A Heterotic Standard Model

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Overview

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

hep-th/0410055: Elliptic Calabi-Yau Threefolds with $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson Lines
hep-th/0501070: A Heterotic Standard Model
hep-th/0502155: A Standard Model from the $E_8 \times E_8$ Heterotic Superstring
hep-th/0505041: Vector Bundle Extensions, Sheaf Cohomology, and the Heterotic Standard Model
hep-th/0509051: Heterotic Standard Model Moduli
hep-th/0510142: Moduli Dependent μ-Terms in a Heterotic Standard Model

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Introduction

Introduction

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

- Geometric compactification of the $E_8 \times E_8$ heterotic string.

Introduction

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

- Geometric compactification of the $E_8 \times E_8$ heterotic string.
- $d = 4, \mathcal{N} = 1$ $\Rightarrow$ stable background.

Introduction

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

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- $d = 4, \mathcal{N} = 1$ $\Rightarrow$ stable background.
- $SU(3)_C \times SU(2)_L \times U(1)_Y$.

Introduction

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

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Introduction

Introduction
- ❖ Introduction
- ❖ An Organizational Principle
- ❖ Wilson Line Breaking
- ❖ More Group Theory
- ❖ Yet More Group Theory
- ❖ Wish List

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

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- $\cancel{SU(3)_C \times SU(2)_L \times U(1)_Y}$.
- $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$
  $\Rightarrow$ proton decay suppressed.

Introduction

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

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- No exotic matter.

Introduction

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

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- $\cancel{SU(3)_C \times SU(2)_L \times U(1)_Y}$.
- $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$
  $\Rightarrow$ proton decay suppressed.
- No exotic matter.
- All of the ordinary matter fields (including right-handed Neutrino).

An Organizational Principle

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Ancient Lore: $\mathrm{Spin}(10)$ GUT with $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson lines "works":

$\underline{\mathbf{16}}$ of $\mathrm{Spin}(10)$: Breaks into one family of quarks and leptons including a right-handed Neutrino.

$\underline{\overline{\mathbf{16}}}$ of $\mathrm{Spin}(10)$: Anti-family.

$\underline{\mathbf{10}} = \underline{\overline{\mathbf{10}}}$ of $\mathrm{Spin}(10)$: Higgs and color triplets.

An Organizational Principle

Introduction
- Introduction
- An Organizational Principle
- Wilson Line Breaking
- More Group Theory
- Yet More Group Theory
- Wish List

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

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$\underline{\mathbf{10}} = \overline{\underline{\mathbf{10}}}$ of $\mathrm{Spin}(10)$: Higgs and color triplets.

Compactification scale $\sim$ GUT scale

... but nice way to package representations.

Wilson Line Breaking

Introduction
- Introduction
- An Organizational Principle
- Wilson Line Breaking
- More Group Theory
- Yet More Group Theory
- Wish List

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

$$\mathrm{Spin}(10) \supset SU(3)\times SU(2)\times U(1)\times U(1)\times \mathbb{Z}_3 \times \mathbb{Z}_3$$

$$\left\{ \begin{array}{c} \text{Standard Model} \\ \text{gauge group} \end{array} \right\} \times U(1)_{B-L} \times \{\text{Wilson lines}\}$$

$\mathbb{Z}_3 \times \mathbb{Z}_3$ is smallest Wilson line possible.

Wilson Line Breaking

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

$$\mathrm{Spin}(10) \supset SU(3)\times SU(2)\times U(1)\times U(1)\times \mathbb{Z}_3 \times \mathbb{Z}_3$$

$$\begin{aligned}\underline{\mathbf{16}} = {}& \chi_1^2\chi_2(\underline{\mathbf{3}}, \underline{\mathbf{2}}, 1, 1) \oplus \chi_1^2(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 6, 3) \oplus \\ & \oplus \chi_1^2\chi_2^2(\underline{\overline{\mathbf{3}}}, \underline{\mathbf{1}}, -4, -1) \oplus \chi_2^2(\underline{\overline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, -1) \oplus \\ & \oplus (\underline{\mathbf{1}}, \underline{\overline{\mathbf{2}}}, -3, -3) \oplus \chi_1(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0, 3)\end{aligned}$$

$$\begin{aligned}\underline{\mathbf{10}} = {}& \chi_1(\underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 0) \oplus \chi_1\chi_2(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -2, -2) \oplus \\ & \oplus \chi_1^2(\underline{\mathbf{1}}, \underline{\overline{\mathbf{2}}}, -3, 0) \oplus \chi_1^2\chi_2^2(\underline{\overline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, 2)\end{aligned}$$

Wilson Line Breaking

Introduction

- Introduction
- An Organizational Principle
- Wilson Line Breaking
- More Group Theory
- Yet More Group Theory
- Wish List

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

$$\mathrm{Spin}(10) \supset SU(3)\times SU(2)\times U(1)\times U(1)\times \mathbb{Z}_3 \times \mathbb{Z}_3$$

$$\begin{aligned}
\underline{\mathbf{16}} = {} & \chi_1^2\chi_2(\underline{\mathbf{3}},\underline{\mathbf{2}},1,1) \oplus \chi_1^2(\underline{\mathbf{1}},\underline{\mathbf{1}},6,3)\oplus \\
& \oplus \chi_1^2\chi_2^2(\underline{\overline{\mathbf{3}}},\underline{\mathbf{1}},-4,-1) \oplus \chi_2^2(\underline{\overline{\mathbf{3}}},\underline{\mathbf{1}},2,-1)\oplus \\
& \oplus (\underline{\mathbf{1}},\underline{\overline{\mathbf{2}}},-3,-3) \oplus \boxed{\chi_1(\underline{\mathbf{1}},\underline{\mathbf{1}},0,3)} \\
\underline{\mathbf{10}} = {} & \chi_1(\underline{\mathbf{1}},\underline{\mathbf{2}},3,0) \oplus \chi_1\chi_2(\underline{\mathbf{3}},\underline{\mathbf{1}},-2,-2)\oplus \\
& \oplus \chi_1^2(\underline{\mathbf{1}},\underline{\overline{\mathbf{2}}},-3,0) \oplus \chi_1^2\chi_2^2(\underline{\overline{\mathbf{3}}},\underline{\mathbf{1}},2,2)
\end{aligned}$$

Right-handed Neutrino

More Group Theory

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

$$G = \mathbb{Z}_3 \times \mathbb{Z}_3 = G_1 \times G_2$$

Fix generators g_1 and g_2.

Characters (=1-d representations): Denote generators by χ_1 and χ_2, where ($\omega = e^{\frac{2\pi i}{3}}$)

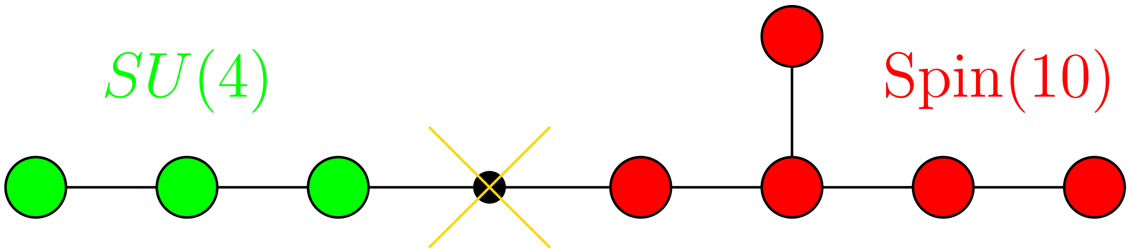
$$\begin{aligned} \chi_1(g_1) &= \omega & \qquad \chi_1(g_2) &= 1 \\ \chi_2(g_1) &= 1 & \qquad \chi_2(g_2) &= \omega\,. \end{aligned}$$

All other characters are products of χ_1 and χ_2.

Yet More Group Theory

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Maximal regular subgroup $SU(4) \times \mathrm{Spin}(10) \subset E_8$:



The adjoint of E_8 (fermions in the $E_8 \times E_8$ heterotic string) decomposes as

$$\underline{\mathbf{248}} = (\underline{\mathbf{1}}, \underline{\mathbf{45}}) \oplus (\underline{\mathbf{15}}, \underline{\mathbf{1}}) \oplus (\underline{\mathbf{4}}, \underline{\mathbf{16}}) \oplus (\underline{\overline{\mathbf{4}}}, \underline{\overline{\mathbf{16}}}) \oplus (\underline{\mathbf{6}}, \underline{\mathbf{10}})$$

Wish List

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

To make use of this group theory, we would like

- A Calabi-Yau threefold X with $\mathbb{Z}_3 \times \mathbb{Z}_3$ fundamental group.

Wish List

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

To make use of this group theory, we would like

- A Calabi-Yau threefold X with $\mathbb{Z}_3 \times \mathbb{Z}_3$ fundamental group.
- The Calabi-Yau should be torus fibered.

Wish List

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

To make use of this group theory, we would like

- A Calabi-Yau threefold X with $\mathbb{Z}_3 \times \mathbb{Z}_3$ fundamental group.
- The Calabi-Yau should be torus fibered.
- A $SU(4) \subset E_8$ instanton leaves $\mathrm{Spin}(10)$ unbroken, so we want a rank 4 stable holomorphic vector bundle V on X.

Wish List

Introduction
❖ Introduction
❖ An Organizational Principle
❖ Wilson Line Breaking
❖ More Group Theory
❖ Yet More Group Theory
❖ Wish List
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

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- A Calabi-Yau threefold X with $\mathbb{Z}_3 \times \mathbb{Z}_3$ fundamental group.
- The Calabi-Yau should be torus fibered.
- A $SU(4) \subset E_8$ instanton leaves $\mathrm{Spin}(10)$ unbroken, so we want a rank 4 stable holomorphic vector bundle V on X.
- With the "right" cohomology groups (low energy spectrum).

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

The Calabi-Yau

Calabi-Yau Introduction

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

Calabi-Yau threefold X with $\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$

Calabi-Yau Introduction

Introduction
The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Work with		Have in mind
Simply connected Calabi-Yau threefold $\widetilde{X}$ with free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action	$=$	Calabi-Yau threefold X with $\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$

Calabi-Yau Introduction

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

Work with		Have in mind
Simply connected Calabi-Yau threefold $\widetilde{X}$ with free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action	$=$	Calabi-Yau threefold X with $\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$
elliptically fibered (torus fibered with section)		torus fibered (assuming $\mathbb{Z}_3 \times \mathbb{Z}_3$ preserves fibration)

Calabi-Yau Construction

Introduction

The Calabi-Yau

❖ Calabi-Yau Introduction

❖ Calabi-Yau Construction

❖ Calabi-Yau Properties

❖ Group Actions on the Base I

❖ Group Actions on the Base II

❖ Invariant Cohomology

❖ Divisors on the Base

❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

Start with two dP_9 surfaces B_1 and B_2.

Calabi-Yau Construction

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

Start with two dP_9 surfaces $\begin{array}{c} B_1 \\ \downarrow \beta_1 \\ \mathbb{P}^1 \end{array}$ and $\begin{array}{c} B_2 \\ \downarrow \beta_2 \\ \mathbb{P}^1 \end{array}$.

Note: dP_9 are elliptically fibered; Fibers over a generic point $x \in \mathbb{P}^1$ are

$$\beta_1^{-1}(x) \simeq T^2 \subset B_1 , \qquad \beta_2^{-1}(x) \simeq T^2 \subset B_2 .$$

Calabi-Yau Construction

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

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Note: dP_9 are elliptically fibered; Fibers over a generic point $x \in \mathbb{P}^1$ are

$$\beta_1^{-1}(x) \simeq T^2 \subset B_1 , \qquad \beta_2^{-1}(x) \simeq T^2 \subset B_2 .$$

The fiber product $B_1 \times_{\mathbb{P}^1} B_2$ is the fibration over $\mathbb{P}^1$ with fiber

$$\beta^{-1}(x) = \beta_1^{-1}(x) \times \beta_2^{-1}(x)$$

Calabi-Yau Construction

Introduction
The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

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Calabi-Yau Properties

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

- $\widetilde{X} \stackrel{\text{def}}{=} B_1 \times_{\mathbb{P}^1} B_2$ is a simply connected Calabi-Yau threefold, $c_1(\widetilde{X}) = 0$.

Calabi-Yau Properties

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

- $\widetilde{X} \stackrel{\text{def}}{=} B_1 \times_{\mathbb{P}^1} B_2$ is a simply connected Calabi-Yau threefold, $c_1(\widetilde{X}) = 0$.
- Every elliptically fibered Calabi-Yau over a dP_9 is such a fiber product.

Calabi-Yau Properties

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

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- Every elliptically fibered Calabi-Yau over a dP_9 is such a fiber product.
- $h^{1,1}(\widetilde{X}) = 19 = h^{2,1}(\widetilde{X})$

Calabi-Yau Properties

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

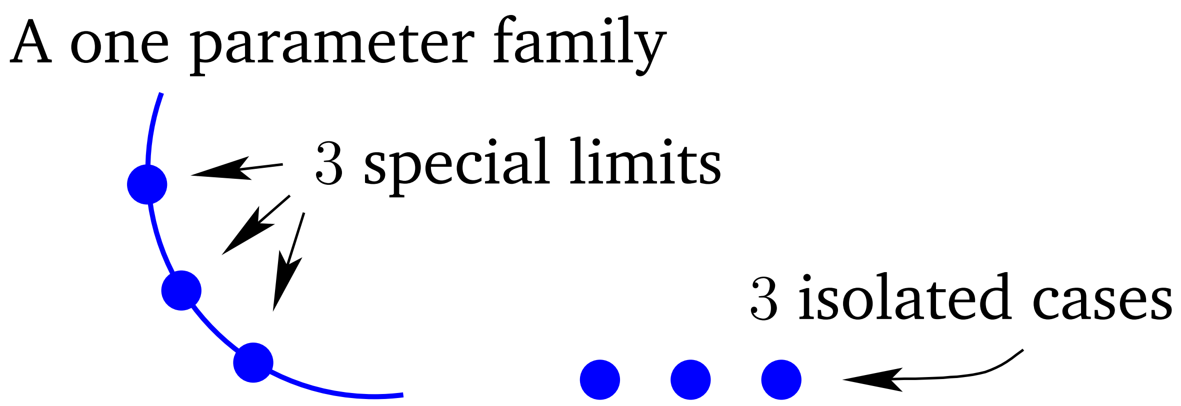
- $\widetilde{X} \stackrel{\text{def}}{=} B_1 \times_{\mathbb{P}^1} B_2$ is a simply connected Calabi-Yau threefold, $c_1(\widetilde{X}) = 0$.
- Every elliptically fibered Calabi-Yau over a dP_9 is such a fiber product.
- $h^{1,1}(\widetilde{X}) = 19 = h^{2,1}(\widetilde{X})$
- Group actions on B_1, B_2 lift to $\widetilde{X}$ if their action on the common base $\mathbb{P}^1$ is identical.

Group Actions on the Base I

Introduction
The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

We classified all $\mathbb{Z}_3 \times \mathbb{Z}_3$ actions on dP_9 surfaces.

The moduli space looks like this:



Group Actions on the Base II

Introduction

The Calabi-Yau

❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

All such dP_9 surfaces with $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action give rise to a G action on $\widetilde{X}$.

Group Actions on the Base II

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

All such dP_9 surfaces with $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action give rise to a G action on $\widetilde{X}$.

- The 3 isolated cases never yield a free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action.

Group Actions on the Base II

Introduction
The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

All such dP_9 surfaces with $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action give rise to a G action on $\widetilde{X}$.

- The 3 isolated cases never yield a free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action.
- The one-parameter family and its limits can give a free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on $\widetilde{X}$.

We only consider this one-parameter family in the following.

Invariant Cohomology

Introduction
The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

$G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action free

$$\Rightarrow \qquad H^{p,q}\big(X\big) = H^{p,q}\big(\widetilde{X}\big)^G$$

Invariant Cohomology

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

$G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action free

$$\Rightarrow \qquad H^{p,q}\big(X\big) = H^{p,q}\big(\widetilde{X}\big)^G$$

Hodge diamond $h^{p,q}(X) =$
$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 3 & & 0 & \\ 1 & & 3 & & 3 & & 1 \\ & 0 & & 3 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

Invariant Cohomology

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

$G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action free

$$\Rightarrow \qquad H^{p,q}\big(X\big) = H^{p,q}\big(\widetilde{X}\big)^G$$

Hodge diamond $h^{p,q}(X) = $

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 3 & & 0 & \\ 1 & & 3 & & 3 & & 1 \\ & 0 & & 3 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

$h^{1,1}\big(X\big) = 3$ dimensional space of divisor classes.

Divisors on the Base

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

$$
\begin{tikzcd}
\dim_{\mathbb{C}} = 3 : & & \widetilde{X} \arrow[dl, "\pi_1"'] \arrow[dr, "\pi_2"] & \\
\dim_{\mathbb{C}} = 2 : & B_1 \arrow[dr, "\beta_1"'] & & B_2 \arrow[dl, "\beta_2"] \\
\dim_{\mathbb{C}} = 1 : & & \mathbb{P}^1 &
\end{tikzcd}
$$

Divisors on the Base

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

$$
\begin{tikzcd}
\dim_{\mathbb{C}} = 3 : & & \widetilde{X} \arrow[dl, "\pi_1"'] \arrow[dr, "\pi_2"] & \\
\dim_{\mathbb{C}} = 2 : & B_1 \arrow[dr, "\beta_1"'] & & B_2 \arrow[dl, "\beta_2"] \\
\dim_{\mathbb{C}} = 1 : & & \mathbb{P}^1 &
\end{tikzcd}
$$

Invariant divisors on the base B_1, B_2:

$$H^{1,1}\big(B_1\big)^G = \mathbb{C} f_1 \oplus \mathbb{C} t_1$$

$$H^{1,1}\big(B_2\big)^G = \mathbb{C} f_2 \oplus \mathbb{C} t_2$$

Divisors on the Calabi-Yau

Introduction

The Calabi-Yau
❖ Calabi-Yau Introduction
❖ Calabi-Yau Construction
❖ Calabi-Yau Properties
❖ Group Actions on the Base I
❖ Group Actions on the Base II
❖ Invariant Cohomology
❖ Divisors on the Base
❖ Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

Pull-back of divisors from the base

$$\pi_1^{-1}(f_1) = \left\{ T^4 \text{ fiber of } \begin{array}{c} \widetilde{X} \\ \downarrow \\ \mathbb{P}^1 \end{array} \right\} = \pi_2^{-1}(f_2) \overset{\text{def}}{=} \phi$$

$$\pi_1^{-1}(t_1) \overset{\text{def}}{=} \tau_1 \qquad \pi_2^{-1}(t_2) \overset{\text{def}}{=} \tau_2$$

Divisors on the Calabi-Yau

Introduction

The Calabi-Yau
- Calabi-Yau Introduction
- Calabi-Yau Construction
- Calabi-Yau Properties
- Group Actions on the Base I
- Group Actions on the Base II
- Invariant Cohomology
- Divisors on the Base
- Divisors on the Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

Pull-back of divisors from the base

$$\pi_1^{-1}(f_1) = \left\{ T^4 \text{ fiber of } \begin{array}{c} \widetilde{X} \\ \downarrow \\ \mathbb{P}^1 \end{array} \right\} = \pi_2^{-1}(f_2) \stackrel{\text{def}}{=} \phi$$

$$\pi_1^{-1}(t_1) \stackrel{\text{def}}{=} \tau_1 \qquad \pi_2^{-1}(t_2) \stackrel{\text{def}}{=} \tau_2$$

$$\begin{aligned} H^{1,1}\big(\widetilde{X}\big)^G &= \mathbb{C}\pi_1^{-1}(f_1) + \mathbb{C}\pi_1^{-1}(t_1) + \\ &\quad + \mathbb{C}\pi_2^{-1}(f_2) + \mathbb{C}\pi_2^{-1}(t_2) = \\ &= \mathbb{C}\phi \oplus \mathbb{C}\tau_1 \oplus \mathbb{C}\tau_2 \end{aligned}$$

Introduction

The Calabi-Yau

The Vector Bundle

❖ Line Bundles

❖ Equivariant Line Bundles I

❖ Equivariant Line Bundles II

❖ Notation

❖ The Serre Construction

❖ Equivariant Vector Bundles

❖ Equivariant Example

❖ Constructing Vector Bundles

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

The Vector Bundle

Line Bundles

Introduction

The Calabi-Yau

The Vector Bundle

❖ Line Bundles

❖ Equivariant Line Bundles I

❖ Equivariant Line Bundles II

❖ Notation

❖ The Serre Construction

❖ Equivariant Vector Bundles

❖ Equivariant Example

❖ Constructing Vector Bundles

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

On any variety Y, we have

$$\Big\{ \text{Divisors } D \Big\} \Big/ \sim \;=\; \Big\{ \text{Line bundles } \mathcal{O}_Y(D) \Big\}$$

Line Bundles

Introduction

The Calabi-Yau

The Vector Bundle

❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

On any variety Y, we have

$$\Big\{\text{Divisors } D\Big\}\Big/\sim \;=\; \Big\{\text{Line bundles } \mathcal{O}_Y(D)\Big\}$$

Linear equivalence

For $\widetilde{X}$, B_1, B_2, $\mathbb{P}^1$ that is just cohomology class of the divisor in $H^{1,1}$.

Line Bundles

Introduction

The Calabi-Yau

The Vector Bundle

❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

On any variety Y, we have

$$\Big\{\text{Divisors } D\Big\}\Big/\sim \;=\; \Big\{\text{Line bundles } \mathcal{O}_Y(D)\Big\}$$

Linear equivalence

For $\widetilde{X}$, B_1, B_2, $\mathbb{P}^1$ that is just cohomology class of the divisor in $H^{1,1}$.

Every line bundle is of the form

- $\mathcal{O}_{\widetilde{X}}(x_1\tau_1 + x_2\tau_2 + x_3\phi)$, $x_1, x_2, x_3 \in \mathbb{Z}$.
- $\mathcal{O}_{B_i}(y_1 t_i + y_2 f_i)$, $y_1, y_2 \in \mathbb{Z}$.
- $\mathcal{O}_{\mathbb{P}^1}(n)$, $n \in \mathbb{Z}$.

Equivariant Line Bundles I

Introduction
The Calabi-Yau
The Vector Bundle
❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Line bundles on

$$X = \widetilde{X}/G$$

Equivariant Line Bundles I

Introduction
The Calabi-Yau
The Vector Bundle
❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Work with		Have in mind
G-equivariant line bundles on $\widetilde{X}$	$=$	Line bundles on $X = \widetilde{X}/G$

Equivariant Line Bundles I

Introduction
The Calabi-Yau
The Vector Bundle
❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Work with		Have in mind
G-equivariant line bundles on $\widetilde{X}$	$=$	Line bundles on $X = \widetilde{X}/G$

An equivariant line bundle is a line bundle $\mathcal{L}$ together with a group action $\gamma : G \times \mathcal{L} \to \mathcal{L}$:

$$\begin{tikzcd} \mathcal{L} \arrow[r, "\gamma_g"'] \arrow[d] & \mathcal{L} \arrow[d] \\ \widetilde{X} \arrow[r, "g"'] & \widetilde{X} \end{tikzcd}$$

Equivariant Line Bundles II

Introduction

The Calabi-Yau

The Vector Bundle

❖ Line Bundles

❖ Equivariant Line Bundles I

❖ Equivariant Line Bundles II

❖ Notation

❖ The Serre Construction

❖ Equivariant Vector Bundles

❖ Equivariant Example

❖ Constructing Vector Bundles

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

- Most line bundles on $\widetilde{X}$ cannot be made equivariant.

Equivariant Line Bundles II

Introduction
The Calabi-Yau
The Vector Bundle
❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

- Most line bundles on $\widetilde{X}$ cannot be made equivariant.
- Only the line bundles $\mathcal{O}_{\widetilde{X}}(x_1\tau_1 + x_2\tau_2 + x_3\phi)$, $x_1, x_2, x_3 \in \mathbb{Z}$ with $x_1 + x_2 \equiv 0 \mod 3$ allow for a $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action.

Equivariant Line Bundles II

Introduction
The Calabi-Yau
The Vector Bundle
❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

- Most line bundles on $\widetilde{X}$ cannot be made equivariant.
- Only the line bundles $\mathcal{O}_{\widetilde{X}}(x_1\tau_1 + x_2\tau_2 + x_3\phi)$, $x_1, x_2, x_3 \in \mathbb{Z}$ with $x_1 + x_2 \equiv 0 \mod 3$ allow for a $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action.
- In these cases, there is always more than one G action

  $\Rightarrow$ Different equivariant line bundles!

Notation

Introduction
The Calabi-Yau
The Vector Bundle
❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Consider the trivial line bundle $\mathcal{O}_{\widetilde{X}} = \widetilde{X} \times \mathbb{C}$.

- Obvious equivariant action $\gamma_g : \widetilde{X} \times \mathbb{C} \to \widetilde{X} \times \mathbb{C},\ (p, v) \mapsto \big(g(p), v\big)$

Notation

Introduction

The Calabi-Yau

The Vector Bundle

❖ Line Bundles

❖ Equivariant Line Bundles I

❖ Equivariant Line Bundles II

❖ Notation

❖ The Serre Construction

❖ Equivariant Vector Bundles

❖ Equivariant Example

❖ Constructing Vector Bundles

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

Consider the trivial line bundle $\mathcal{O}_{\widetilde{X}} = \widetilde{X} \times \mathbb{C}$.

- Obvious equivariant action
  $\gamma_g : \widetilde{X} \times \mathbb{C} \to \widetilde{X} \times \mathbb{C},\ (p, v) \mapsto \big(g(p), v\big)$
- Different equivariant action by multiplying with a character
  $\chi\gamma_g : \widetilde{X} \times \mathbb{C} \to \widetilde{X} \times \mathbb{C},\ (p, v) \mapsto \big(g(p), \chi(g)v\big)$

Notation

Introduction
The Calabi-Yau
The Vector Bundle
❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Consider the trivial line bundle $\mathcal{O}_{\widetilde{X}} = \widetilde{X} \times \mathbb{C}$.

- Obvious equivariant action
  $\gamma_g : \widetilde{X} \times \mathbb{C} \to \widetilde{X} \times \mathbb{C},\ (p, v) \mapsto \big(g(p), v\big)$
- Different equivariant action by multiplying with a character
  $\chi\gamma_g : \widetilde{X} \times \mathbb{C} \to \widetilde{X} \times \mathbb{C},\ (p, v) \mapsto \big(g(p), \chi(g)v\big)$
- We write $\chi\mathcal{O}_{\widetilde{X}}$ for this different equivariant line bundle.

The Serre Construction

Introduction
The Calabi-Yau
The Vector Bundle
❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

A way to construct may stable rank 2 vector bundles on a surface (here: B_1 and B_2).

- Take two line bundles $\mathcal{L}_1, \mathcal{L}_2$.
- An ideal sheaf I (sheaf of functions vanishing at some fixed points).
- Define $\mathcal{S}$ as an extension

$$0 \longrightarrow \mathcal{L}_1 \longrightarrow \mathcal{S} \longrightarrow \mathcal{L}_2 \otimes I \longrightarrow 0$$

- Cayley-Bacharach property $\Rightarrow$ generic extension is a rank 2 vector bundle.

Equivariant Vector Bundles

Introduction

The Calabi-Yau

The Vector Bundle

❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

$$\boxed{\text{Vector bundles on } X = \widetilde{X}/G}$$

Equivariant Vector Bundles

Introduction

The Calabi-Yau

The Vector Bundle

❖ Line Bundles

❖ Equivariant Line Bundles I

❖ Equivariant Line Bundles II

❖ Notation

❖ The Serre Construction

❖ Equivariant Vector Bundles

❖ Equivariant Example

❖ Constructing Vector Bundles

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

Work with		Have in mind
G-equivariant vector bundles on $\widetilde{X}$	$=$	Vector bundles on $X = \widetilde{X}/G$

Equivariant Vector Bundles

Introduction

The Calabi-Yau

The Vector Bundle

❖ Line Bundles

❖ Equivariant Line Bundles I

❖ Equivariant Line Bundles II

❖ Notation

❖ The Serre Construction

❖ Equivariant Vector Bundles

❖ Equivariant Example

❖ Constructing Vector Bundles

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

Work with		Have in mind
G-equivariant vector bundles on $\widetilde{X}$	$=$	Vector bundles on $X = \widetilde{X}/G$

Problem: Even if $\mathcal{E}$, $\mathcal{F}$ are equivariant,

$$0 \longrightarrow \mathcal{E} \longrightarrow \mathcal{V} \longrightarrow \mathcal{F} \longrightarrow 0$$

Extension is not necessarily equivariant!

Only extensions in $\mathrm{Ext}^1\left(\mathcal{F}, \mathcal{E}\right)^G$ are equivariant.

Equivariant Example

$$0 \longrightarrow \mathcal{O}_{B_2}(-2f_2) \longrightarrow \mathcal{W} \longrightarrow \chi_2 \mathcal{O}_{B_2}(2f_2) \otimes I_9 \longrightarrow 0$$

Equivariant Example

$$0 \longrightarrow \mathcal{O}_{B_2}(-2f_2) \longrightarrow \mathcal{W} \longrightarrow \chi_2 \mathcal{O}_{B_2}(2f_2) \otimes I_9 \longrightarrow 0$$

- $\mathcal{O}_{B_2}(-2f_2)$, $\chi_2 \mathcal{O}_{B_2}(2f_2)$ are equivariant.

Equivariant Example

$$0 \longrightarrow \mathcal{O}_{B_2}(-2f_2) \longrightarrow \mathcal{W} \longrightarrow \chi_2 \mathcal{O}_{B_2}(2f_2) \otimes I_9 \longrightarrow 0$$

- $\mathcal{O}_{B_2}(-2f_2)$, $\chi_2 \mathcal{O}_{B_2}(2f_2)$ are equivariant.
- I_9 is the ideal sheaf of one G orbit.

Equivariant Example

$$0 \longrightarrow \mathcal{O}_{B_2}(-2f_2) \longrightarrow \mathcal{W} \longrightarrow \chi_2 \mathcal{O}_{B_2}(2f_2) \otimes I_9 \longrightarrow 0$$

- $\mathcal{O}_{B_2}(-2f_2)$, $\chi_2 \mathcal{O}_{B_2}(2f_2)$ are equivariant.
- I_9 is the ideal sheaf of one G orbit.
- Has the Cayley-Bacharach property.

Equivariant Example

$$0 \longrightarrow \mathcal{O}_{B_2}(-2f_2) \longrightarrow \mathcal{W} \longrightarrow \chi_2\mathcal{O}_{B_2}(2f_2) \otimes I_9 \longrightarrow 0$$

- $\mathcal{O}_{B_2}(-2f_2)$, $\chi_2\mathcal{O}_{B_2}(2f_2)$ are equivariant.
- I_9 is the ideal sheaf of one G orbit.
- Has the Cayley-Bacharach property.
- $\mathrm{Ext}^1\left(\chi_2\mathcal{O}_{B_2}(2f_2) \otimes I_9,\ \mathcal{O}_{B_2}(-2f_2)\right) = \mathbb{C}^9$

Equivariant Example

$$0 \longrightarrow \mathcal{O}_{B_2}(-2f_2) \longrightarrow \mathcal{W} \longrightarrow \chi_2 \mathcal{O}_{B_2}(2f_2) \otimes I_9 \longrightarrow 0$$

- $\mathcal{O}_{B_2}(-2f_2)$, $\chi_2 \mathcal{O}_{B_2}(2f_2)$ are equivariant.
- I_9 is the ideal sheaf of one G orbit.
- Has the Cayley-Bacharach property.
- $\mathrm{Ext}^1\Big(\chi_2 \mathcal{O}_{B_2}(2f_2) \otimes I_9,\ \mathcal{O}_{B_2}(-2f_2)\Big) = \mathrm{Reg}(G) =$
  $1 \ \oplus \chi_1 \oplus \chi_1^2 \oplus \chi_2 \oplus \chi_1\chi_2 \oplus \chi_1^2\chi_2 \oplus \chi_2^2 \oplus \chi_1\chi_2^2 \oplus \chi_1^2\chi_2^2$

Equivariant Example

$$0 \longrightarrow \mathcal{O}_{B_2}(-2f_2) \longrightarrow \mathcal{W} \longrightarrow \chi_2 \mathcal{O}_{B_2}(2f_2) \otimes I_9 \longrightarrow 0$$

- $\mathcal{O}_{B_2}(-2f_2)$, $\chi_2 \mathcal{O}_{B_2}(2f_2)$ are equivariant.
- I_9 is the ideal sheaf of one G orbit.
- Has the Cayley-Bacharach property.
- $\mathrm{Ext}^1\Big(\chi_2 \mathcal{O}_{B_2}(2f_2) \otimes I_9,\ \mathcal{O}_{B_2}(-2f_2)\Big) = \mathrm{Reg}(G) =$
  $1 \oplus \chi_1 \oplus \chi_1^2 \oplus \chi_2 \oplus \chi_1\chi_2 \oplus \chi_1^2\chi_2 \oplus \chi_2^2 \oplus \chi_1\chi_2^2 \oplus \chi_1^2\chi_2^2$
  so there exist extensions.

Constructing Vector Bundles

Introduction
The Calabi-Yau
The Vector Bundle
❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Building blocks:

- Line bundles on $\widetilde{X}$.
- Rank 2 bundles pulled back from B_1, B_2.

Constructing Vector Bundles

Introduction
The Calabi-Yau
The Vector Bundle
❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Building blocks:

- Line bundles on $\widetilde{X}$.
- Rank 2 bundles pulled back from B_1, B_2.

Operations:

- Tensor product of bundles.

Constructing Vector Bundles

Introduction

The Calabi-Yau

The Vector Bundle
❖ Line Bundles
❖ Equivariant Line Bundles I
❖ Equivariant Line Bundles II
❖ Notation
❖ The Serre Construction
❖ Equivariant Vector Bundles
❖ Equivariant Example
❖ Constructing Vector Bundles

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

Building blocks:

- Line bundles on $\widetilde{X}$.
- Rank 2 bundles pulled back from B_1, B_2.

Operations:

- Tensor product of bundles.
- Sums of bundles.

Constructing Vector Bundles

Introduction

The Calabi-Yau

The Vector Bundle

❖ Line Bundles

❖ Equivariant Line Bundles I

❖ Equivariant Line Bundles II

❖ Notation

❖ The Serre Construction

❖ Equivariant Vector Bundles

❖ Equivariant Example

❖ Constructing Vector Bundles

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

Building blocks:

- Line bundles on $\widetilde{X}$.
- Rank 2 bundles pulled back from B_1, B_2.

Operations:

- Tensor product of bundles.
- ~~*Sums of bundles.*~~ Never (slope-) stable!
- Extensions of bundles.

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
❖ The Gauge Bundle
❖ Particle Spectrum
❖ The Lagrangian
❖ The String Miracle
Spectral Sequences
A New Heterotic Standard Model
Conclusion

A First Heterotic Standard Model

The Gauge Bundle

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
❖ The Gauge Bundle
❖ Particle Spectrum
❖ The Lagrangian
❖ The String Miracle
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Define these two rank 2 vector bundles

$$\begin{aligned}\mathcal{V}_1 &\stackrel{\text{def}}{=} \chi_2 \mathcal{O}_{\widetilde{X}}(-\tau_1+\tau_2) \oplus \chi_2 \mathcal{O}_{\widetilde{X}}(-\tau_1+\tau_2) = \\ &= 2\chi_2 \mathcal{O}_{\widetilde{X}}(-\tau_1+\tau_2) \\ \mathcal{V}_2 &\stackrel{\text{def}}{=} \mathcal{O}_{\widetilde{X}}(\tau_1-\tau_2) \otimes \pi_2^*(\mathcal{W})\end{aligned}$$

We define the rank 4 bundle $\mathcal{V}$ finally as a generic extension

$$0 \longrightarrow \mathcal{V}_2 \longrightarrow \mathcal{V} \longrightarrow \mathcal{V}_1 \longrightarrow 0$$

hep-th/0501070: A Heterotic Standard Model
hep-th/0502155: A Standard Model from the $E_8 \times E_8$ Heterotic Superstring
hep-th/0505041: Vector Bundle Extensions, Sheaf Cohomology, and the Heterotic Standard Model

Particle Spectrum

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

❖ The Gauge Bundle

❖ Particle Spectrum

❖ The Lagrangian

❖ The String Miracle

Spectral Sequences

A New Heterotic Standard Model

Conclusion

- 3 families of quarks and leptons.
- Zero anti-families.
- 4 Higgs (twice MSSM).
- Doublets and triplets are completely split, all triplets are projected out.
- Hidden pure E_7 or $\mathrm{Spin}(12)$ with 2 matter fields.
- 6 geometric moduli, 19 vector bundle moduli, some hidden E_8 bundle moduli.

hep-th/0509051: Heterotic Standard Model Moduli

The Lagrangian

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
❖ The Gauge Bundle
❖ Particle Spectrum
❖ The Lagrangian
❖ The String Miracle
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Of course, we do not know the Kähler potential. What can we learn from the superpotential W?

- Higgs μ-terms $\phi H \bar{H}$
- Yukawa couplings $Q_i H \bar{Q}_i + Q_i \bar{H} \bar{Q}_i$

Field	Name
ϕ	Vector bundle moduli
H	Higgs
$\bar{H}$	Higgs-conjugate
Q_i	Quarks & leptons of the i-th family
$\bar{Q}_i$	Anti-Q_i

The String Miracle

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

❖ The Gauge Bundle

❖ Particle Spectrum

❖ The Lagrangian

❖ The String Miracle

Spectral Sequences

A New Heterotic Standard Model

Conclusion

Compactifying on $(\widetilde{X}, V)/G$, we found

The String Miracle

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
❖ The Gauge Bundle
❖ Particle Spectrum
❖ The Lagrangian
❖ The String Miracle
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Compactifying on $(\widetilde{X}, \mathcal{V})/G$, we found

- Higgs μ-terms $\phi H \bar{H}$ with 4 out of the 19 vector bundle moduli.

hep-th/0510142: Moduli Dependent μ-Terms in a Heterotic Standard Model

The String Miracle

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
❖ The Gauge Bundle
❖ Particle Spectrum
❖ The Lagrangian
❖ The String Miracle
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Compactifying on $(\widetilde{X}, \mathcal{V})/G$, we found

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hep-th/0510142: Moduli Dependent μ-Terms in a Heterotic Standard Model

The String Miracle

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
❖ The Gauge Bundle
❖ Particle Spectrum
❖ The Lagrangian
❖ The String Miracle
Spectral Sequences
A New Heterotic Standard Model
Conclusion

Compactifying on $(\widetilde{X}, \mathcal{V})/G$, we found

- Higgs μ-terms $\phi H \bar{H}$ with 4 out of the 19 vector bundle moduli.
- No Yukawa couplings.

$\boxed{\text{Yukawa textures without symmetries!?!}}$

hep-th/0510142: Moduli Dependent μ-Terms in a Heterotic Standard Model

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
❖ Leray Spectral Sequence
❖ An Example
❖ Leray Degrees
❖ Leray Degree Table
❖ The Superpotential
❖ More on Leray Degrees
A New Heterotic Standard Model
Conclusion

Spectral Sequences

Leray Spectral Sequence

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
❖ Leray Spectral Sequence
❖ An Example
❖ Leray Degrees
❖ Leray Degree Table
❖ The Superpotential
❖ More on Leray Degrees
A New Heterotic Standard Model
Conclusion

How did we compute all these cohomology groups?

Leray spectral sequence for any sheaf $\mathcal{F}$ on $\widetilde{X} \to B_2$:

$$E_2^{p,q} = H^p\Big(B_2,\ R^q \pi_{2*}\mathcal{F}\Big) \quad \Rightarrow \quad H^{p+q}\Big(\widetilde{X},\ \mathcal{F}\Big)$$

Leray Spectral Sequence

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
❖ Leray Spectral Sequence
❖ An Example
❖ Leray Degrees
❖ Leray Degree Table
❖ The Superpotential
❖ More on Leray Degrees
A New Heterotic Standard Model
Conclusion

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$R^q\pi_{2*}$ is just the degree q cohomology along the fiber.

Leray Spectral Sequence

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

❖ Leray Spectral Sequence

❖ An Example

❖ Leray Degrees

❖ Leray Degree Table

❖ The Superpotential

❖ More on Leray Degrees

A New Heterotic Standard Model

Conclusion

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$R^q\pi_{2*}$ is just the degree q cohomology along the fiber.

Think of $E_2^{p,q}$ as the “forms with p legs along the base and q legs along the fiber”.

An Example

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences
❖ Leray Spectral Sequence
❖ An Example
❖ Leray Degrees
❖ Leray Degree Table
❖ The Superpotential
❖ More on Leray Degrees

A New Heterotic Standard Model

Conclusion

Example: $H^1\Big(\widetilde{X}, \wedge^2\mathcal{V}\Big) = H^1\Big(\widetilde{X}, 2\chi_2\pi_2^*(\mathcal{W})\Big)$

$$\pi_{2*}\Big(2\chi_2\pi_2^*(\mathcal{W})\Big) = 2\chi_2\mathcal{W}$$

$$R^1\pi_{2*}\Big(2\chi_2\pi_2^*(\mathcal{W})\Big) = 2\chi_1\chi_2\mathcal{W} \otimes \mathcal{O}_{B_2}(-f_2)$$

Compute $H^p(B_1, \cdots)$ by two more Leray SS...

$$\Rightarrow E_2^{p,q} = \begin{array}{r|ccc} q{=}1 & 0 & 2{\oplus}2\chi_1{\oplus}2\chi_2{\oplus}2\chi_1^2{\oplus}2\chi_2^2{\oplus}2\chi_1\chi_2^2{\oplus}2\chi_1^2\chi_2 & 0 \\ q{=}0 & 0 & 2{\oplus}2\chi_1{\oplus}2\chi_2{\oplus}2\chi_1^2{\oplus}2\chi_2^2{\oplus}2\chi_1\chi_2^2{\oplus}2\chi_1^2\chi_2 & 0 \\ \hline & p{=}0 & p{=}1 & p{=}2 \end{array}$$

Leray Degrees

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
❖ Leray Spectral Sequence
❖ An Example
❖ Leray Degrees
❖ Leray Degree Table
❖ The Superpotential
❖ More on Leray Degrees
A New Heterotic Standard Model
Conclusion

The two fibrations

$$
\begin{array}{ll}
\dim_{\mathbb{C}} = 3 : & \\
\dim_{\mathbb{C}} = 2 : & \\
\dim_{\mathbb{C}} = 1 : &
\end{array}
\qquad
\begin{tikzcd}
 & \widetilde{X} \arrow[dl, "\pi_1"'] \arrow[dr, "\pi_2"] & \\
B_1 \arrow[dr, "\beta_1"'] & & B_2 \arrow[dl, "\beta_2"] \\
 & \mathbb{P}^1 &
\end{tikzcd}
$$

allow us to refine the cohomology degree according to # of legs in the π_1 fiber, the base, and the π_2 fiber direction.

Leray Degree Table

Field	Cohomology	Fiber 1	Base	Fiber 2
$Q_i,\ \bar{Q}_i$	$H^1\big(\widetilde{X}, \mathcal{V}\big)$	0	0	1
$H_1,\ H_2$	$H^1\big(\widetilde{X}, \wedge^2\mathcal{V}\big)$	0	1	0
$\bar{H}_1,\ \bar{H}_2$	$H^1\big(\widetilde{X}, \wedge^2\mathcal{V}\big)$	0	0	1
$\phi_1, \ldots, \phi_4$	$H^1\big(\widetilde{X}, \mathcal{V} \otimes \mathcal{V}^\vee\big)$	1	0	0
$\phi_5, \ldots, \phi_{19}$	$H^1\big(\widetilde{X}, \mathcal{V} \otimes \mathcal{V}^\vee\big)$	0	0	1

Leray Degree Table

Field	Cohomology	Fiber 1	Base	Fiber 2
Q_i, $\bar{Q}_i$	$H^1\big(\widetilde{X}, \mathcal{V}\big)$	0	0	1
H_1, H_2	$H^1\big(\widetilde{X}, \wedge^2\mathcal{V}\big)$	0	1	0
$\bar{H}_1$, $\bar{H}_2$	$H^1\big(\widetilde{X}, \wedge^2\mathcal{V}\big)$	0	0	1
$\phi_1, \ldots, \phi_4$	$H^1\big(\widetilde{X}, \mathcal{V} \otimes \mathcal{V}^\vee\big)$	1	0	0
$\phi_5, \ldots, \phi_{19}$	$H^1\big(\widetilde{X}, \mathcal{V} \otimes \mathcal{V}^\vee\big)$	0	0	1
$\bar{\Omega}$	$H^3\big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\big)$	1	1	1

The Superpotential

The cubic terms in the superpotential are

The Superpotential

The cubic terms in the superpotential are

- Higgs μ-terms (note: $\wedge^2\mathcal{V} = \wedge^2\mathcal{V}^\vee$)

$$H^1\Big(\widetilde{X}, \mathcal{V} \otimes \mathcal{V}^\vee\Big) \otimes H^1\Big(\widetilde{X}, \wedge^2\mathcal{V}\Big) \otimes H^1\Big(\widetilde{X}, \wedge^2\mathcal{V}^\vee\Big) \longrightarrow H^3\Big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\Big) = \mathbb{C}$$

The Superpotential

The cubic terms in the superpotential are

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$$H^1\Big(\widetilde{X}, \mathcal{V}\otimes\mathcal{V}^\vee\Big) \otimes H^1\Big(\widetilde{X}, \wedge^2\mathcal{V}\Big) \otimes H^1\Big(\widetilde{X}, \wedge^2\mathcal{V}^\vee\Big) \longrightarrow H^3\Big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\Big) = \mathbb{C}$$

- Yukawa couplings

$$H^1\Big(\widetilde{X}, \mathcal{V}\Big) \otimes H^1\Big(\widetilde{X}, \mathcal{V}\Big) \otimes H^1\Big(\widetilde{X}, \wedge^2\mathcal{V}^\vee\Big) \longrightarrow H^3\Big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\Big) = \mathbb{C}$$

More on Leray Degrees

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

❖ Leray Spectral Sequence

❖ An Example

❖ Leray Degrees

❖ Leray Degree Table

❖ The Superpotential

❖ More on Leray Degrees

A New Heterotic Standard Model

Conclusion

The products respect the additional Leray degrees!

Field	Fiber 1	Base	Fiber 2
H_1, H_2	0	1	0
$\bar{H}_1, \bar{H}_2$	0	0	1
$\phi_1, \ldots, \phi_4$	1	0	0

More on Leray Degrees

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
❖ Leray Spectral Sequence
❖ An Example
❖ Leray Degrees
❖ Leray Degree Table
❖ The Superpotential
❖ More on Leray Degrees
A New Heterotic Standard Model
Conclusion

The products respect the additional Leray degrees!

Field	Fiber 1	Base	Fiber 2
$H_1,\ H_2$	0	1	0
$\bar{H}_1,\ \bar{H}_2$	0	0	1
$\phi_1, \ldots, \phi_4$	1	0	0

The only allowed cubic coupling is

$$W = \sum_{\substack{i=1..4 \\ a,b=1,2}} \lambda_{iab}\, \phi_i H_a \bar{H}_b$$

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

❖ Ideal Sheaves

❖ Serre Construction

❖ The Gauge Bundle

❖ Low Energy Spectrum

❖ Gauge Group Breaking

❖ Vector Bundle Breaking

❖ The Higgs Sector

❖ Cohomology

❖ Doublet-Triplet Splitting

❖ Leray Degrees

Conclusion

A New Heterotic Standard Model

Ideal Sheaves

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
❖ Ideal Sheaves
❖ Serre Construction
❖ The Gauge Bundle
❖ Low Energy Spectrum
❖ Gauge Group Breaking
❖ Vector Bundle Breaking
❖ The Higgs Sector
❖ Cohomology
❖ Doublet-Triplet Splitting
❖ Leray Degrees
Conclusion

I thought your solution was unique!

Whats new?

Ideal Sheaves

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
❖ Ideal Sheaves
❖ Serre Construction
❖ The Gauge Bundle
❖ Low Energy Spectrum
❖ Gauge Group Breaking
❖ Vector Bundle Breaking
❖ The Higgs Sector
❖ Cohomology
❖ Doublet-Triplet Splitting
❖ Leray Degrees
Conclusion

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Whats new?

- On $\widetilde{X}$ the $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action is free.

Ideal Sheaves

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

❖ Ideal Sheaves

❖ Serre Construction

❖ The Gauge Bundle

❖ Low Energy Spectrum

❖ Gauge Group Breaking

❖ Vector Bundle Breaking

❖ The Higgs Sector

❖ Cohomology

❖ Doublet-Triplet Splitting

❖ Leray Degrees

Conclusion

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Whats new?

- On $\widetilde{X}$ the $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action is free.
- But on B_1, B_2 there are orbits of length 3 and 9.

Ideal Sheaves

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
❖ Ideal Sheaves
❖ Serre Construction
❖ The Gauge Bundle
❖ Low Energy Spectrum
❖ Gauge Group Breaking
❖ Vector Bundle Breaking
❖ The Higgs Sector
❖ Cohomology
❖ Doublet-Triplet Splitting
❖ Leray Degrees
Conclusion

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Whats new?

- On $\widetilde{X}$ the $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action is free.
- But on B_1, B_2 there are orbits of length 3 and 9.

Observation: We can split up the ideal sheaf of 9 points in $3 + 6$ points!

Ideal Sheaves

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
❖ Ideal Sheaves
❖ Serre Construction
❖ The Gauge Bundle
❖ Low Energy Spectrum
❖ Gauge Group Breaking
❖ Vector Bundle Breaking
❖ The Higgs Sector
❖ Cohomology
❖ Doublet-Triplet Splitting
❖ Leray Degrees
Conclusion

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Whats new?

- On $\widetilde{X}$ the $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action is free.
- But on B_1, B_2 there are orbits of length 3 and 9.

Observation: We can split up the ideal sheaf of 9 points in $3 + 6$ points! Define

I_3 Ideal sheaf on B_1, 3 points in 3 fibers.

I_6 Ideal sheaf on B_2,
Singular point in $3I_1$ with multiplicity 2.
(i.e. function and a first derivative $= 0$)

Serre Construction

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
❖ Ideal Sheaves
❖ Serre Construction
❖ The Gauge Bundle
❖ Low Energy Spectrum
❖ Gauge Group Breaking
❖ Vector Bundle Breaking
❖ The Higgs Sector
❖ Cohomology
❖ Doublet-Triplet Splitting
❖ Leray Degrees
Conclusion

Define rank 2 bundles $\mathcal{W}_i$ on B_i

$$0 \longrightarrow \chi_1 \mathcal{O}_{B_1}(-f_1) \longrightarrow \mathcal{W}_1 \longrightarrow \chi_1^2 \mathcal{O}_{B_1}(f_1) \otimes I_3 \longrightarrow 0$$

$$0 \longrightarrow \chi_2^2 \mathcal{O}_{B_2}(-2f_2) \longrightarrow \mathcal{W}_2 \longrightarrow \chi_2 \mathcal{O}_{B_2}(2f_2) \otimes I_6 \longrightarrow 0$$

The Gauge Bundle

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
❖ Ideal Sheaves
❖ Serre Construction
❖ The Gauge Bundle
❖ Low Energy Spectrum
❖ Gauge Group Breaking
❖ Vector Bundle Breaking
❖ The Higgs Sector
❖ Cohomology
❖ Doublet-Triplet Splitting
❖ Leray Degrees
Conclusion

Define these two rank 2 vector bundles

$$\begin{aligned} \mathcal{V}_1 &\stackrel{\text{def}}{=} \mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2) \otimes \pi_1^*(\mathcal{W}_1) \\ \mathcal{V}_2 &\stackrel{\text{def}}{=} \mathcal{O}_{\widetilde{X}}(\tau_1 - \tau_2) \otimes \pi_2^*(\mathcal{W}_2) \end{aligned}$$

We define the rank 4 bundle $\mathcal{V}$ finally as a generic extension

$$0 \longrightarrow \mathcal{V}_1 \longrightarrow \mathcal{V} \longrightarrow \mathcal{V}_2 \longrightarrow 0$$

Low Energy Spectrum

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

❖ Ideal Sheaves

❖ Serre Construction

❖ The Gauge Bundle

❖ Low Energy Spectrum

❖ Gauge Group Breaking

❖ Vector Bundle Breaking

❖ The Higgs Sector

❖ Cohomology

❖ Doublet-Triplet Splitting

❖ Leray Degrees

Conclusion

The massless spectrum

$= \text{zero modes of } \not{D}_{E_8}$

$= H^1 \text{ cohomology of the adjoint bundle } \mathcal{E}_8^{\mathcal{V}/G}.$

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
❖ Ideal Sheaves
❖ Serre Construction
❖ The Gauge Bundle
❖ Low Energy Spectrum
❖ Gauge Group Breaking
❖ Vector Bundle Breaking
❖ The Higgs Sector
❖ Cohomology
❖ Doublet-Triplet Splitting
❖ Leray Degrees
Conclusion

The massless spectrum

$= $ zero modes of $\not{D}_{E_8}$

$= H^1$ cohomology of the adjoint bundle $\mathcal{E}_8^{\mathcal{V}/G}$.

$$\boxed{\begin{aligned} &H^1\Big(X,\ \mathcal{E}_8^{\mathcal{V}/G}\Big) = \\ &= H^1\Big(X,\ \mathcal{E}_8^{\mathcal{V}}/G\Big) \end{aligned}}$$

Low Energy Spectrum

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
❖ Ideal Sheaves
❖ Serre Construction
❖ The Gauge Bundle
❖ Low Energy Spectrum
❖ Gauge Group Breaking
❖ Vector Bundle Breaking
❖ The Higgs Sector
❖ Cohomology
❖ Doublet-Triplet Splitting
❖ Leray Degrees
Conclusion

The massless spectrum
$= $ zero modes of $\not{D}_{E_8}$
$= H^1$ cohomology of the adjoint bundle $\mathcal{E}_8^{\mathcal{V}/G}$.

Work with | Have in mind

$$\boxed{H^1\left(\widetilde{X},\ \mathcal{E}_8^{\mathcal{V}}\right)^G} = \boxed{\begin{array}{c} H^1\left(X,\ \mathcal{E}_8^{\mathcal{V}/G}\right) = \\ = H^1\left(X,\ \mathcal{E}_8^{\mathcal{V}}/G\right) \end{array}}$$

Gauge Group Breaking

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

❖ Ideal Sheaves

❖ Serre Construction

❖ The Gauge Bundle

❖ Low Energy Spectrum

❖ Gauge Group Breaking

❖ Vector Bundle Breaking

❖ The Higgs Sector

❖ Cohomology

❖ Doublet-Triplet Splitting

❖ Leray Degrees

Conclusion

$$\underline{\mathbf{248}} = (\underline{\mathbf{1}}, \underline{\mathbf{45}}) \oplus (\underline{\mathbf{15}}, \underline{\mathbf{1}}) \oplus \oplus (\underline{\mathbf{4}}, \underline{\mathbf{16}}) \oplus (\overline{\underline{\mathbf{4}}}, \overline{\underline{\mathbf{16}}}) \oplus (\underline{\mathbf{6}}, \underline{\mathbf{10}})$$

$$\underline{\mathbf{10}} = \chi_2(\underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 0) \oplus \chi_1^2\chi_2(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -2, -2) \oplus \oplus \chi_2^2(\underline{\mathbf{1}}, \overline{\underline{\mathbf{2}}}, -3, 0) \oplus \chi_1\chi_2^2(\overline{\underline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, 2)$$

Correspondingly, the fermions split as...

Vector Bundle Breaking

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

❖ Ideal Sheaves

❖ Serre Construction

❖ The Gauge Bundle

❖ Low Energy Spectrum

❖ Gauge Group Breaking

❖ Vector Bundle Breaking

❖ The Higgs Sector

❖ Cohomology

❖ Doublet-Triplet Splitting

❖ Leray Degrees

Conclusion

$$\mathcal{E}_8^V = \Big(\mathcal{O}_{\tilde{X}} \otimes \theta(\underline{\mathbf{45}})\Big) \oplus \Big(\mathrm{ad}(\mathcal{V}) \otimes \theta(\underline{\mathbf{1}})\Big) \oplus$$
$$\oplus \Big(\mathcal{V} \otimes \theta(\underline{\mathbf{16}})\Big) \oplus \Big(\mathcal{V}^\vee \otimes \theta(\underline{\overline{\mathbf{16}}})\Big) \oplus \Big(\wedge^2 \mathcal{V} \otimes \theta(\underline{\mathbf{10}})\Big)$$

where $\theta(\cdots)$ is the trivial bundle.

$$\theta(\underline{\mathbf{10}}) = \Big[\chi_2 \theta\big(\underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 0\big)\Big] \oplus \Big[\chi_1^2 \chi_2 \theta\big(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -2, -2\big)\Big] \oplus$$
$$\oplus \Big[\chi_2^2 \theta\big(\underline{\mathbf{1}}, \underline{\overline{\mathbf{2}}}, -3, 0\big)\Big] \oplus \Big[\chi_1 \chi_2^2 \theta\big(\underline{\overline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, 2\big)\Big]$$

The Higgs Sector

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
❖ Ideal Sheaves
❖ Serre Construction
❖ The Gauge Bundle
❖ Low Energy Spectrum
❖ Gauge Group Breaking
❖ Vector Bundle Breaking
❖ The Higgs Sector
❖ Cohomology
❖ Doublet-Triplet Splitting
❖ Leray Degrees
Conclusion

For example, focus on the fields in the $\underline{\mathbf{10}}$:

$$
\begin{aligned}
H^1\Big(\widetilde{X},\, \mathcal{E}_8^V\Big)^G = &\ (\text{lots of other fields}) \oplus \\
&\oplus \Big[\chi_2 \otimes H^1\Big(\widetilde{X},\, \wedge^2\mathcal{V}\Big)\Big]^G \otimes (\underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 0) \oplus \\
&\oplus \Big[\chi_1^2\chi_2 \otimes H^1\Big(\widetilde{X},\, \wedge^2\mathcal{V}\Big)\Big]^G \otimes (\underline{\mathbf{3}}, \underline{\mathbf{1}}, -2, -2) \oplus \\
&\oplus \Big[\chi_2^2 \otimes H^1\Big(\widetilde{X},\, \wedge^2\mathcal{V}\Big)\Big]^G \otimes (\underline{\mathbf{1}}, \underline{\overline{\mathbf{2}}}, -3, 0) \oplus \\
&\oplus \Big[\chi_1\chi_2^2 \otimes H^1\Big(\widetilde{X},\, \wedge^2\mathcal{V}\Big)\Big]^G \otimes (\underline{\overline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, 2)\,.
\end{aligned}
$$

Cohomology

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
❖ Ideal Sheaves
❖ Serre Construction
❖ The Gauge Bundle
❖ Low Energy Spectrum
❖ Gauge Group Breaking
❖ Vector Bundle Breaking
❖ The Higgs Sector
❖ Cohomology
❖ Doublet-Triplet Splitting
❖ Leray Degrees
Conclusion

The necessary cohomology groups for $\mathcal{V}$ are

$$
\begin{aligned}
H^1\Big(\widetilde{X}, \mathcal{V}\Big) &= 3\,\mathrm{Reg}(G) \\
H^1\Big(\widetilde{X}, \mathcal{V}^\vee\Big) &= 0 \\
H^1\Big(\widetilde{X}, \wedge^2\mathcal{V}\Big) &= H^1\Big(\widetilde{X}, \mathcal{V}_1 \otimes \mathcal{V}_2\Big) = \\
&= \chi_1\chi_2 \oplus \chi_1^2\chi_2^2 \oplus \chi_2 \oplus \chi_2^2
\end{aligned}
$$

Doublet-Triplet Splitting

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

❖ Ideal Sheaves

❖ Serre Construction

❖ The Gauge Bundle

❖ Low Energy Spectrum

❖ Gauge Group Breaking

❖ Vector Bundle Breaking

❖ The Higgs Sector

❖ Cohomology

❖ Doublet-Triplet Splitting

❖ Leray Degrees

Conclusion

$$H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right) = \chi_1\chi_2 \oplus \chi_1^2\chi_2^2 \oplus \chi_2 \oplus \chi_2^2$$

Doublet-Triplet Splitting

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

❖ Ideal Sheaves

❖ Serre Construction

❖ The Gauge Bundle

❖ Low Energy Spectrum

❖ Gauge Group Breaking

❖ Vector Bundle Breaking

❖ The Higgs Sector

❖ Cohomology

❖ Doublet-Triplet Splitting

❖ Leray Degrees

Conclusion

$$H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right) = \chi_1\chi_2 \oplus \chi_1^2\chi_2^2 \oplus \chi_2 \oplus \chi_2^2$$

$$1 = \left[\chi_2 \otimes H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right)\right]^G \qquad \text{up Higgs}$$

Doublet-Triplet Splitting

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
❖ Ideal Sheaves
❖ Serre Construction
❖ The Gauge Bundle
❖ Low Energy Spectrum
❖ Gauge Group Breaking
❖ Vector Bundle Breaking
❖ The Higgs Sector
❖ Cohomology
❖ Doublet-Triplet Splitting
❖ Leray Degrees
Conclusion

$$H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right) = \chi_1\chi_2 \oplus \chi_1^2\chi_2^2 \oplus \chi_2 \oplus \chi_2^2 \oplus 0\chi_1\chi_2^2$$

$$1 = \left[\chi_2 \otimes H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right)\right]^G \qquad \text{up Higgs}$$

$$0 = \left[\chi_1^2\chi_2 \otimes H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right)\right]^G \qquad \underline{\mathbf{3}}$$

Doublet-Triplet Splitting

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
❖ Ideal Sheaves
❖ Serre Construction
❖ The Gauge Bundle
❖ Low Energy Spectrum
❖ Gauge Group Breaking
❖ Vector Bundle Breaking
❖ The Higgs Sector
❖ Cohomology
❖ Doublet-Triplet Splitting
❖ Leray Degrees
Conclusion

$$H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right) = \chi_1\chi_2 \oplus \chi_1^2\chi_2^2 \oplus \chi_2 \oplus \chi_2^2$$

$$1 = \left[\chi_2 \otimes H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right)\right]^G$$ up Higgs

$$0 = \left[\chi_1^2\chi_2 \otimes H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right)\right]^G$$ $\underline{\mathbf{3}}$

$$1 = \left[\chi_2^2 \otimes H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right)\right]^G$$ down Higgs

Doublet-Triplet Splitting

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

❖ Ideal Sheaves

❖ Serre Construction

❖ The Gauge Bundle

❖ Low Energy Spectrum

❖ Gauge Group Breaking

❖ Vector Bundle Breaking

❖ The Higgs Sector

❖ Cohomology

❖ Doublet-Triplet Splitting

❖ Leray Degrees

Conclusion

$$H^1\Big(\widetilde{X}, \wedge^2 \mathcal{V}\Big) = \chi_1\chi_2 \oplus \chi_1^2\chi_2^2 \oplus \chi_2 \oplus \chi_2^2 \oplus 0\chi_1^2\chi_2$$

$$1 = \Big[\chi_2 \otimes H^1\Big(\widetilde{X}, \wedge^2 \mathcal{V}\Big)\Big]^G \qquad \text{up Higgs}$$

$$0 = \Big[\chi_1^2\chi_2 \otimes H^1\Big(\widetilde{X}, \wedge^2 \mathcal{V}\Big)\Big]^G \qquad \underline{\mathbf{3}}$$

$$1 = \Big[\chi_2^2 \otimes H^1\Big(\widetilde{X}, \wedge^2 \mathcal{V}\Big)\Big]^G \qquad \text{down Higgs}$$

$$0 = \Big[\chi_1\chi_2^2 \otimes H^1\Big(\widetilde{X}, \wedge^2 \mathcal{V}\Big)\Big]^G \qquad \underline{\overline{\mathbf{3}}}$$

Doublet-Triplet Splitting

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

❖ Ideal Sheaves

❖ Serre Construction

❖ The Gauge Bundle

❖ Low Energy Spectrum

❖ Gauge Group Breaking

❖ Vector Bundle Breaking

❖ The Higgs Sector

❖ Cohomology

❖ Doublet-Triplet Splitting

❖ Leray Degrees

Conclusion

$$H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right) = \chi_1\chi_2 \oplus \chi_1^2\chi_2^2 \oplus \chi_2 \oplus \chi_2^2$$

$$1 = \left[\chi_2 \otimes H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right)\right]^G \qquad \text{up Higgs}$$

$$0 = \left[\chi_1^2\chi_2 \otimes H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right)\right]^G \qquad \underline{\mathbf{3}}$$

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Leray Degrees

Field	Cohomology	Fiber 1	Base	Fiber 2
$Q_1, \bar{Q}_1$	$H^1\big(\widetilde{X}, \mathcal{V}\big)$	1	0	0
$Q_2, Q_3, \bar{Q}_2, \bar{Q}_3$	$H^1\big(\widetilde{X}, \mathcal{V}\big)$	0	0	1
$H_1, \bar{H}_1$	$H^1\big(\widetilde{X}, \wedge^2\mathcal{V}\big)$	0	1	0
$\phi_1, \ldots ?$	$H^1\big(\widetilde{X}, \mathcal{V} \otimes \mathcal{V}^\vee\big)$	?	?	?

Leray Degrees

Field	Cohomology	Fiber 1	Base	Fiber 2
$Q_1, \bar{Q}_1$	$H^1\big(\widetilde{X}, \mathcal{V}\big)$	1	0	0
$Q_2, Q_3, \bar{Q}_2, \bar{Q}_3$	$H^1\big(\widetilde{X}, \mathcal{V}\big)$	0	0	1
$H_1, \bar{H}_1$	$H^1\big(\widetilde{X}, \wedge^2\mathcal{V}\big)$	0	1	0
$\phi_1, \ldots ?$	$H^1\big(\widetilde{X}, \mathcal{V} \otimes \mathcal{V}^\vee\big)$	?	?	?

- No μ-terms, $H_1 \wedge \bar{H}_1 = 0$.

Leray Degrees

Field	Cohomology	Fiber 1	Base	Fiber 2
$Q_1, \bar{Q}_1$	$H^1\big(\widetilde{X}, \mathcal{V}\big)$	1	0	0
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$H_1, \bar{H}_1$	$H^1\big(\widetilde{X}, \wedge^2\mathcal{V}\big)$	0	1	0
$\phi_1, \ldots ?$	$H^1\big(\widetilde{X}, \mathcal{V} \otimes \mathcal{V}^\vee\big)$	?	?	?

- No μ-terms, $H_1 \wedge \bar{H}_1 = 0$.
- Yukawa couplings.

Introduction

The Calabi-Yau

The Vector Bundle

A First Heterotic Standard Model

Spectral Sequences

A New Heterotic Standard Model

Conclusion

❖ Summary

❖ Important Lessons

❖ Future Directions

Conclusion

Summary

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion
❖ Summary
❖ Important Lessons
❖ Future Directions

The "new" Heterotic Standard Model has

- 3 families of quarks and leptons.
- Zero anti-families.
- 1 Higgs–Higgs conjugate pair (exact MSSM).
- Doublets and triplets are completely split, all triplets are projected out.
- Yukawa couplings.
- No Higgs μ-terms, but can get those from D-terms.

Important Lessons

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion
❖ Summary
❖ Important Lessons
❖ Future Directions

- Discrete symmetries are important

Important Lessons

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion
❖ Summary
❖ Important Lessons
❖ Future Directions

- Discrete symmetries are important
  - Doublet-triplet splitting.

Important Lessons

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion
❖ Summary
❖ Important Lessons
❖ Future Directions

- Discrete symmetries are important
  - Doublet-triplet splitting.
  - Moduli reduction, e.g. $h^{1,1}(\widetilde{X}) = 19 \longrightarrow 3 = h^{1,1}(X)$

Important Lessons

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion
❖ Summary
❖ Important Lessons
❖ Future Directions

- Discrete symmetries are important
  - Doublet-triplet splitting.
  - Moduli reduction, e.g. $h^{1,1}(\widetilde{X}) = 19 \longrightarrow 3 = h^{1,1}(X)$
- Not at a special point in moduli space $\Rightarrow$ no enhanced spectrum.

Important Lessons

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion
❖ Summary
❖ Important Lessons
❖ Future Directions

- Discrete symmetries are important
  - Doublet-triplet splitting.
  - Moduli reduction, e.g. $h^{1,1}(\widetilde{X}) = 19 \longrightarrow 3 = h^{1,1}(X)$
- Not at a special point in moduli space $\Rightarrow$ no enhanced spectrum.
- Unique solution?

Important Lessons

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion
❖ Summary
❖ Important Lessons
❖ Future Directions

- Discrete symmetries are important
  - Doublet-triplet splitting.
  - Moduli reduction, e.g. $h^{1,1}(\widetilde{X}) = 19 \longrightarrow 3 = h^{1,1}(X)$
- Not at a special point in moduli space $\Rightarrow$ no enhanced spectrum.
- Unique solution?
- Equivariant actions are the key.

Future Directions

Introduction
The Calabi-Yau
The Vector Bundle
A First Heterotic Standard Model
Spectral Sequences
A New Heterotic Standard Model
Conclusion
❖ Summary
❖ Important Lessons
❖ Future Directions

- Supersymmetry breaking.
- $U(1)_{B-L}$ breaking.
- Instanton corrections to Yukawa couplings.
- Moduli stabilization.
- Revisit $SU(5)$ with $\mathbb{Z}_2$ Wilson line: no $U(1)_{B-L}$.