8 Nov. 2007

'A simple BRST system with quadratically constrained ghosts'

Michael Chesterman (UNC)

Overview

- Motivation
- Pure spinor superparticle review
- The simple system
  - Cohomology
  - "b" - field problem
- Character / Partition function
- Conclusions

Motivation

- Toy model for Berkovits' pure spinor superstring.

  Pure spinor superstring has some complicated features: i) 'b'-field in minimal formalism?
  ii) Covariant quantization of pure spinor fields requires infinitely many ghosts-for-ghosts.

- Simple system as starting point to study general systems with quadratically constrained ghosts.

Berkovits Superparticle $D=10$ $\mathcal{N}=1$

Superparticle - zero modes of superstring.

- Target space: $(\underbrace{X^a, \theta^\alpha}_{\substack{D=10\ \mathcal{N}=1 \\ \text{superspace}}}, \underbrace{\lambda^\alpha}_{\text{Bosonic ghost}})$

  $a=0,\dots,9 \qquad \alpha=1,\dots,16$

  Spinors: θ^α - Majorana-Weyl, fermionic.

  λ^α - Complex-Weyl, bosonic.

  $\lambda^\alpha \Gamma^a_{\alpha\beta} \lambda^\beta = 0$ — quadratic 'pure spinor' constraint

- BRST operator $\quad Q = \lambda^\alpha D_\alpha$, $\quad D_\alpha = \dfrac{\partial}{\partial\theta^\alpha} - i\Gamma^a_{\alpha\beta}\theta^\beta \dfrac{\partial}{\partial X^a}$

  λ^α: ghosts, D_α: constraints

  $$Q^2 = -i \lambda^\alpha \Gamma^a_{\alpha\beta} \lambda^\beta \frac{\partial}{\partial X^a} = 0$$

  $H^*(Q) \equiv \{$Physical, classical fields of linearised SYM$\}$
  BV

- Not BRST quantisation of a gauge theory.

The simple system

- Simplest non-trivial quadratic constraint:

$$u^a u^a = 0 \qquad a = 1,2 \qquad u^a - \text{complex bosonic.}$$

- BRST operator: $\Omega = u^a D_a$

ghosts — fermionic constraints

$$\{D_a, D_b\} = -2i\,\delta_{ab}\,\frac{\partial}{\partial X}$$

$$\Omega^2 = -i u^a u_a \frac{\partial}{\partial X} = 0$$

- Target space: $(\underbrace{X, \theta^a}_{D=1,\,N=2}, \underset{\sim}{u^a})$

$D=1, N=2$ Superspace, Bosonic ghosts, $SO(2)$ R-sym

- $D_a = \dfrac{\partial}{\partial \theta^a} - i\,\theta_a \dfrac{\partial}{\partial X}$ — covariant derivative

$Q_a = \dfrac{\partial}{\partial \theta^a} + i\,\theta_a \dfrac{\partial}{\partial X}$ — Supersym. generator

$[Q_a, \Omega] = 0 \Rightarrow Q_a$ - physical operator

Zero momentum cohomology

- BRST charge $\rightarrow$ $\Omega_0 = u^a \dfrac{\partial}{\partial \theta^a}$

  $\Omega_0^2 = 0$ even for unconstrained u^a.

- Relate cohomology $H(\Omega_0 \,|\, u^a u^a = 0)$

  to cohomology $H(\Omega_0)$ for unconstrained u^a.

- BRST closed wavefunction $F_0(u,\theta)$ obeys

  $$\Omega_0 F_0 = u^a u^a F_1(u,\theta)$$

  $$\Omega_0 F_1 = 0$$

  BRST transformations: $\delta F_0 = \Omega_0 G_0$

  $\delta F_1 = \Omega_0 G_1 \quad \delta F_0 = u^a u^a G_1$

- Note $H^*(\Omega_0) \equiv$ const. for u^a unconstrained

If $F_1 = 0$, then $F_0 \in H^*(\Omega_0) =$ const.

Otherwise $F_1 \in H^*(\Omega_0) \equiv$ const

$F_1 = \tilde{C} \Rightarrow F_0 = u^a \theta^a \tilde{C}$

$H(\Omega_0 \,|\, u^a u^a = 0) = \boxed{\Psi = C + u^a \theta^a \tilde{C}}$

Zero momentum cohomology for Berkovits superparticle

$$\Omega_0 = \lambda^\alpha \frac{\partial}{\partial \theta^\alpha} \qquad \lambda^\alpha \Gamma^a_{\alpha\beta} \lambda^\beta = 0$$

$$\Omega_0 F_0 = \lambda \Gamma_a \lambda \, F_1^a$$

$$\Omega_0 F_1^a = (\lambda \Gamma^a)_\alpha F_2^\alpha$$

$$\Omega_0 F_2^\alpha = (\lambda \Gamma_b \lambda \Gamma^{b\,\alpha\beta} - 2\lambda^\alpha \lambda^\beta) F_{3\beta}$$

$$\Omega_0 F_{3\beta} = (\lambda \Gamma^c)_\beta F_{4c}$$

$$\Omega_0 F_{4c} = \lambda \Gamma_c \lambda \, F_5$$

$$\Omega_0 F_5 = 0$$

$\{F_i\}$ - Fields of BV $N=1$ SYM

$$\begin{array}{cccccc} F_0 & F_1^a & F_2^\alpha & F_{3\alpha} & F_{4c} & F_5 \\ C & A^a & \chi^\alpha & \tilde{\chi}_\alpha & \tilde{A}_a & \tilde{C} \end{array}$$

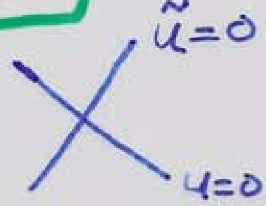
$$\Psi = C + \lambda\theta A + \lambda\theta^2 \chi + \lambda^2 \theta^3 \tilde{\chi} + \lambda^2 \theta^4 \tilde{A} + \lambda^3 \theta^5 \tilde{C}$$

Full Cohomology

- $H^*(\Omega)$ can be solved directly at all ghost numbers for this model

- Define $u = u^1 + iu^2$, $\boxed{u\tilde{u} = 0}$
  $\tilde{u} = u^1 - iu^2$

  $D=1$ cone



- BRST operator $\Omega = uD + \tilde{u}\bar{D}$

$$D = \frac{\partial}{\partial \theta} - \frac{i}{2}\bar{\theta}\frac{\partial}{\partial x}, \quad \bar{D} = \frac{\partial}{\partial \bar{\theta}} - \frac{i}{2}\theta\frac{\partial}{\partial x}$$

$$D^2 = \bar{D}^2 = 0$$

- General wavefnc. holomorphic in $u, \tilde{u}$ on constraint surface

$$\Psi = \underline{f_0(x,\theta)} + \sum_{p=1}^{\infty} u^p f_p(x,\theta) + \tilde{u}^p \tilde{f}_p(x,\theta)$$

- $H^*(\Psi)$: $p \geq 2$: $f_p \in H(D) = 0$, $\tilde{f}_p \in H(\bar{D}) = 0$

  $p = 1$: $\delta f_1 = D\underline{g_0}$, $\delta \tilde{f}_1 = \bar{D}\underline{g_0}$

$$\Rightarrow \Psi_1 = \frac{1}{2}(u\bar{\theta} + \tilde{u}\theta)\tilde{c}(x)$$

$$\delta \tilde{c}(x) = \frac{\partial}{\partial x} g(x)$$

Equivalence to a standard BRST model

- Physical wavefnc $\Psi = C + u^a \Theta^a \tilde{C}(x)$

  where $C = \text{const}$, $\delta \tilde{C} = \frac{\partial}{\partial x} g(x)$

- Equivalent model:

  BRST charge: $\Omega = \underbrace{c}_{\text{fermionic ghost}} \underbrace{\frac{\partial}{\partial x}}_{\text{constraint}}$

  wavefnc: $\Psi = C(x) + c\,\tilde{C}(x)$

  $H^*(\Omega)$: $\frac{\partial}{\partial x} C = 0$, $\delta \tilde{C}(x) = \frac{\partial}{\partial x} g(x)$

- Cohomologies match $\Rightarrow$ equivalence

Operator Cohomology and the 'b'-field problem

- For standard simple BRST system with BRST charge $\Omega = c\dfrac{\partial}{\partial x}$

'Fundamental' gh no. 0 operator which is Ω-exact

$$— \quad \frac{1}{i}\frac{\partial}{\partial x} = \{\underbrace{\frac{1}{i}\frac{\partial}{\partial c}}_{b}, \Omega\} \equiv \text{1st class constraint}$$

- Our system: $\Omega = u^a D_a \qquad u^a u^a = 0$

Gauge symmetry transformation: $\delta_\varepsilon \dfrac{\partial}{\partial u^a} = [\varepsilon \dfrac{\partial}{\partial u^a}, \frac{1}{2} u^b u^b]$

$$= \varepsilon(u)\, u^a$$

$$\not\approx 0$$

$\dfrac{1}{i}\dfrac{\partial}{\partial u^a}$ - ghost momenta are not gauge invariant.

- Action $[\Omega, \cdot\,]$ is on space of gauge invariant operators.

D_a are not Ω-exact $\therefore$ are not the real constraints of the theory

- What are the fundamental BRST-exact operators?
- They must be BRST-closed:

  Candidates: $\frac{1}{i}\frac{\partial}{\partial x}$, $\underbrace{Q_a}_{\text{Supersym generator}}$

- Must be Ω-exact because $\frac{1}{i}\frac{\partial}{\partial x}\Psi_{phys} \sim 0$

  $Q_a \Psi_{phys} \sim 0$

  But what is b – $\{b, \Omega\} = \frac{1}{i}\frac{\partial}{\partial x}$?

  , f_a – $[f_a, \Omega] = Q_a$?

  We know how b acts on Ω-closed states....

  $$\frac{1}{i}\frac{\partial}{\partial x} u^p f_p = \Omega\, b\, u^p f_p = \Omega \underbrace{\frac{1}{u}\bar{D}}_{b} u^p f_p \quad p \geq 1$$

  Similarly $$\Omega \underbrace{\frac{1}{\tilde{u}} D}_{b} \tilde{u}^p \tilde{f}_p = \frac{1}{i}\frac{\partial}{\partial x}\tilde{u}^p f_p \quad p \geq 1$$

  $$b f_0 = 0$$

- b acts differently in the $u=0$ and $\tilde{u}=0$ sectors : define projection operators

- Projection operators: $P\Psi = \sum_{p=1}^{\infty} u^p f_p(x,\theta)$

$$\tilde{P}\Psi = \sum_{p=1}^{\infty} \tilde{u}^p \tilde{f}_p(x,\theta)$$

$$P = \int d\tilde{u}\, \delta(\tilde{u}) - \int du \int d\tilde{u}\, \delta(u)\,\delta(\tilde{u})$$

Guess: $b = \frac{1}{u} P \bar{D} + \frac{1}{\tilde{u}} \tilde{P} D$?

- $\{b, \Omega\} = \frac{1}{i}\frac{\partial}{\partial x}\Big(\underbrace{P + \tilde{P} + \int du \int d\tilde{u}\, \delta(u)\,\delta(\tilde{u})}_{\approx \mathbb{1}}\Big)$ ✓

- More simply: $\boxed{b = \frac{1}{2}\int_{C(u^1,u^2)} du^a D_a \; \delta(u^b u^b)}$

C – $\left\{\begin{array}{l}\text{paths } \perp\text{r to cone } u\tilde{u}=0 \\ \quad \text{excluding path through cone tip: } u=\tilde{u}=0\end{array}\right\}$

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b-field in Berkovits superparticle/string

Particle: $\{b, Q\} = \frac{1}{2} P^2$ What is b?

String: $\{b^{(z)}_{zz}, Q\} = T_{zz}(z)$

b_{zz} is important for string loop amplitudes

- Workaround: $\{\underbrace{b_{zz}(u,v)}_{gh\# 0}, Q\} = T_{zz}(u) \underbrace{Z(v)}_{gh\# 1}$

  $Z(v)$ is Picture changing operator

- Non-minimal formalism

- Tonin b-field $b = \frac{u_\beta}{(\lambda^\alpha u_\alpha)} \Gamma^{a\beta\alpha} P_a D_\alpha$

  Simple, but $\frac{1}{\lambda \cdot u}$?

- Try to find b-field as in simple system

(13) Partition function (Character)

- Implement $u^a u^a = 0$ constraint using BRST charge

$$\Delta = C\, u^a u^a$$

$\uparrow$ fermionic ghost

- Ω BRST charge acts on space $H^*(\Delta)$

$$\Delta^2 = 0 \ , \quad \Omega^2 = -i\left\{\frac{\partial}{\partial C}\frac{\partial}{\partial x}\Delta\right\} \simeq 0 \ , \quad \{\Delta, \Omega\} = 0$$

Ghost no's: $\quad G_\Delta = C\dfrac{\partial}{\partial C} \qquad G_\Omega = u^a \dfrac{\partial}{\partial u^a} - 2C\dfrac{\partial}{\partial C}$

- Advantage of BRST: all ~~states~~ fields are unconstrained. Cohomology $\Rightarrow$ constraints

- Zero-momentum Character — information about physical spectrum, without direct calculation

$$\chi = \mathrm{Tr}_{\mathrm{Phys}} (-)^F t^K = \mathrm{Tr}_{\mathrm{All}} (-)^F t^K$$

F = fermion # $\qquad$ K = physical operator

K is a number operator: $K = \theta^a \frac{\partial}{\partial \theta^a} + u^a \frac{\partial}{\partial u^a} - 2c\frac{\partial}{\partial c} + 2$

$$\chi = -\underbrace{(1-t)^2}_{\theta^a} \underbrace{t^2(1-t^{-2})}_{c} \underbrace{(1-t)^{-2}}_{u^a}$$

$$= 1 - t^2$$

Recall $\Psi_{phys} = (\underbrace{\phi}_{1} + \underbrace{u^a\theta^a}_{-t^2}\tilde{\phi})\,c$ ✓

- For Berkovits superparticle, Δ requires infinitely many ghosts-for-ghosts.

Can work backwards from cohomology to calculate character contribution from these ghosts.

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Conclusions

- This simple model is useful toy to help better understand BRST systems with quadratic ghost constraints.
  - b-ghost Berkovits superstring?
- Other useful models?
- This model is useful, because it can be solved completely. - even the 'b'-ghost problem