Evaluating Kinematical Factors of Pure Spinor Scattering Amplitudes

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Outline

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What do I want to do?

- Compute the kinematical factors of amplitudes obtained with the pure spinor formalism
  - PRL 96 (2006),011602 (Berkovits,C.M.)
  - JHEP 0601 (2006), 075 (C.M.)
  - JHEP 0611 (2006), 079 (Berkovits,C.M.)
- Check if they agree with RNS and GS results
  - For example, the 4-point 1-loop kinematical factor:

$$\langle (\lambda A^1)(\lambda \gamma^m W^2)(\lambda \gamma^n W^3)\mathcal{F}^4_{mn}\rangle + \text{perm}(234) = t_8 F^4 + \text{fermions}$$

  - Or the 4-point amplitude at 2-loops

$$\langle (\lambda \gamma^{mnpqr}\lambda)\mathcal{F}^1_{mn}\mathcal{F}^2_{pq}\mathcal{F}^3_{rs}(\lambda\gamma^s W^4)\rangle + \text{perm}(1234) = (t-u)t_8 F^4 + \ldots$$

  - Or the gauge variation of the 6-point amplitude at 1-loop

$$\langle (\lambda\gamma^m W)(\lambda\gamma^n W)(\lambda\gamma^p W)(W\gamma_{mnp}W)\rangle = \epsilon_{10} F^5$$

What do I want to do?

- Or how to prove the following interesting identity

$$\langle(\lambda\gamma^r\gamma^{m_1n_1}\theta)(\lambda\gamma^s\gamma^{m_2n_2}\theta)(\lambda\gamma^t\gamma^{m_3n_3}\theta)(\theta\gamma^m\gamma^n\gamma_{rst}\gamma^{m_4n_4}\theta)\rangle =$$

$$= -\frac{2}{45}\left(\eta^{mn}t_8^{m_1n_1m_2n_2m_3n_3m_4n_4} - \frac{1}{2}\epsilon_{10}^{mnm_1n_1m_2n_2m_3n_3m_4n_4}\right)$$

- Find tricks and shortcuts to compute general scattering amplitudes
- Note: Compact notation t_8F^4 means

$$t_8F^4 \equiv 4(F^1F^2F^3F^4) + 4(F^1F^3F^2F^4) + 4(F^1F^2F^4F^3)$$

$$-(F^1F^2)(F^3F^4) - (F^1F^3)(F^2F^4) - (F^1F^2)(F^4F^3)$$

The pure spinor formalism is a CFT based on the following action

Action (Minimal Pure Spinor Formalism)

$$S = \int d^2z \left(\frac{1}{2} \partial X^m \overline{\partial} X_m + p_\alpha \overline{\partial} \theta^\alpha - w_\alpha \overline{\partial} \lambda^\alpha \right)$$

With a bosonic pure spinor λ^α

Constraints

$$(\lambda \gamma^m \lambda) = 0$$

Pure Spinor Formalism

Some important definitions for amplitude computations:

- Lorentz current

$$N^{mn} = \frac{\alpha\prime}{4}(w\gamma^{mn}\lambda)$$

- Supersymmetric momentum

$$\Pi^m = \partial X^m + \frac{1}{2}(\theta\gamma^m\partial\theta)$$

- Supersymmetric derivative

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + \frac{1}{2}(\theta\gamma^m)_\alpha\partial_m$$

Pure Spinor Formalism

- Supersymmetric Green-Schwarz constraint

$$d_\alpha = \frac{\alpha'}{2} p_\alpha - \frac{1}{2}(\gamma^m\theta)_\alpha \partial X_m - \frac{1}{8}(\gamma^m\theta)_\alpha(\theta\gamma_m\partial\theta)$$

Pure Spinor Formalism

Relevant OPE's

$$X^m(z,\overline{z})X^n(w,\overline{w}) \longrightarrow -\frac{1}{2}\eta^{mn}\ln|z-w|^2$$

$$N^{mn}(z)\lambda^\alpha(y) \longrightarrow \frac{\alpha\prime}{4}\frac{(\gamma^{mn}\lambda)^\alpha}{z-y}$$

$$d_\alpha(z)V(y,\theta) \longrightarrow \frac{D_\alpha V(y,\theta)}{z-y}$$

$$\Pi^m(z)V(y,\theta) \longrightarrow \frac{\partial^m V(y,\theta)}{z-y}$$

Issues of RNS and GS not present

Space-time SUSY

The pure spinor formalism has manifest space-time supersymmetry

Covariant BRST Quantization

$$Q_{\text{BRST}} = \oint \lambda^{\alpha} d_{\alpha}$$

Prescription for Scattering Amplitudes

- Massless Vertex Operators:
  - Unintegrated

$$V = \lambda^\alpha A_\alpha(X, \theta)$$

  - Integrated

$$U = \int dz \left(\partial\theta^\alpha A_\alpha + A_m \Pi^m + d_\alpha W^\alpha + \frac{1}{2} N^{mn} \mathcal{F}_{mn} \right)$$

- Where $A_\alpha(x, \theta), A_m(x, \theta), W^\alpha(x, \theta)$ and $\mathcal{F}_{mn}(x, \theta)$ are the SYM superfields.

Tree-level Amplitudes

- The prescription for tree-level amplitudes is given by

Tree-level N-point

$$\mathcal{A}_N = \langle V_1(z_1) V_2(z_2) V_3(z_3) \int dz_4 U_4(z_4) \ldots \int dz_N U_N(z_N) \rangle$$

- Computation proceeds as usual in a CFT
- Use OPE's to integrate out conformal weight 1 variables
- Then integrate out zero-modes

Tree-level Amplitudes

- For our purposes now, integration over λ^α and θ^α zero-modes is done with the rule

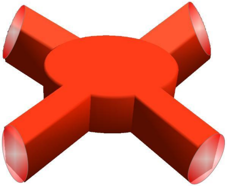
$\lambda^3\theta^5$ prescription

$$\langle(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta)\rangle = 1$$

Pure Spinor Superspace

- The computation of scattering amplitudes gives rise to pure spinor superspace expressions
- Compact way of writing the full amplitude
  - Contain all possible contributions of fermionic and bosonic external states
- To compare results with RNS/GS one has to express these pure spinor expressions in terms of polarization and momenta
- This is now a solved problem:
  - Systematic procedure to evaluate pure spinor superspace expressions in components
  - I have made Mathematica functions that make this job

Four gravitons at tree-level



Example

$$\mathcal{A} = \langle V^1(z_1, \overline{z}_1) V^2(z_2, \overline{z}_2) V^3(z_3, \overline{z}_3) \int_{\mathbb{C}} d^2 z U^4(z, \overline{z}) \rangle$$

where $V^i(z, \overline{z}) = V^i(z) \otimes \tilde{V}^i(\overline{z}) e^{ik \cdot X}$ and $U(z, \overline{z}) = U(z) \otimes \tilde{U}(\overline{z}) e^{ik \cdot X}$

Tree-level 4-graviton computation (unpublished)

Sidenote

Previous computation (Policastro, Tsimpis 2006) were done in a way that hided the simplicity of the result. Cancellations were overlooked and no simple pure spinor expression was written down for the kinematical factor.

- We have to compute

$$\langle(\lambda A^1)(z_1)(\lambda A^2)(z_2)(\lambda A^3)(z_3)\int d^2z(\Pi^m A^4_m + (dW^4) + \frac{1}{2}N^{mn}\mathcal{F}_{mn})\rangle$$

$$\otimes(\text{right-moving part})$$

- SL(2,C) invariance allows the fixing $z_1 = 0$, $z_2 = 1$ and $z_3 \to \infty$

Tree-level 4-graviton computation

- $\Pi^m A^4_m$ term of integrated vertex contribute only with

$$\langle(\lambda A^1)(\lambda A^2)(\lambda A^3)A^4_m\Pi^m : e^{ik_1 X} :: e^{ik_2 X} : e^{ik_3 X} : e^{ik_4 X} :\rangle =$$

$$= \sum_{i=1}^{2} \frac{\alpha'}{2} \frac{ik_i^m}{z_i - z_4} \langle(\lambda A^1)(\lambda A^2)(\lambda A^3)A^4_m\rangle \otimes \Pi(z_{ij})$$

Tree-level 4-graviton computation

- One can use some identities to simplify result of other OPE's
- Delay as long as possible explicit evaluation of pure spinor integrals

Lemma

One can show the OPE identity

$$\langle(\lambda A^1)(\lambda A^2)(\lambda A^3)((dW^4) + \frac{1}{2}N^{mn}\mathcal{F}_{mn})\rangle =$$

$$+\frac{\alpha'}{2(z_1 - z_4)}\langle A^1_m(\lambda A^2)(\lambda A^3)(\lambda\gamma^m W^4)\rangle - (1 \leftrightarrow 2) + (1 \leftrightarrow 3)$$

Tree-level 4-graviton computation

- We organize the computation as

$$\mathcal{A} = \text{const} \int d^2 z_4 \left(\frac{F_1}{z_1 - z_4} + \frac{F_2}{z_2 - z_4} \right) \otimes \left(\frac{\tilde{F}_1}{\overline{z}_1 - \overline{z}_4} + \frac{\tilde{F}_2}{\overline{z}_2 - \overline{z}_4} \right)$$

$$\cdot |z_4|^{-\alpha' t/2} |1 - z_4|^{-\alpha' u/2}$$

where

$$F_1 = i k_m^1 \langle (\lambda A^1)(\lambda A^2)(\lambda A^3) A_m^4 \rangle + \langle A_m^1 (\lambda A^2)(\lambda A^3)(\lambda \gamma^m W^4) \rangle$$

$$F_2 = i k_m^2 \langle (\lambda A^1)(\lambda A^2)(\lambda A^3) A_m^4 \rangle - \langle (\lambda A^1) A_m^2 (\lambda A^3)(\lambda \gamma^m W^4) \rangle$$

Tree-level 4-graviton computation

- Using the general formula

$$\int d^2z z^A (1-z)^B \overline{z}^{\tilde{A}} (1-\overline{z})^{\tilde{B}} = 2\pi \frac{\Gamma(1+A)\Gamma(1+B)}{\Gamma(2+A+B)} \cdot \frac{\Gamma(-1-\tilde{A}-\tilde{B})}{\Gamma(-\tilde{A})\Gamma(-\tilde{B})}$$

we get

$$\mathcal{A} = K\tilde{K} \frac{\Gamma(-\alpha' t/4)\Gamma(-\alpha' u/4)\Gamma(-\alpha' s/4)}{\Gamma(1+\alpha' s/4)\Gamma(1+\alpha' t/4)\Gamma(1+\alpha' u/4)}$$

where

$$K = uF_1 - tF_2 \qquad \tilde{K} = u\tilde{F}_1 - t\tilde{F}_2$$

Tree-level 4-graviton result

Pure Spinor Superspace Result

$$\mathcal{A} = K \otimes \tilde{K} \frac{\Gamma(-s/4)\Gamma(-t/4)\Gamma(-u/4)}{\Gamma(1+s/4)\Gamma(1+t/4)\Gamma(1+u/4)}$$

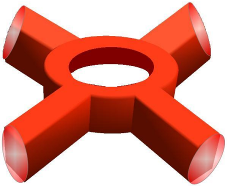
where the kinematical factor is given by

$$K = \langle \partial^n(\lambda A^1)\partial^m(\lambda A^2)(\lambda A^3)\mathcal{F}^4_{mn}\rangle$$

$$+\langle(\partial_p A^1_m)(\lambda A^2)\partial^p(\lambda A^3)(\lambda\gamma^m W^4)\rangle$$

$$+\langle(\lambda A^1)(\partial_p A^2_m)\partial^p(\lambda A^3)(\lambda\gamma^m W^4)\rangle$$

Massless 4-point one-loop amplitude



Prescription

$$\mathcal{A}_N = \langle \mathcal{N} \left(\int \mu \cdot b \right) V_1(z_1) \int U_2 \int U_3 \int U_4 \rangle$$

Massless 4-point one-loop amplitude

- This amplitude was computed with the minimal pure spinor formalism (Berkovits 2004) and shown to agree with the RNS and GS results (C.M. 2005).
- Computed also in the non-minimal pure spinor formalism (Berkovits 2005, Berkovits & C.M. 2006)

Pure Spinor Superspace Result

4-gravitons interaction at one-loop order

$$\mathcal{A} = K \otimes \tilde{K} \int \frac{d^2\tau}{(\mathrm{Im}\tau)^2} F(\tau)$$

- Minimal Pure Spinor Formalism

$$K_{\text{one-loop}} = \langle (\lambda A)(\lambda\gamma^m W)(\lambda\gamma^n W)\mathcal{F}_{mn}\rangle$$

- Now one has to show that $K_{\text{one-loop}}$ is proportional to $t_8 F^4$
- How to do that?

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Evaluating Pure Spinor Superspace Expressions

Evaluating Pure Spinor Superspace Expressions

- Pure spinor superspace expressions are compact and elegant
- However, until the Pure Spinor Formalism becomes the *de facto* standard superstring formalism, one needs to check the results in components
- Straightforward to do with the $(\lambda^3\theta^5)$ rule

$\lambda^3\theta^5$ prescription

$$\langle(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta)\rangle = 1$$

Evaluating Pure Spinor Superspace Expressions

- Suppose one wants to compute the 1-loop pure spinor superspace integral

$$\langle (\lambda A)(\lambda\gamma^m W)(\lambda\gamma^n W)\mathcal{F}_{mn}\rangle$$

- We first expand superfields in θ's as follows

Evaluating Pure Spinor Superspace Expressions

SYM Superfields θ-Expansion

$$A_\alpha(x,\theta) = \frac{1}{2}a_m(\gamma^m\theta)_\alpha - \frac{1}{3}(\xi\gamma_m\theta)(\gamma^m\theta)_\alpha - \frac{1}{32}F_{mn}(\gamma_p\theta)_\alpha(\theta\gamma^{mnp}\theta) + \dots$$

$$A_m(x,\theta) = a_m - (\xi\gamma_m\theta) - \frac{1}{8}(\theta\gamma_m\gamma^{pq}\theta)F_{pq} + \frac{1}{12}(\theta\gamma_m\gamma^{pq}\theta)(\partial_p\xi\gamma_q\theta) + \dots$$

$$W^\alpha(x,\theta) = \xi^\alpha - \frac{1}{4}(\gamma^{mn}\theta)^\alpha F_{mn} + \frac{1}{4}(\gamma^{mn}\theta)^\alpha(\partial_m\xi\gamma_n\theta)$$
$$+\frac{1}{48}(\gamma^{mn}\theta)^\alpha(\theta\gamma_n\gamma^{pq}\theta)\partial_m F_{pq} + \dots$$

$$\mathcal{F}_{mn}(x,\theta) = F_{mn} - 2(\partial_{[m}\xi\gamma_{n]}\theta) + \frac{1}{4}(\theta\gamma_{[m}\gamma^{pq}\theta)\partial_{n]}F_{pq} + \dots,$$

Evaluating Pure Spinor Superspace Expressions

- Remember that correlator must have 5 θ's to be non-zero
- If we want the bosonic contribution we distribute θ's as follows

$A_\alpha(\theta)$	$W^\alpha(\theta)$	$W^\alpha(\theta)$	$\mathcal{F}_{mn}(\theta)$
1	1	1	2
1	1	3	0
1	3	1	0
3	1	1	0

Evaluating Pure Spinor Superspace Expressions

Sidenote

Previous computation (Anguelova, Grassi, Vanhove 2004) was wrong. Omitted first three lines of above table.

- In JHEP 0601 (2006) (C.M.) it was shown that to get the right result one also has to include the first three lines.
- In JHEP 0705 (2007) (C. Stahn) the fermionic contributions were also computed.

Evaluating Pure Spinor Superspace Expressions

- Considering all lines of the table we get

$$K_1^{NS} = +\frac{15}{64} F^1_{mn} F^2_{pq} F^3_{rs} F^4_{tu} \langle (\lambda\gamma^{[t|}\gamma^{pq}\theta)(\lambda\gamma^{|u]}\gamma^{rs}\theta)(\lambda\gamma_a\theta)(\theta\gamma^{mna}\theta)\rangle +$$

$$+\frac{15}{16}(k^4_m e^1_n) F^2_{pq} F^3_{rs} F^4_{tu} \langle (\lambda\gamma^{[m|}\gamma^{pq}\theta)(\lambda\gamma^{|a]}\gamma^{rs}\theta)(\lambda\gamma^n\theta)(\theta\gamma_a\gamma^{tu}\theta)\rangle +$$

$$+\frac{5}{16}(k^2_m e^1_n) F^2_{pq} F^3_{rs} F^4_{tu} \langle (\lambda\gamma^{[t|}\gamma^{ma}\theta)(\lambda\gamma^{|u]}\gamma^{rs}\theta)(\lambda\gamma^n\theta)(\theta\gamma_a\gamma^{pq}\theta)\rangle +$$

$$+\frac{5}{16}(k^3_m e^1_n) F^2_{pq} F^3_{rs} F^4_{tu} \langle (\lambda\gamma^{[t|}\gamma^{pq}\theta)(\lambda\gamma^{|u]}\gamma^{ma}\theta)(\lambda\gamma^n\theta)(\theta\gamma_a\gamma^{rs}\theta)\rangle .$$

Evaluating Pure Spinor Superspace Expressions

Practical Question

How do we compute $\langle (\lambda\gamma^{[t|}\gamma^{pq}\theta)(\lambda\gamma^{|u]}\gamma^{rs}\theta)(\lambda\gamma_a\theta)(\theta\gamma^{mna}\theta)\rangle$ or

$$\langle (\lambda\gamma^m\gamma^{m_1n_1}\theta)(\lambda\gamma^n\gamma^{m_2n_2}\theta)(\lambda\gamma^p\gamma^{m_3n_3}\theta)(\theta\gamma^{m_4n_4}\gamma_{mnp}\gamma^{m_5n_5}\theta)\rangle$$

$$\langle (\lambda\gamma^m\theta)(\lambda\gamma^a\gamma^{m_1n_1}\theta)(\lambda\gamma^{bcn}\gamma^{m_2n_2}\theta)(\theta\gamma^{m_3n_3}\gamma_{abc}\gamma^{m_4n_4}\theta)\rangle$$

in general?

- One has to relate a general pure spinor superspace expression to $\langle (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta)\rangle$
- One can always do that by symmetry arguments
- Example:

$$\langle (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{ijk}\theta)\rangle = \frac{1}{120}\delta^{mnp}_{ijk}$$

Identities

- $(\lambda\gamma^m\gamma^{np}\theta) = (\lambda\gamma^{mnp}\theta) + \eta^{mn}(\lambda\gamma^p\theta) - \eta^{mp}(\lambda\gamma^n\theta)$
- $(\lambda\gamma^{abc}\gamma^{de}\theta) = +(\lambda\gamma^{abcde}\theta) - 2\delta^{bc}_{de}(\lambda\gamma^a\theta) + 2\delta^{ac}_{de}(\lambda\gamma^b\theta) - 2\delta^{ab}_{de}(\lambda\gamma^c\theta)$

$$-\delta^c_e(\lambda\gamma^{abd}\theta) + \delta^c_d(\lambda\gamma^{abe}\theta) + \delta^b_e(\lambda\gamma^{acd}\theta) - \delta^b_d(\lambda\gamma^{ace}\theta)$$

$$-\delta^a_e(\lambda\gamma^{bcd}\theta) + \delta^a_d(\lambda\gamma^{bce}\theta)$$

- $(\theta\gamma^{m_4 n_4}\gamma_{mnp}\gamma^{m_5 n_5}\theta) = G^{m_4 n_4 m_5 n_5}_{mnpr_1 r_2 r_3}(\theta\gamma^{r_1 r_2 r_3}\theta)$ where

$$G^{m_4 n_4 m_5 n_5}_{mnpr_1 r_2 r_3} = +\frac{1}{6}\epsilon^{mm_4 m_5 nn_4 n_5 pr_1 r_2 r_3} - 24\delta^{np}_{n_4 n_5}\delta^{mm_4 m_5}_{r_1 r_2 r_3} + 12\delta^{m_5 n_5}_{n_4 p}\delta^{mm_4 n}_{r_1 r_2 r_3}$$

$$-6\delta^{m_5 n_5}_{np}\delta^{mm_4 n_4}_{r_1 r_2 r_3} + 12\delta^{m_4 n_4}_{n_5 p}\delta^{mm_5 n}_{r_1 r_2 r_3} - 6\delta^{m_4 n_4}_{np}\delta^{mm_5 n_5}_{r_1 r_2 r_3} - 2\delta^{m_4 n_4}_{m_5 n_5}\delta^{mnp}_{r_1 r_2 r_3}$$

$$+[mnp] + [m_4 n_4] + [m_5 n_5],$$

Identities

- $(\theta\gamma^{abc}\gamma^{mn}\theta) = (\theta\gamma^{r_1 r_2 r_3}\theta)K^{abcmn}_{r_1 r_2 r_3}$ where

$$K^{abcmn}_{r_1 r_2 r_3} = -\eta^{cn}\delta^{abm}_{r_1 r_2 r_3} + \eta^{cm}\delta^{abn}_{r_1 r_2 r_3} + \eta^{bn}\delta^{acm}_{r_1 r_2 r_3}$$
$$-\eta^{bm}\delta^{acn}_{r_1 r_2 r_3} - \eta^{an}\delta^{bcm}_{r_1 r_2 r_3} + \eta^{am}\delta^{bcn}_{r_1 r_2 r_3}$$

$$(\gamma^{mnp})_{\alpha\beta}(\gamma_{mnp})^{\gamma\delta} = 48\left(\delta^{\gamma}_{\alpha}\delta^{\delta}_{\beta} - \delta^{\gamma}_{\beta}\delta^{\delta}_{\alpha}\right), \quad (\lambda\gamma_m\psi)(\lambda\gamma^m\xi) = 0 \quad \forall\psi^\alpha,\ \xi^\alpha$$

$$(\lambda\gamma^{mnpqr}\lambda)(\lambda\gamma_{mna}\theta) = 0, \quad (\lambda\gamma^{mnpqr}\lambda)(\lambda\gamma_m\theta) = 0$$

- The identities bellow are necessary to get recursion relations between different pure spinor superspace identities

$$(\lambda\gamma^{amn}\theta)(\lambda\gamma_a\theta) = 2(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)$$
$$(\lambda\gamma^{abm}\theta)(\lambda\gamma^{abn}\theta) = -4(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)$$

Identities

$$(\lambda\gamma^{mabcn}\lambda)(\theta\gamma_{abc}\theta) = 96(\lambda\gamma^m\theta)(\lambda\gamma^n\theta),$$

$$(\lambda\gamma^{abcmn}\theta)(\lambda\gamma_{abc}\theta) = -36(\lambda\gamma^m\theta)(\lambda\gamma^n\theta),$$

$$(\lambda\gamma^a\gamma^{bcmn}\theta)(\lambda\gamma_{abc}\theta) = -28(\lambda\gamma^m\theta)(\lambda\gamma^n\theta),$$

$$\begin{aligned}(\lambda\gamma^{abc}\theta)(\lambda\gamma^{ade}\theta) = &-(\lambda\gamma^{cde}\theta)(\lambda\gamma^b\theta) + (\lambda\gamma^{bde}\theta)(\lambda\gamma^c\theta)\\ &+(\lambda\gamma^{bce}\theta)(\lambda\gamma^d\theta) - (\lambda\gamma^{bcd}\theta)(\lambda\gamma^e\theta)\\ &-\eta^{ce}(\lambda\gamma^b\theta)(\lambda\gamma^d\theta) + \eta^{cd}(\lambda\gamma^b\theta)(\lambda\gamma^e\theta)\\ &+\eta^{be}(\lambda\gamma^c\theta)(\lambda\gamma^d\theta) - \eta^{bd}(\lambda\gamma^c\theta)(\lambda\gamma^e\theta)\end{aligned}$$

Identities

- And many many more...

Evaluating Pure Spinor Superspace Expressions

Pure Spinor Superspace Identities

$$\langle(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{ijk}\theta)\rangle = \frac{1}{120}\delta^{mnp}_{ijk}$$

$$\langle(\lambda\gamma^{mnp}\theta)(\lambda\gamma_q\theta)(\lambda\gamma_t\theta)(\theta\gamma_{ijk}\theta)\rangle = \frac{1}{70}\delta^{[m}_{[q}\eta_{t][i}\delta^n_j\delta^{p]}_{k]}$$

$$\langle(\lambda\gamma_t\theta)(\lambda\gamma^{mnp}\theta)(\lambda\gamma^{qrs}\theta)(\theta\gamma_{ijk}\theta)\rangle = \frac{1}{8400}\epsilon^{ijkmnpqrst}$$

$$+\frac{1}{140}\Big[\delta^{[m}_t\delta^n_{[i}\eta^{p][q}\delta^r_j\delta^{s]}_{k]} - \delta^{[q}_t\delta^r_{[i}\eta^{s][m}\delta^n_j\delta^{p]}_{k]}\Big]$$

$$-\frac{1}{280}\Big[\eta_{t[i}\eta^{v[q}\delta^r_j\eta^{s][m}\delta^n_{k]}\delta^{p]}_v - \eta_{t[i}\eta^{v[m}\delta^n_j\eta^{p][q}\delta^r_{k]}\delta^{s]}_v\Big]$$

Evaluating Pure Spinor Superspace Expressions

- It is straightforward to compute pure spinor superspace expressions in components (although tedious):
  1. Substitute SYM superfields by their theta expansions
  2. Use above PS superspace identities (and a lot more!)
  3. Write everything in terms of polarization (e_i^m, ξ^α) and momenta
- Doing that we finally get

$$\langle(\lambda A^1)(\lambda\gamma^m W^2)(\lambda\gamma^n W^3)\mathcal{F}^4_{mn}\rangle + \text{perm} = t_8 F^4 + \text{fermions}$$

PS survived

Pure spinor formalism is equivalent to RNS/GS at 1-loop order

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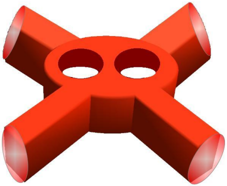
Massless 4-point two-loop amplitude



Prescription

$$\mathcal{A}_N = \langle \mathcal{N} \left(\int \mu \cdot b\right) \left(\int \mu \cdot b\right) \left(\int \mu \cdot b\right) \int U_2 \int U_3 \int U_4 \rangle$$

Pure Spinor Superspace Result

4 gravitons at two-loop order

$$\mathcal{A} = K \otimes \tilde{K} \int d^2\Omega_{11} d^2\Omega_{12} d^2\Omega_{22} \prod_{i=1}^{4} \int d^2 z_i \frac{\exp\left(-\Sigma_{i,j=1}^{4} k_i \cdot k_j G(z_i, z_j)\right)}{(\det Im\Omega)^5}$$

where

$$K_{\text{two-loop}} = \langle (\lambda\gamma^{mnpqr}\lambda)\mathcal{F}^1_{mn}\mathcal{F}^2_{pq}\mathcal{F}^3_{rs}(\lambda\gamma^s W^4)\rangle \Delta(z_1, z_3)\Delta(z_2, z_4)$$

$$+\text{perm}(1234)$$

Pure Spinor Superspace Result

- Using the above procedure it was shown (Berkovits,C.M.,2005) that

$$\langle(\lambda\gamma^{mnpqr}\lambda)\mathcal{F}^1_{mn}\mathcal{F}^2_{pq}\mathcal{F}^3_{rs}(\lambda\gamma^s W^4)\rangle\Delta(z_1,z_3)\Delta(z_2,z_4)$$

$$+\text{perm}(1234) = (t-u)t_8 F^4\Delta(z_1,z_2)\Delta(z_3,z_4) + \ldots$$

- Fermionic terms were also computed (C.Stahn,2007)
- Comparing it with the RNS result (D'Hoker, Phong, 2005)...

PS survived again

Pure spinor formalism is equivalent to RNS at 2-loop order

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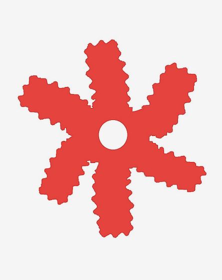
Gauge Variation of Massless 6-point one-loop amplitude



Example

$$\delta \mathcal{A}_N = \langle \mathcal{N} \left(\int \mu \cdot b \right) Q_{BRST} \Omega \int U_2 \int U_3 \int U_4 \int U_5 \int U_6 \rangle$$

Gauge Variation of Massless 6-point one-loop amplitude

- Gauge variation of unintegrated vertex is $\delta(\lambda^\alpha A_\alpha) = Q_{BRST}\Omega$
- Computed in the non-minimal pure spinor formalism (Berkovits & C.M. 2006)

Pure Spinor Superspace Result

$$\delta\mathcal{A} = K_{\text{anom}} \times \text{moduli space part}$$

where

$$K_{\text{anom}} = \langle(\lambda\gamma^m W)(\lambda\gamma^n W)(\lambda\gamma^p W)(W\gamma_{mnp}W)\rangle$$
$$= \epsilon_{10}F^5$$

Summary

Example

- Four gravitons at tree-level

$$K_{\text{tree}} = (t_8 \cdot F^4)$$

- Four gravitons at one-loop

$$K_{\text{one-loop}} = (t_8 \cdot F^4)$$

- Four gravitons at two-loop

$$K_{\text{two-loop}} = (t_8 \cdot F^4)\left[(t-u)\Delta(z_1, z_2)\Delta(z_3, z_4) + \text{permutations}\right]$$

- Anomaly kinematic factor

$$K_{\text{anom}} = (\epsilon_{10} \cdot F^5)$$

Results are OK

- Everything done so far agrees with standard RNS and GS results
  - General proof for tree-level equivalence (Berkovits, Valillo 2000)
  - Equivalence at one- and two-loop level by explicit computation (Berkovits, C.M. 2005/2006)
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  - Two-loop computation with the RNS: 200 pages
  - With pure spinors: 10 pages

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TODO list

- Compute the coefficients and check unitarity (work in progress)
- Compute higher-point amplitudes
- Study the properties of pure spinor superspace integrals

THE END