - i\,x^-\,\partial + i\,\frac{\partial}{\partial^+}\Big\{\theta^m\,\bar{\partial}_m + \frac{i}{4\sqrt{2}\,\partial^+}\,(d^p\,\bar{d}_p - \bar{d}_p\,d^p) \\
- \frac{1}{4}\,\frac{\bar{\partial}_{pq}}{\partial^+}\Big\{\frac{\partial^+}{\sqrt{2}}\,\theta^p\,\theta^q - \frac{\sqrt{2}}{\partial^+}\,\partial^p\,\partial^q + \frac{1}{\sqrt{2}\partial^+}\,d^p\,d^q\Big\}
\end{aligned}$$

$SO(8)$ Algebra

The remaining boosts are,

$$J^{-\,mn} \;=\; [\,J^-\,,\,J^{mn}\,]\;;\qquad \bar{J}^-{}_{mn} \;=\; [\,\bar{J}^-\,,\,\bar{J}_{mn}\,]\;.$$

The four supersymmetries in four dimensions turn into one supersymmetry in ten dimensions.

$$i\,[\,\bar{J}^-\,,\,q_+^m\,] \;\equiv\; \mathcal{Q}^m\;,\qquad i\,[\,J^-\,,\,\bar{q}_{+\,m}\,] \;\equiv\; \overline{\mathcal{Q}}_m\;,$$

where

$$\mathcal{Q}^m \;=\; \frac{\bar{\partial}}{\partial^+}\,q_+{}^m \;+\; \frac{1}{2}\,\frac{\partial^{mn}}{\partial^+}\,\bar{q}_{+\,n}\;,$$

$$\overline{\mathcal{Q}}_m \;=\; \frac{\partial}{\partial^+}\,\bar{q}_{+\,m} \;+\; \frac{1}{2}\,\frac{\bar{\partial}_{mn}}{\partial^+}\,q_+^{\;n}\;.$$

They satisfy the supersymmmetry algebra

$$\{\,\mathcal{Q}^m\,,\,\overline{\mathcal{Q}}_n\,\} \;=\; i\,\sqrt{2}\,\delta^m_{\;n}\,\frac{1}{\partial^+}\left(\partial\,\bar{\partial} \;+\; \frac{1}{4}\,\bar{\partial}_{pq}\,\partial^{pq}\right) \qquad *$$

Meet, $\overline{\nabla}$

The cubic interaction in the $\mathcal{N}=4$ Lagrangian contains explicitly, the derivative operators ∂ and $\bar{\partial}$.

To achieve covariance in ten dimensions, these must be generalized. We propose the following operator

$$\overline{\nabla} \;\equiv\; \bar{\partial} + \frac{i\,\alpha}{4\sqrt{2}\,\partial^+}\,\bar{d}_p\,\bar{d}_q\,\partial^{pq} \;,$$

which naturally incorporates the rest of the derivatives ∂^{pq}, with α as an arbitrary parameter.

We define its rotated partner as,

$$\nabla^{mn} \;\equiv\; \left[\,\overline{\nabla}\,,\,J^{mn}\,\right] \,,$$

where

$$\nabla^{mn} \;=\; \partial^{mn} \;-\; \frac{i\,\alpha}{4\sqrt{2}\,\partial^+}\,\bar{d}_r\,\bar{d}_s\,\epsilon^{mnrs}\,\partial \;.$$

Transformation Properties of $\overline{\nabla}$

If we now apply the inverse transformation to ∇^{mn}, it goes back to the original form

$$\left[\,\bar{J}_{pq}\,,\,\nabla^{mn}\,\right] \;=\; (\,\delta_p{}^m\,\delta_q{}^n\,-\delta_q{}^m\,\delta_p{}^n\,)\,\overline{\nabla}\ ,$$

Thus, these operators transform under $SO(8)/(SO(2)\times SO(6))$, and $SO(2)\times SO(6)$ as the components of an 8-vector.

We introduce the conjugate operator ∇ by requiring that

$$\nabla\,\bar{\phi}\;\equiv\;\overline{(\,\overline{\overline{\nabla}}\,\phi\,)}\ ,$$

Covariance(& helicity) requirements are insufficient to fix α.

Instead, we choose to fix this parameter, by requiring invariance of the action.

Invariance

The kinetic term is trivially made $SO(8)$-invariant by including the six extra transverse derivatives in the d'Alembertian.

The quartic interactions are obviously invariant since they do not contain any transverse derivative operators. Hence we focus exclusively, on the cubic vertex.

Since it is obviously invariant under $SO(6) \times SO(2)$, we need only consider the coset variations.

$$\delta_J \phi \equiv \overline{\omega}_{mn} J^{mn} \phi = i\sqrt{2}\, \overline{\omega}_{mn} \partial^+ \theta^m \theta^n \phi$$

$$\delta_{\bar{J}} \phi \equiv \omega^{pq} \overline{J}_{pq} \phi = \omega^{pq} \left\{ \frac{i}{\sqrt{2}} \partial^+ \bar{\theta}_p \bar{\theta}_q - i\sqrt{2} \frac{1}{\partial^+} \bar{\partial}_p \bar{\partial}_q + \frac{i}{\sqrt{2}} \frac{1}{\partial^+} \bar{d}_p \bar{d}_q \right.$$

$$\delta_J \overline{\nabla} = \overline{\omega}_{mn} [J^{mn} , \overline{\nabla}] = - \overline{\omega}_{mn} \nabla^{mn} ,$$

$$\delta_{\bar{J}} \overline{\nabla} = \omega^{pq} [\overline{J}_{pq} , \overline{\nabla}] = \frac{i\alpha}{2\sqrt{2}} \omega^{pq} \bar{d}_p \bar{d}_q \frac{\overline{\partial}}{\partial^+} .$$

Invariance under $SO(8)$ is checked by doing a δ_J variation on the cubic vertex, *including* its complex conjugate.

In terms of

$$\mathcal{V} + \overline{\mathcal{V}} \equiv f_{abc} \int \left(\frac{1}{\partial^+} \,\bar{\phi}^a \;\; \phi^b \;\, \overline{\nabla}\, \phi^c \; + \; \frac{1}{\partial^+} \,\phi^a \;\, \bar{\phi}^b \;\, \nabla \,\bar{\phi}^c \right) ,$$

explicit calculations yield

$$\delta_J \left(\mathcal{V} + \overline{\mathcal{V}} \right) \;\; = \;\; (\alpha \, - \, 1)\, f_{abc}\, \overline{\omega}_{mn} \times$$

$$\times \int \left(\frac{1}{\partial^+} \,\bar{\phi}^a \phi^b \partial^{mn} \, \phi^c + \frac{i}{\sqrt{2}\,\partial^+} \,\bar{\phi}^a \phi^b d^m \, d^n \, \frac{\partial}{\partial^+} \,\bar{\phi}^c \right) .$$

The cubic vertex is $SO(8)$ invariant only for $\alpha = 1$.

Thus the generalized derivative reads,

$$\overline{\nabla} \; = \; \bar{\partial} \, + \, \frac{i}{4\,\sqrt{2}\,\partial^+} \, \bar{d}_p \, \bar{d}_q \, \partial^{pq}$$

Checks

To be sure, we have checked invariance by performing the Grassmann integrations, and looking at components.

To obtain this result, we have used the antisymmetry of the structure functions, the chiral constraints, the "inside-out" constraints, and performed integrations by parts on the coordinates and the Grassmann variables.

In this light-cone form, the Lorentz invariance in ten dimensions is automatic once the little group invariance has been established.

We have therefore shown ten-dimensional invariance(since the quartic term remained unaltered).

What is $\overline{\nabla}$?

$$\overline{\nabla} \equiv \bar{\partial} + \frac{i}{4\sqrt{2}\,\partial^+}\,\bar{d}_p\,\bar{d}_q\,\partial^{pq} \ ,$$

- The field theory on a D3-brane is oxidized into the field theory on a space-filling brane. Clearly, this involves the relaxation of open-string boundary conditions.

- Is there a curvature at play?

$$\overline{\nabla} \equiv \quad 1 \cdot \bar{\partial} + \frac{i}{4\sqrt{2}\partial^+}\,\bar{d}_p\,\bar{d}_q\,\partial^{pq} = \qquad e \cdot \bar{\partial} + e_{pq} \cdot \partial^{pq}$$

where the vielbein is simply, $E \,=\, (\,e\,,\,e_{pq}\,)$

$\mathcal{N} = 8$ Supergravity

The other maximally supersymmetric theory in $d = 4$ is the $\mathcal{N} = 8$ supergravity theory.

Obtained by reducing eleven-dimensional supergravity

$$SO(9) \supset SO(2) \times SO(7)$$

The $\mathcal{N} = 8$ theory was formulated in light-cone superspace by Bengtsson, Bengtsson and Brink(1982).

Oxidize this theory by simply introducing the additional 7-derivatives.

This offers a complete superspace description of ($\mathcal{N} = 1, d = 11$) Supergravity (up to order κ);

hep-th/0501079 (with Pierre Ramond and Lars Brink)

Next Step

Numerous aspects worth examining,

- Physical significance of $\overline{\nabla}$
- Extending the Supergravity result to order κ^2
- SuperFeynman rules
- Compute 1-loop graphs and track indices
- Couple the "oxidizing" derivative to an external field
- NC superspace: light-cone approach - different
- "Oxidize" theories that people have left lying around
- Evaluate 3-loop 4-point function in $\mathcal{N} = 8$ Supergravity