

Dimension quenching and c-duality

Ian Swanson
Institute for Advanced Study

based on work with Simeon Hellerman:

hep-th/0611317

hep-th/0612051

hep-th/0612116

arxiv:0705.0980 [hep-th]

UNC / Duke, Nov 15, 2007

Preface

It would be useful to understand how string theories in **supercritical dimensions** are related to the more familiar incarnations of string theory.

In this talk I will describe cosmological solutions of string theory that interpolate in time between theories with **different numbers of spacetime dimensions** and **different amounts of worldsheet supersymmetry**.

These cosmologies connect supercritical string theories to the more familiar string duality web in ten dimensions.

They also provide a precise link between **supersymmetric** and **purely bosonic** string theory.

Outline

Overview of quintessent cosmology and linear dilaton backgrounds

Supercritical string theory: spacetime effective action

Stability in time-dependent backgrounds

Exact solutions with nonzero tachyon

Type 0 tachyon condensation

Conclusions

Outline

Overview of quintessent cosmology and linear dilaton backgrounds

Supercritical string theory: spacetime effective action

Stability in time-dependent backgrounds

Exact solutions with nonzero tachyon

Type 0 tachyon condensation

Conclusions

Quintessent cosmology in D dimensions

Let's start with a **toy model**.

The metric for a spatially flat ($k = 0$) FRW cosmology is

$$ds^2 = -dt^2 + a(t)^2 dx^i dx^i$$

where $i = 1, \dots, D - 1$.

The equation of motion for the scale factor $a(t)$:

$$\frac{\ddot{a}}{a} = -\frac{D - 3 + w(D - 1)}{(D - 1)(D - 2)}\rho$$

where $w \equiv p/\rho$ is the **state equation**.

Consider a theory of a real scalar field ϕ with Lagrangian

$$\mathcal{L}_\phi = \frac{1}{\kappa^2} \sqrt{-\det G} \left[\frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - \mathcal{V}(\phi) \right]$$

where

$$\mathcal{V}(\phi) \equiv c \exp(\gamma\phi), \quad c, \gamma > 0$$

Linear dilaton background

At this point, we adopt the ansatz that our solution exhibits a **constant equation of state w** .

It follows that $\dot{\phi}^2$, H^2 and \mathcal{V} all scale as t^{-2} , so we find the general expressions

$$\begin{aligned}\phi(t) &= \lambda \log(t/t_1) \\ a(t) &= a_0 \left(\frac{t}{t_0} \right)^\alpha\end{aligned}$$

for some α, λ .

This amounts to a direct relation between \dot{a} and a , which can be integrated to yield the following:

$$\begin{aligned}\alpha &= \frac{2}{(1+w)(D-1)} \\ \gamma^2 &= \frac{2(D-1)(w+1)}{D-2}\end{aligned}$$

Linear dilaton background

Because $c > 0$, the energy density ρ is positive, and the cosmological scale accelerates as a function of FRW time if $-1 \leq w < w_{\text{crit}}$, where

$$w_{\text{crit}} = -\frac{D-3}{D-1}$$

in D spacetime dimensions.

The **global causal structure** of the solution therefore depends on whether w is less than, greater than, or equal to the critical value w_{crit} .

The spatial slice $t = 0$ defines an **initial singularity** in all three cases.

The precise nature of this singularity and the nature of the **asymptotic future** $t \rightarrow +\infty$, however, depend on the state equation of the cosmology.

Linear dilaton background

We can rewrite the metric in a canonical form for a **conformally flat spacetime**. We define a new time coordinate \bar{t} via the equation

$$\bar{t} \equiv \left(\frac{(D-1)(1+w)}{(D-1)w + (D-3)} t_0^{\frac{2}{(D-1)(1+w)}} a_0^{-1} \right) t^{\frac{(D-1)w + (D-3)}{(D-1)(1+w)}}$$

In these coordinates, the metric takes the form

$$ds^2 = \omega(\bar{t})^2 \left[-d\bar{t}^2 + dx^i dx^i \right] = \omega(\bar{t})^2 \left[-d\bar{t}^2 + dr^2 + r^{D-2} d\Omega_{D-2}^2 \right]$$

where we have defined

$$\omega(\bar{t}) \equiv l \left(\frac{(D-1)w + (D-3)}{(D-1)(1+w)} \bar{t} \right)^{\frac{2}{(D-1)w + (D-3)}}$$
$$l \equiv a_0 \left(\frac{a_0}{t_0} \right)^{\frac{2}{(D-1)w + (D-3)}}$$

Linear dilaton background

To construct Penrose diagrams, one ignores the $(D - 2)$ -sphere fibered over each diagram and conformally compactifies the (r, t) plane using the transformation

$$r \equiv \frac{\sin \chi}{\cos \chi + \cos \tau} \quad \bar{t} \equiv \frac{\sin \tau}{\cos \chi + \cos \tau}$$

In these coordinates, the metric on the (r, t) plane becomes

$$ds^2 = \frac{l^2}{4} \frac{(\sin |\tau|)^{2\Delta}}{\left[\cos\left(\frac{\chi+\tau}{2}\right)\cos\left(\frac{\chi-\tau}{2}\right)\right]^{2+2\Delta}} \left| \frac{(D-1)w + (D-3)}{2(D-1)(1+w)} \right|^{2\Delta} \\ \times (-d\tau^2 + d\chi^2)$$

with

$$\Delta \equiv \frac{2}{(D-1)w + (D-3)}$$

Linear dilaton background

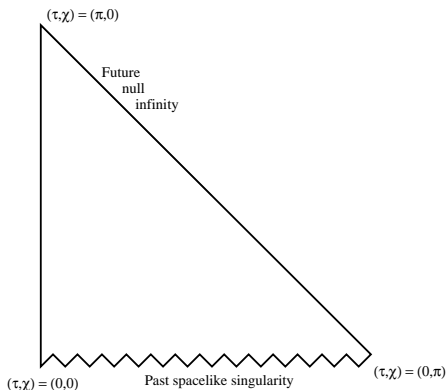
For $-1 < w < w_{\text{crit}}$, the constant Δ is **negative** and the range of the τ and χ coordinates is

$$\tau \in [-\pi, 0] \quad \chi \in [0, \tau + \pi] \quad (\text{accelerating universe}).$$

For $w > w_{\text{crit}}$, the quantity Δ becomes **positive**, and we have

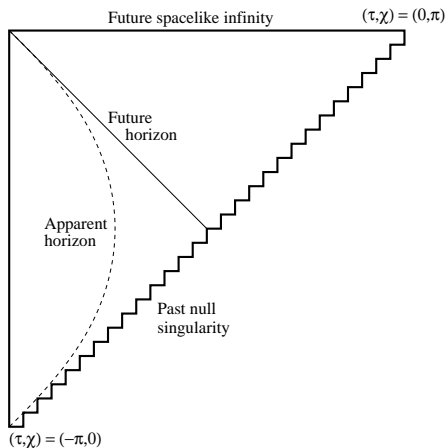
$$\tau \in [0, \pi] \quad \chi \in [0, \pi - \tau] \quad (\text{decelerating universe}).$$

Linear dilaton background



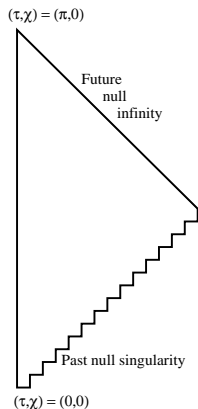
Penrose diagram of the **decelerating universe** ($w > w_{\text{crit}}$). The initial singularity is spacelike, and the future boundary is null.

Linear dilaton background



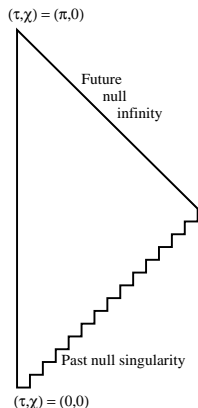
Penrose diagram of the **accelerating** ($-1 < w < w_{\text{crit}}$) universe. The initial singularity is null, and the future spacelike boundary is obscured from observers by a horizon.

Linear dilaton background



Penrose diagram of the universe with **critical equation of state** ($w = w_{\text{crit}}$).

Linear dilaton background



Penrose diagram of the universe with **critical equation of state** ($w = w_{\text{crit}}$). The initial singularity is null, as is the future boundary. It is **conformally equivalent to Minkowski space** (conventional big bang, asymptotic infinity, and ordinary final states).

Outline

Overview of quintessent cosmology and linear dilaton backgrounds

Supercritical string theory: spacetime effective action

Stability in time-dependent backgrounds

Exact solutions with nonzero tachyon

Type 0 tachyon condensation

Conclusions

Supercritical string theory

For the **bosonic string** in $D > 26$, the effective action for the metric and dilaton appears as

$$S_{\text{eff}} = \frac{1}{2\kappa^2} \int d^D x \sqrt{-\det G^{(S)}} \exp(-2\Phi) \left[-\frac{D-26}{3\alpha'} + R^{(S)} + 4(\partial\Phi)^2 \right]$$

Higher dimension terms are dropped: such terms in the tree-level action are suppressed by powers of $\alpha' = 1/(2\pi T_{\text{string}})$, where T_{string} is the fundamental string tension.

We may rewrite the action in terms of the **Einstein metric** using the field redefinition

$$G_{\mu\nu}^{(S)} = \exp\left(\frac{4\Phi}{D-2}\right) G_{\mu\nu}^{(E)}$$

We may also rescale $\Phi \rightarrow \frac{1}{2}\sqrt{D-2} \phi \dots$

Supercritical string theory

We obtain:

$$S_{\text{eff}} = \frac{1}{2\kappa^2} \int d^D X \sqrt{-\det G^{(E)}} \left[-\frac{2(D-26)}{3\alpha'} \exp\left(\frac{2\phi}{\sqrt{D-2}}\right) + R^{(E)} - (\partial\phi)^2 \right]$$

This is the action for a quintessent cosmology, with coefficients now defined by the following values:

$$\gamma = \frac{2}{\sqrt{D-2}} \quad c = \frac{D-26}{3\alpha'}$$

We therefore recover a quintessent solution with equation of state

$$w = -\frac{D-3}{D-1}$$

Supercritical string theory

The tree-level potential of the string theory gives rise to an equation of state at the boundary between accelerating and decelerating cosmological backgrounds.

The resulting spacetime is globally conformally equivalent to Minkowski space.

In retrospect, this must have been the case: The worldsheet theory of the string in this background is *defined* to have a target space with string frame metric $\eta_{\mu\nu}$, and coordinates X^μ that are infinite in extent.

Supercritical string theory

The general quintessent solution is of the form

$$\Phi = \Phi_0 - \frac{D-2}{2} \ln \left(\frac{t}{t_0} \right)$$

$$a = \frac{a_0}{t_0} t$$

We have the relation

$$t_{\text{FRW}} = t_0 \exp \left(+ \frac{2q t_{\text{conf}}}{D-2} \right)$$

So when we move to coordinates in which the metric is manifestly conformally flat and we decanonicalize the scalar field, we find that the dilaton is **logarithmic** as a function of FRW time, and **linear** as a function of conformal time:

$$ds^2 = \frac{a_0^2}{t_0^2} t^2 (-dt_{\text{conf}}^2 + dx^i dx^i) = a^2 \eta_{\mu\nu} dx^\mu dx^\nu \quad (1)$$

$$\Phi = \Phi_0 - q t_{\text{conf}}$$

Outline

Overview of quintessent cosmology and linear dilaton backgrounds

Supercritical string theory: spacetime effective action

Stability in time-dependent backgrounds

Exact solutions with nonzero tachyon

Type 0 tachyon condensation

Conclusions

Stability

In time-dependent backgrounds there is no obvious or natural definition of stability.

One must take into account the coupling of the background fields to the string modes, which is suppressed by a factor of the string coupling $g_s \sim e^\phi$.

Exponentially growing modes will have decreasing effect on the remaining degrees of freedom in the theory if they grow more slowly than g_s^{-1} .

A useful definition, therefore, is that unstable modes must grow faster than g_s^{-1} at late times.

Stability

Consider a massless scalar coupled to the string:

$$\mathcal{L}_\sigma = -\frac{1}{2\kappa^2} \sqrt{-\det G^{(S)}} e^{-2\Phi} (\partial\sigma)^2 = -\frac{1}{2\kappa^2} \sqrt{-\det G^{(E)}} (\partial\sigma)^2$$

The scalar field has solutions of the form

$$\sigma = \sigma_\infty - \xi t^{-(D-2)}$$

where σ_∞ and ξ are constants of motion that can take arbitrary real values.

The modes of the field σ asymptote to the constant value σ_∞ as $t \rightarrow \infty$.

From the point of view of the Einstein frame, this effect is due to [Hubble friction](#); in the string frame this behavior is understood to be caused by the drag force arising from the interaction between σ and the linear dilaton.

Stability

Quanta of the string are necessarily **coupled to the flat metric**. To recover fluctuations in such a form we must introduce a rescaled field $\tilde{\sigma}$:

$$\tilde{\sigma} \equiv e^{-\Phi} \sigma$$

This induces a mass term for the rescaled field that represents a proper quantum of string:

$$e^{-2\Phi} (\partial\sigma)^2 = (\partial\tilde{\sigma})^2 + \tilde{\sigma}^2 (\partial\Phi)^2 + 2\tilde{\sigma} (\partial\tilde{\sigma}) [(\partial\Phi)_{\text{background}} + (\partial\Phi)_{\text{fluctuation}}]$$

- ▶ The fluctuation term represents a trilinear vertex that we discard.
- ▶ The background term is constant, so its product with $\tilde{\sigma} \partial\tilde{\sigma}$ amounts to a total derivative.

Stability

The mass term for the rescaled field is **tachyonic**, and equal to $-q^2$:

$$\mathcal{L}_{\tilde{\sigma}} \sim -\frac{1}{2\kappa^2} [(\partial\tilde{\sigma})^2 - q^2\tilde{\sigma}^2]$$

The criterion for stability we proposed was that **g_s times the canonical field $\tilde{\sigma}$ not increase exponentially with time.**

Since $g_s\tilde{\sigma}$ is just σ (the original field appearing in the spacetime action in front of $\exp(-2\Phi)$), the requirement for physical stability is that **modes normalized to have the factor $\exp(-2\Phi)$ in their kinetic term should shrink exponentially in the future (or at least remain constant).**

So, canonical worldsheet modes may grow exponentially in time, but they do not necessarily represent a physical instability: **the exponential growth is countered by the shrinking string coupling.**

Stability

We are ultimately interested in “truly tachyonic” perturbations in the theory that are marginal on the worldsheet.

We have found a large class of such perturbations that turn out to be exactly solvable at the quantum level, despite the fact that the underlying theories are fully interacting.

Outline

Overview of quintessent cosmology and linear dilaton backgrounds

Supercritical string theory: spacetime effective action

Stability in time-dependent backgrounds

Exact solutions with nonzero tachyon

Type 0 tachyon condensation

Conclusions

Exact solutions with nonzero tachyon

Consider a theory with stress tensor

$$T_{++} = -\frac{1}{\alpha'} : \partial_{\sigma^+} X^\mu \partial_{\sigma^+} X_\mu : + \partial_{\sigma^+}^2 (V_\mu X^\mu)$$

$$T_{--} = -\frac{1}{\alpha'} : \partial_{\sigma^-} X^\mu \partial_{\sigma^-} X_\mu : + \partial_{\sigma^-}^2 (V_\mu X^\mu)$$

where colons represent normal ordering of the 2D theory. Here, σ^\pm are particular light-cone combinations of the worldsheet coordinates $\sigma^{0,1}$:

$$\sigma^\pm = -\sigma^0 \pm \sigma^1$$

Physical states of the string correspond to local operators \mathcal{U} that are Virasoro primaries of weight one. That is, their operator product expansion (OPE) with the stress tensor satisfies:

$$T_{++}(\sigma)\mathcal{U}(\tau) \simeq \frac{\mathcal{U}(\tau)}{(\sigma^+ - \tau^+)^2} + \frac{\partial_+ \mathcal{U}(\tau)}{\sigma^+ - \tau^+}$$

and similarly for T_{--} ,

Exact solutions with nonzero tachyon

A profile $\mathcal{T}(X)$ for the tachyon corresponds to the vertex operator

$$\mathcal{U}_M \equiv: \mathcal{T}(X) :$$

and admits the following on-shell condition:

$$\partial_\mu \partial^\mu \mathcal{T}(X) - 2V^\mu \partial_\mu \mathcal{T}(X) + \frac{4}{\alpha'} \mathcal{T}(X) = 0$$

For tachyon profiles of the form

$$\mathcal{T}(X) = \mu^2 \exp(B_\mu X^\mu)$$

this condition is

$$B^2 - 2V \cdot B = -4/\alpha'$$

A general value of B_μ will lead to a **nontrivial interacting theory** when the strength μ^2 of the perturbation is treated as non-infinitesimal.

Exact solutions with nonzero tachyon

There is a special set of choices for B_μ that renders the 2D theory **well-defined** and **conformal** to all orders in perturbation theory.

We choose the first term in the linearized tachyon equation of motion to vanish separately.

This is tantamount to choosing the vector B_μ to be null. **This renders the vertex operator** : $\exp(B_\mu X^\mu)$: **non-singular in the vicinity of itself.**

We therefore put B_μ in the form

$$\begin{aligned} B_0 &= B_1 \equiv \beta/\sqrt{2} \\ B_i &= 0, \quad i \geq 2 \end{aligned}$$

Exact solutions with nonzero tachyon

The initial singularity of the cosmology lies in the strong-coupling region, and the tachyon increases into the future.

This gives rise to a particularly simple quantum theory. The kinetic term for X^\pm appears as

$$\mathcal{L} \sim -\frac{1}{2\pi\alpha'} \left[(\partial_{\sigma 0} X^+) (\partial_{\sigma 0} X^-) - (\partial_{\sigma 1} X^+) (\partial_{\sigma 1} X^-) \right]$$

The propagator for the X^\pm fields is therefore **oriented**.

$$X^+ \longrightarrow X^-$$

Exact solutions with nonzero tachyon

- ▶ The X field has oriented propagators.
- ▶ All the interaction vertices in the theory depend only on X^+ .
- ▶ There are no non-trivial Feynman diagrams in the theory.
- ▶ This constitutes an interacting quantum theory, **without quantum corrections**.

(In conformal gauge, prior to enforcing gauge constraints, the theory is not unitary.)

The tachyon couples to the worldsheet in the term

$$\mathcal{L} \sim -\frac{1}{2\pi}\mu^2 \exp(\beta X^+)$$

Classically, X^+ is harmonic, and acts as a source for X^- .

Exact solutions with nonzero tachyon

By writing the solution to the Laplace equation for X^+ as

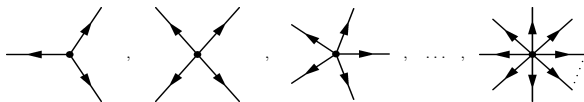
$$X^+ = f_+(\sigma^+) + f_-(\sigma^-)$$

the general solution for X^- can be expressed as follows:

$$X^- = g_+(\sigma^+) + g_-(\sigma^-) + \frac{\alpha' \beta \mu^2}{4} \left[\int_{\sigma^+}^{\infty} dy^+ \exp(\beta f_+(y^+)) \right] \left[\int_{\sigma^-}^{\infty} dy^- \exp(\beta f_-(y^-)) \right]$$

We thus see that the theory is **exactly solvable**.

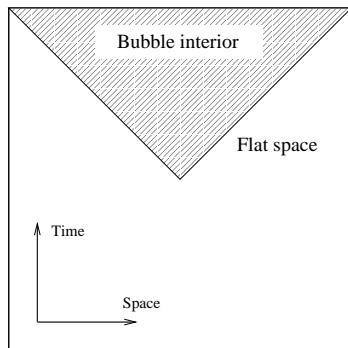
All interaction vertices in the theory depend only on X^+ , and therefore correspond to diagrams composed **strictly from outgoing lines**:



Physical interpretation

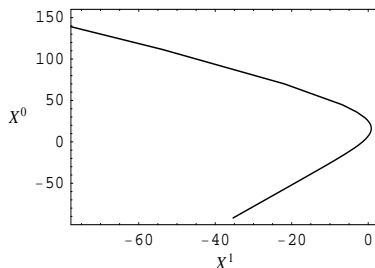
The solution can be thought of as a **phase boundary** in spacetime between the $\mathcal{T} = 0$ phase and the $\mathcal{T} > 0$ phase.

The spacetime picture is therefore a phase bubble expanding out from a nucleation point:



Physical interpretation

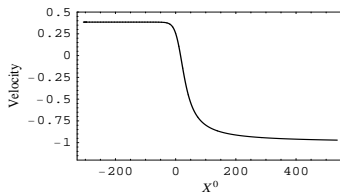
To see what happens to states in the neighborhood of the bubble we can place a string state near the phase boundary.



The state collides with the bubble wall and is **forced out of the region with nonzero tachyon**. (The solution has $\mu^2 = 1$, $\beta = .1$, and the trajectory corresponds to $p^+ = 3$, $H_{\perp} \equiv \frac{\alpha' p_i^2}{2} = 4$.)

Physical interpretation

We can also plot the velocity of the particle as a function of time:

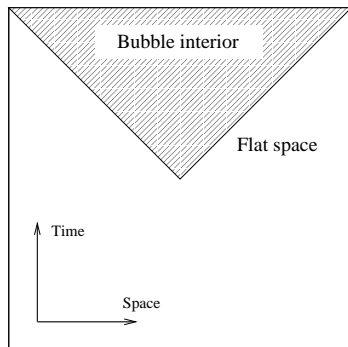


So the particle propagates until it hits the bubble wall, where the exponential term becomes important. At that point, the speed of the particle rapidly goes to -1 .

Physical interpretation

Absolutely no matter (including gravitons) can enter the region of nonzero tachyon.

The solution can be thought of as a **bubble of nothing**.



Exact solutions with nonzero tachyon

Let's now introduce some dependence on a **third direction**:

$$\mathcal{T}(X) = \mu_0^2 \exp(\beta X^+) - \mu_k^2 \cos(kX_2) \exp(\beta_k X^+)$$

with

$$q\beta_k = \sqrt{2} \left(\frac{2}{\alpha'} - \frac{1}{2} k^2 \right)$$

This is exactly marginal when the wavelength k^{-1} of the tachyon is long compared to the string scale:

$$\mathcal{T}(X^+, X_2) = + \frac{\mu^2}{2\alpha'} \exp(\beta X^+) : X_2^2 : + \mathcal{T}_0(X^+)$$

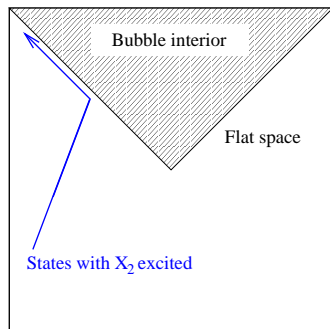
$$\mathcal{T}_0(X^+) = \frac{\mu^2 X^+}{\alpha' q \sqrt{2}} \exp(\beta X^+) + \mu'^2 \exp(\beta X^+)$$

Exact solutions with nonzero tachyon

States with modes of X_2 excited are pushed out along the bubble wall: the physics is **essentially the same as the bubble of nothing**.

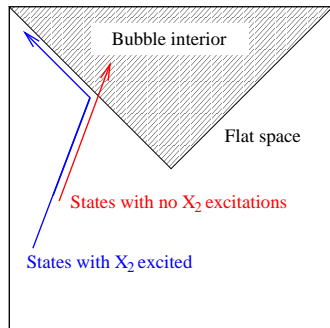
At late times, the adiabatic theorem is satisfied to a better approximation, and these modes become **frozen in an excited state**.

So these string states are pushed out to infinity and disappear from the theory in the late-time limit:



Exact solutions with nonzero tachyon

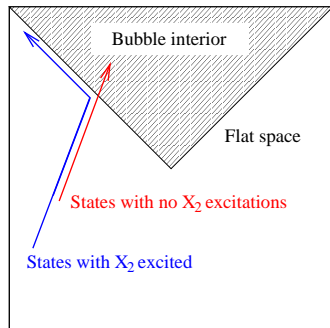
There is a less generic class of states with no energy in the X_2 direction.



These propagate *through the domain wall and into the bubble region.*

Exact solutions with nonzero tachyon

There is a less generic class of states with no energy in the X_2 direction.



These propagate *through the domain wall and into the bubble region.*

The result is that the amount of matter on the worldsheet decreases dynamically as a function of time.

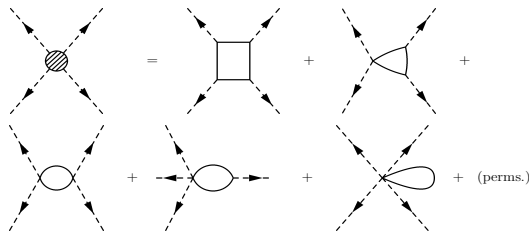
Exact solutions with nonzero tachyon

In other words, the number of dimensions in the target space decreases as a function of time.

Question: What happens to the central charge if the spacetime dimension shifts? How can the perturbation be marginal?

The theory is solvable, so we should be able to answer this question exactly.

In fact, quantum corrections in this theory truncate at one-loop order:



Exact solutions with nonzero tachyon

The one-loop diagrams can be thought of as a set of **effective vertices** for X^+ , associated with integrating out the massive field X_2 .

As you approach the domain wall, there are a number of massive string modes that acquire expectation values, and there are higher-derivative operators dressed with factors of $\exp(X^+)$.

Most of these decay exponentially in the future.

In fact: in the far future, all corrections coming from integrating out X_2 decay away, except for three contributions:

- ▶ the effective tachyon,
- ▶ the dilaton,
- ▶ the string-frame metric.

Exact solutions with nonzero tachyon

The **effective tachyon** can be fine-tuned away in the future.

The remaining contributions are always nonzero, coming from the following diagrams:

$$\Delta(\partial_+ \Phi) = \text{Diagram 1}$$
A Feynman diagram consisting of a circle with a dashed line entering from the left and a dashed line exiting to the right with an arrowhead pointing right.

$$\Delta G_{++} = \text{Diagram 2}$$
A Feynman diagram consisting of a circle with a dashed line entering from the left with an arrowhead pointing left, and a dashed line exiting to the right with an arrowhead pointing right.

Write the **renormalized** dilaton gradient and string-frame metric as:

$$\begin{aligned}\hat{V}_\mu &\equiv V_\mu + \Delta V_\mu \\ \hat{G}^{\mu\nu} &\equiv G_{\mu\nu} + \Delta G_{\mu\nu}\end{aligned}$$

Exact solutions with nonzero tachyon

We obtain:

$$\hat{V}_- = V_- = -\frac{q}{\sqrt{2}}$$

$$\hat{V}_+ = -\frac{q}{\sqrt{2}} + \frac{\beta}{12}$$

$$\hat{G}^{+-} = \hat{G}^{-+} = -1 \quad \hat{G}^{--} = -\frac{\alpha'\beta^2}{24} \quad \hat{G}^{++} = 0$$

q and β take values such that $q^2 = (D - 26)/6\alpha'$ and $q\beta = \frac{2\sqrt{2}}{\alpha'}$.

In the $X^+ \rightarrow \infty$ limit, we therefore get

$$c^{\text{dilaton}} = 6\alpha' \hat{G}^{\mu\nu} \hat{V}_\mu \hat{V}_\nu = -(D - 26) + 1$$

The result is that the shift in central charge contribution from the dilaton **precisely cancels** the central charge shift due to the reduction in spacetime dimension.

Exact solutions with nonzero tachyon

This mechanism of **central charge transfer** works equally well when the tachyon has a quadratic minimum in several transverse directions:

$$\mathcal{T}(X) = \frac{\mu^2}{2\alpha'} \exp(\beta X^+) \sum_{i=2}^{n+1} : X_i^2 : + \mathcal{T}_0(X^+)$$
$$\mathcal{T}_0(X^+) = \frac{n\mu^2 X^+}{\alpha' q \sqrt{2}} \exp(\beta X^+) + \mu'^2 \exp(\beta X^+)$$

In this case the renormalization of the metric and dilaton leads to a central charge contribution in the $X^+ \rightarrow \infty$ limit given by:

$$c^{\text{dilaton}} = 6\alpha' \hat{G}^{\mu\nu} \hat{V}_\mu \hat{V}_\nu = -6\alpha' q^2 + \frac{nq\beta\alpha'}{\sqrt{2}} - \frac{n\alpha'^2 q^2 \beta^2}{8} = -(D - 26) + n$$

Outline

Overview of quintessent cosmology and linear dilaton backgrounds

Supercritical string theory: spacetime effective action

Stability in time-dependent backgrounds

Exact solutions with nonzero tachyon

Type 0 tachyon condensation

Conclusions

Type 0 tachyon condensation

In all examples so far, **the basic kind of string theory is unchanged** between the initial and final configurations.

We now turn to a related model of lightlike tachyon condensation in **type 0 string theory**, where the tachyon depends only on X^+ , and is independent of the $D - 2$ dimensions transverse to X^\pm .

We start with the Lagrangian for a timelike linear dilaton theory on a flat worldsheet, describing D free, massless fields and their superpartners:

$$\mathcal{L}_{\text{kin}} = \frac{1}{2\pi} G_{MN} \left[\frac{2}{\alpha'} (\partial_+ X^M)(\partial_- X^N) - i\psi^M(\partial_- \psi^N) - i\tilde{\psi}^M(\partial_+ \tilde{\psi}^N) \right]$$

Type 0 tachyon condensation

The dilaton gradient V_M must satisfy $4\alpha' V^2 = -(D - 10)$, so we take

$$\begin{aligned}V_+ &= V_- = -\frac{q}{\sqrt{2}} \\V_i &= 0, \quad i = 2, \dots, D - 1 \\q &\equiv \sqrt{\frac{D - 10}{4\alpha'}}\end{aligned}$$

assuming the dilaton rolls to weak coupling in the future.

We would like to consider solutions for which [the type 0 tachyon condenses](#), growing exponentially in the lightlike direction X^+ .

We again take

$$\mathcal{T} \equiv \tilde{\mu} \exp(\beta X^+)$$

Type 0 tachyon condensation

The linearized equation of motion

$$\partial^2 \mathcal{T} - 2V \cdot \partial \mathcal{T} + \frac{2}{\alpha'} \mathcal{T} = 0$$

fixes

$$\beta \mathbf{q} = \frac{\sqrt{2}}{\alpha'}$$

The tachyon couples to the worldsheet as a (1, 1) superpotential:

$$\mathcal{L}_{\text{int}} = \frac{i}{2\pi} \int d\theta_+ d\theta_- \mathcal{T}(X)$$

This gives rise to a potential and Yukawa term:

$$\mathcal{L}_{\text{int}} = -\frac{\alpha'}{8\pi} G^{MN} \partial_M \mathcal{T} \partial_N \mathcal{T} + \frac{i\alpha'}{4\pi} \partial_M \partial_N \mathcal{T} \tilde{\psi}^M \psi^N$$

Type 0 tachyon condensation

We also get a **modified supersymmetry transformation** for the fermions:

$$\begin{aligned}\{Q_-, \psi^M\} &= -\{Q_+, \tilde{\psi}^M\} = F^M \\ F^M &\equiv -\sqrt{\frac{\alpha'}{8}} G^{MN} \partial_N T\end{aligned}$$

Since the gradient of the tachyon is null, the worldsheet potential

$$\frac{\alpha'}{16\pi} G^{MN} \partial_M T \partial_N T$$

is zero.

But there is a **nonvanishing F -term** and **Yukawa coupling** between the lightlike fermions:

$$\begin{aligned}F^- &= +\frac{q\sqrt{\alpha'}\mu}{2} \exp(\beta X^+) \\ \mathcal{L}_{\text{Yukawa}} &= \frac{i\mu}{4\pi} \exp(\beta X^+) \tilde{\psi}^+ \psi^+\end{aligned}$$

where $\mu \equiv \beta^2 \alpha' \tilde{\mu}$.

Type 0 tachyon condensation

We want to determine the $X^+ \rightarrow \infty$ limit of this theory.

There is no worldsheet potential, so **no string states are expelled from the interior of the bubble.**

The Lagrangian for the light-cone multiplets X^μ , ψ^μ , $\tilde{\psi}^\mu$ is:

$$\begin{aligned}\mathcal{L}_{\text{LC}} = & \frac{i}{\pi} \tilde{\psi}^+ \partial_+ \tilde{\psi}^- + \frac{i}{\pi} \psi^+ \partial_- \psi^- + \frac{iM}{2\pi} \tilde{\psi}^+ \psi^+ \\ & - \frac{1}{\pi\alpha'} (\partial_+ X^+) (\partial_- X^-) - \frac{1}{\pi\alpha'} (\partial_+ X^-) (\partial_- X^+)\end{aligned}$$

where $M \equiv \mu \exp(\beta X^+)$.

Type 0 tachyon condensation

The stress tensor of the light-cone sector of the theory is

$$T^{\text{LC}} = T^{X^\mu} + T^{\psi^\mu}$$

$$T^{X^\mu} \equiv -\frac{1}{\alpha'} G_{\mu\nu} : \partial_+ X^\mu \partial_+ X^\nu : + V_\mu \partial_+^2 X^\mu$$

$$T^{\psi^\mu} = +\frac{i}{2} G_{\mu\nu} : \psi^\mu \partial_+ \psi^\nu :$$

with supercurrent

$$\begin{aligned} G^{\text{LC}}(\sigma^+) &\equiv \sqrt{\frac{2}{\alpha'}} \psi_\mu (\partial_+ X^\mu) - \sqrt{2\alpha'} V_\mu \partial_+ \psi^\mu \\ &= -\sqrt{\frac{2}{\alpha'}} \psi^+ \partial_+ X^- - \sqrt{\frac{2}{\alpha'}} \psi^- \partial_+ X^+ + \sqrt{\alpha'} q \partial_+ \psi^+ + \sqrt{\alpha'} q \partial_+ \psi^- \end{aligned}$$

Analogous equations apply for the left-moving stress tensor and supercurrent, replacing ψ with $\tilde{\psi}$ and ∂_+ with ∂_- .

Type 0 tachyon condensation

As $M \rightarrow \infty$, the massive interaction becomes large and the theory is **strongly coupled** in the variables X^μ , ψ^μ , $\tilde{\psi}^\mu$.

We would like to define an effective field theory useful for analyzing the large- M regime, described by **free effective fields** whose interactions are proportional to **negative** rather than **positive** powers of M .

Type 0 tachyon condensation

We will **not integrate out degrees of freedom**. Instead, we will perform a canonical change of variables such that the **new set of variables has interaction terms *inversely* proportional to M** .

Nothing is integrated out and no information is lost as $M \rightarrow \infty$, but **the theory becomes free** in this limit, when expressed in terms of the new variables.

Type 0 tachyon condensation

First, consider an approximation in which the perturbation M is treated as a fixed constant M_0 .

As $M_0 \rightarrow \infty$, the conformal invariance of the original ψ^\pm , $\tilde{\psi}^\pm$ theory is broken.

We would like to find a new set of variables in which the theory is approximately conformal, with corrections that vanish in the $M_0 \rightarrow \infty$ limit:

$$\begin{aligned}\psi^+ &= 2c'_5 - M_0^{-1}\tilde{b}_5 & \psi^- &= M_0\tilde{c}_5 \\ \tilde{\psi}^+ &= -2\tilde{c}'_5 + M_0^{-1}b_5 & \tilde{\psi}^- &= -M_0c_5\end{aligned}$$

Type 0 tachyon condensation

This change of variables is **canonical**, but **not manifestly Lorentz invariant**.

The Lagrangian becomes

$$\begin{aligned}\mathcal{L}_{\text{fermi}} = & -\frac{i}{\pi}\tilde{b}_5\partial_+\tilde{c}_5 - \frac{i}{\pi}b_5\partial_-\tilde{c}_5 - \frac{i}{2\pi M_0}b_5\tilde{b}_5 \\ & -\frac{1}{\pi\alpha'}(\partial_+X^+)(\partial_-\tilde{X}^-) - \frac{1}{\pi\alpha'}(\partial_+\tilde{X}^-)(\partial_-\tilde{X}^+)\end{aligned}$$

Enforcing the equations of motion, the change of variables is

$$\begin{aligned}\psi^+ &= 2\partial_+c_5, & \psi^- &= M_0\tilde{c}_5, \\ \tilde{\psi}^+ &= 2\partial_-\tilde{c}_5, & \tilde{\psi}^- &= -M_0c_5\end{aligned}$$

The transformation is therefore **Lorentz invariant** if we assign to b_5 a Lorentz weight of $3/2$, and to c_5 a weight of $-1/2$.

Type 0 tachyon condensation

So the $M_0 \rightarrow \infty$ limit of the original theory has a **renormalization group flow** to a **ghost system** with spins $(3/2, -1/2)$.

The RG flow induced by the massive perturbation $M_0\psi^+\tilde{\psi}^+$ **decreases the central charge by 12 units**.

The central charge of the original ψ^\pm system is 1, but the central charge of a *bc* ghost system with weights $(3/2, -1/2)$ is -11 .

Promoting M to a dynamical object

We now want to find a **canonical change of variables** that generalizes what we have done to the case for which M is defined as $\mu \exp(\beta X^+)$, where X^+ is a **dynamical field**.

We define a *new* set of variables $b_4, c_4, \tilde{b}_4, \tilde{c}_4$:

$$\psi^+ = 2c_4' - M^{-1}\tilde{b}_4 + 2\beta(\partial_+ X^+)c_4$$

$$\psi^- = M\tilde{c}_4$$

$$\tilde{\psi}^+ = -2\tilde{c}_4' + M^{-1}b_4 + 2\beta(\partial_- X^+)\tilde{c}_4$$

$$\tilde{\psi}^- = -Mc_4$$

Perform a corresponding redefinition of the bosons X^\pm :

$$X^+ \equiv Y^+$$

$$X^- \equiv Y^- + i\beta\alpha' \mu \exp(\beta X^+) c_4 \tilde{c}_4$$

Promoting M to a dynamical object

This yields the following Lagrangian

$$\begin{aligned}\mathcal{L} = & -\frac{i}{\pi}\tilde{b}_4\partial_+\tilde{c}_4 - \frac{i}{\pi}b_4\partial_-c_4 - \frac{i}{2\pi M}b_4\tilde{b}_4 \\ & -\frac{1}{\pi\alpha'}(\partial_+Y^+)(\partial_-Y^-) - \frac{1}{\pi\alpha'}(\partial_+Y^-)(\partial_-Y^+)\end{aligned}$$

The stress tensor becomes:

$$T^{Y^\mu} + T^{\psi^\mu} = -\frac{1}{\alpha'}G_{\mu\nu}\partial_+Y^\mu\partial_+Y^\nu + V_\mu\partial^2Y^\mu - \frac{3i}{2}\partial_+c_4b_4 - \frac{i}{2}c_4\partial_+b_4$$

As M grows, the stress tensor becomes **free in canonical variables**, with all interaction terms going to zero as M^{-1} .

Promoting M to a dynamical object

We refer to the variables Y^μ , b_4 , c_4 , \tilde{b}_4 , \tilde{c}_4 as the *IR variables*, and the X^μ , ψ^μ , $\tilde{\psi}^\mu$ as the *UV variables*.

The IR fields are legitimate, weakly interacting variables, suitable for describing the $X^+ \rightarrow +\infty$ limit of the theory.

There is an exact *duality* between the UV description and the IR description.

In the case at hand, loop corrections are trivial on both sides, and *the duality inverts the expansion parameter for conformal perturbation theory* rather than for the loop expansion.

Promoting M to a dynamical object

However, the **central charge of the fermion theory** has dropped from its original value of 1 in the ψ^\pm description, to a central charge of -11 for a bc ghost system with weights $(3/2, -1/2)$.

In fact, this is a **quantum effect**.

This is a subtle point, since the theory has **no nontrivial dynamical Feynman diagrams that might generate quantum corrections**.

Question: How does this work?

Renormalization of the dilaton gradient

It turns out that the natural **normal-ordering prescription** for the UV variables agrees only up to finite terms with the natural orderings for composite operators in the IR variables.

The effect of these finite differences will be to **renormalize the dilaton gradient** of the system by an amount $\Delta V_+ = \beta$, $\Delta V_- = 0$.

Normal ordering in the UV variables

Using the properties of Feynman diagrams and the equations of motion, we can derive modified OPEs for the UV fields.

The natural basis for operators in the **UV** description is a basis of normal-ordered products

$$: X^{\mu_1}(\rho_1) \cdots X^{\mu_m}(\rho_m) \psi^{\nu_1}(\sigma_1) \cdots \psi^{\nu_n}(\sigma_n) \tilde{\psi}^{\pi_1}(\tau_1) \cdots \tilde{\psi}^{\pi_p}(\tau_p) :$$

- ▶ The normal-ordered operator is nonsingular when any of the arguments in the normal ordering symbol approach one another;
- ▶ The normal-ordered operators obey the equations of motion. For instance:

$$\partial_{\tau^+} \partial_{\tau^-} : X^-(\sigma) X^-(\tau) : = -\frac{i\beta\alpha'\mu}{4} : X^-(\sigma) \exp(\beta X^+(\tau)) \tilde{\psi}^+(\tau) \psi^+(\tau) :$$

- ▶ The normal ordered product of two “+” operators is equal to the ordinary product;

Normal ordering in the UV variables

- ▶ The normal ordered product of a “+” field and a “-” field is defined with the subtraction prescription of the free theory;
- ▶ The normal ordered product of two “-” fields has only “+” fields on the right-hand side, and scales as a single power of M ;
- ▶ In the limit $M \rightarrow 0$, the structure of the algebra of the operators becomes that of the free theory (this property is implied by the three previous properties).

Given these properties, we can derive the full structure of the OPE for UV fields.

Normal ordering in the IR variables

The normal-ordering prescription defined for UV fields is **not useful for the IR description**.

The UV normal ordering : : subtracts terms from the time-ordered product that are proportional to M , which is very large in the IR.

Define a second normal ordering prescription, appropriate to the IR limit of the theory. In this case we take our basis of operators to be

$$\circ Y^{\mu_1}(\rho_1) \cdots Y^{\mu_m}(\rho_m) b_4(\sigma_1) \cdots b_4(\sigma_n) \tilde{b}_4(\tau_1) \cdots \tilde{b}_4(\tau_p) c_4(\zeta_1) \cdots c_4(\zeta_q) \tilde{c}_4(\omega_1) \cdots \tilde{c}_4(\omega_r) \circ,$$

- ▶ The normal-ordered operator is nonsingular when any of the arguments of operators in the normal ordering symbol approach one another;
- ▶ The normal-ordered operators obey the equations of motion. For instance:

$$\partial_{\tau^+} \partial_{\tau^-} \circ Y^-(\sigma) Y^-(\tau) \circ = -\frac{i\beta\alpha'}{4\mu} \circ Y^-(\sigma) \exp(-\beta Y^+(\tau)) b_4(\tau) \tilde{b}_4(\tau) \circ ;$$

Normal ordering in the IR variables

- ▶ The normal ordered product of two operators from the set b_4, \tilde{b}_4, Y^+ is equal to the ordinary product;
- ▶ The normal ordered product of a field from the set c_4, \tilde{c}_4, Y^- with a field from the set b_4, \tilde{b}_4, Y^+ is defined with the subtraction prescription of the free theory;
- ▶ The normal ordered product of two fields from the set c_4, \tilde{c}_4, Y^- has only fields from the set b_4, \tilde{b}_4, Y^+ on the right-hand side, and scales as a single power of M^{-1} ;
- ▶ In the limit $M \rightarrow \infty$, the structure of the algebra of the operators becomes that of the free theory of the IR fields.

Normal ordering in the IR variables

The bosonic stress tensor turns out to transform unproblematically, but the fermionic stress tensor picks up a **quantum correction** due to the mismatch between : : and $\begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix}$ normal ordering prescriptions.

The corrections amounts to a **renormalization of the dilaton gradient**:

$$\begin{aligned}\hat{V}_\mu &\equiv V_\mu + \Delta V_\mu \\ \Delta V_+ &= +\beta \quad \Delta V_- = 0\end{aligned}$$

We are left with a contribution to the central charge equal to

$$\begin{aligned}c^{\text{dilaton}} &= 6\alpha' \eta^{\mu\nu} \hat{V}_\mu \hat{V}_\nu = -6\alpha' q^2 + 6\sqrt{2}\alpha' \beta q \\ &= 27 - \frac{3D}{2}\end{aligned}$$

Quantum corrections

We have the remaining central charge contributions

- ▶ +2 from the Y^μ
- ▶ -11 from the $b_4 c_4$ system
- ▶ $\frac{3}{2}(D - 2)$ from the transverse degrees of freedom X^i, ψ^i
- ▶ total free-field contribution of $\frac{3D}{2} - 12$

The total central charge in the theory is therefore equal to 15.

As one moves in the target space from the original theory to $X^+ = +\infty$, **twelve units of central charge** are transferred from the **light cone fermions** ψ^\pm to the **dilaton gradient**.

The central charge being transferred to the dilaton gradient does *not* occur through a loop diagram of massive fields being integrated out.

Instead, the central charge is transferred through a **mismatch of normal-ordering prescriptions** appropriate to the free field theories in the two limits $X^+ \rightarrow \pm\infty$.

Quantum corrections

Break up the supercurrent: $G^{\text{LC}} = \mathbf{1} + \mathbf{2} + \mathbf{3} + \mathbf{4}$, with

$$\begin{aligned}\mathbf{1} &\equiv -\sqrt{\frac{2}{\alpha'}}\psi^+(\partial_+X^-) & \mathbf{2} &\equiv -\sqrt{\frac{2}{\alpha'}}\psi^-(\partial_+X^+) \\ \mathbf{3} &\equiv \sqrt{\alpha'}q\partial_+\psi^+ & \mathbf{4} &\equiv \sqrt{\alpha'}q\partial_+\psi^-\end{aligned}$$

The full transformation of the supercurrent from **UV** to **IR** variables is

$$\begin{aligned}\mathbf{1}_{\text{classical}} &= -2\sqrt{\frac{2}{\alpha'}} \circ \left[(\partial_+c_4)(\partial_+Y^-) + \beta c_4(\partial_+Y^+)(\partial_+Y^-) - \frac{i\beta\alpha'}{2}(\partial_+c_4)b_4c_4 \right] \circ \\ \mathbf{1}_{\text{quantum}} &= -\beta\sqrt{\frac{\alpha'}{2}}\partial_+^2c_4 - 2\beta^2\sqrt{\frac{\alpha'}{2}}c_4\partial_+^2Y^+ \\ \mathbf{2} + \mathbf{4} &= \frac{q}{2}\sqrt{\alpha'}b_4 \\ \mathbf{3} &= 2q\sqrt{\alpha'}(\partial_+^2c_4) + 2\sqrt{\frac{2}{\alpha'}}(\partial_+Y^+)(\partial_+c_4) + 2\sqrt{\frac{2}{\alpha'}}c_4(\partial_+^2Y^+)\end{aligned}$$

Quantum corrections

Expressed in b_4 , c_4 , Y variables, the supercurrent is **manifestly finite** in the limit $X^+ \rightarrow +\infty$ (as is the stress tensor).

The b_4 , c_4 , Y fields can indeed be regarded as dual variables that render the theory free in the $M \rightarrow \infty$ limit.

The IR limit

We now focus strictly on the limiting regime of the IR theory.

In practice, this means that, when written in IR variables, we discard the $\exp(-\beta Y^+) \tilde{b}_4 b_4$ term in the action, as well as any $\exp(-\beta Y^+)$ terms in the supercurrent and stress tensor.

Rescale the b_4 field so that the new b fermion appears in the supercurrent with unit normalization. To preserve all canonical commutators, however, we will rescale the c_4 field oppositely:

$$b_4 = \frac{2}{q\sqrt{\alpha'}} b_3 = \beta\sqrt{2\alpha'} b_3$$
$$c_4 = \frac{q\sqrt{\alpha'}}{2} c_3 = \frac{1}{\beta\sqrt{2\alpha'}} c_3$$

The IR limit

The invariance properties of the system under spatial reflection are still unclear.

The stress tensor is invariant under the discrete symmetry reflecting the spacelike vector orthogonal to \hat{V}_μ .

The supercurrent is not, however, since V_μ and ΔV_μ appear independently in G^{LC} .

We would like to find field variables that **render this discrete symmetry more manifest**, such that only the vector \hat{V}_μ enters G^{LC} .

We therefore define **new variables** b_2 , c_2 , Z^μ by:

$$Y^\pm = Z^\pm \pm \frac{i}{2\beta} c_2 \partial_+ c_2$$

$$b_3 = b_2 - \frac{2}{\beta\alpha'} (\partial_+ c_2) \left(\partial_+ Z^+ - \partial_+ Z^- \right) - \frac{1}{\beta\alpha'} c_2 \left(\partial_+^2 Z^+ - \partial_+^2 Z^- \right) \\ + \frac{i}{2\beta^2\alpha'} c_2 (\partial_+ c_2) (\partial_+^2 c_2)$$

$$c_3 = c_2$$

The IR limit

The worldsheet supersymmetry is now realized **nonlinearly**.

The bosons Z^μ transform into their own derivatives, times a goldstone fermion:

$$[Q, Z^\mu] = ic_2 \partial_+ Z^\mu \quad \{Q, c_2\} = 1 + ic_2 \partial_+ c_2$$

where

$$Q \equiv \frac{1}{2\pi} \int d\sigma_1 G(\sigma)$$

In the sector involving the transverse fields X_i , ψ^i , supersymmetry is realized in the usual **linear** fashion:

$$\begin{aligned} [Q, X_i] &= i\sqrt{\frac{\alpha'}{2}} \psi^i \\ \{Q, \psi^i\} &= \sqrt{\frac{2}{\alpha'}} \partial_+ X_i \end{aligned}$$

The IR limit

At first sight, our realization of supersymmetry in the full theory is unfamiliar, with **worldsheet supersymmetry realized linearly in one sector and nonlinearly in another**.

However, it turns out that this realization is equivalent to one for which worldsheet supersymmetry is realized **completely nonlinearly** in *all* sectors.

We now perform a **final transformation** on the system. Defining the Hermitian infinitesimal generator

$$g \equiv -\frac{i}{2\pi} \int d\sigma_1 c_2(\sigma) G^\perp(\sigma)$$

we transform all operators in the theory according to

$$\mathcal{O} \rightarrow U \mathcal{O} U^{-1}$$

with

$$U \equiv \exp(ig)$$

The IR limit

The total final supercurrent $G \equiv G^{\text{LC}} + G^\perp$ is then

$$G = b_1 + ic'_1 b_1 c_1 - c_1 T^{\text{mat}} + c_1'' \left(-\frac{1}{6} c^\perp - \frac{1}{2} + \alpha' q^2 \right) \\ + c_1 c_1' c_1'' \left(-\frac{i}{4} \alpha' q^2 - \frac{i}{2} + \frac{i}{24} c^\perp \right)$$

And the total transformed stress tensor is

$$T = T^{\text{mat}} + T^{b_1 c_1}$$

with

$$T^{b_1 c_1} = -\frac{3i}{2} \partial_+ c_1 b_1 - \frac{i}{2} c_1 \partial_+ b_1 + \frac{i}{2} \partial_+ (c_1 \partial_+^2 c_1)$$

Plugging in $q = \sqrt{\frac{D-10}{4\alpha'}}$ and $c^\perp = \frac{3}{2}(D-2)$:

$$G = b_1 + ic'_1 b_1 c_1 - c_1 T^{\text{mat}} - \frac{5}{2} c_1''$$

The IR limit

The $X^+ \rightarrow \infty$ limit of our solution is described by a **free worldsheet theory** with a bc ghost system of weights $(3/2, -1/2)$, D free scalars Z^M and $D - 2$ free fermions ψ^{Z^i} .

The total central charge of the Z^M, ψ^{Z^i} system is 26, and the contribution of -11 from the b_1c_1 system brings the total central charge to 15.

The theory has **critical central charge** for a SCFT interpreted as the worldsheet theory of a RNS superstring in conformal gauge.

Berkovits-Vafa construction

This type of superconformal field theory belongs to a class of constructions introduced by Berkovits and Vafa, in which the bosonic string is embedded in the solution space of the superstring.

[hep-th/9310170]

For a conformal field theory T^{mat} with a central charge of 26, it is possible to construct a corresponding superconformal field theory defined by G , T with central charge 15.

Upon treating the superconformal theory as a superstring theory, the resulting physical states and scattering amplitudes are identical to those of the theory defined by T^{mat} when treated as a bosonic string theory.

Berkovits-Vafa construction

The construction can be summarized as follows:

- ▶ Given a conformal stress tensor T^{mat} with central charge 26, a ghost system $b_1 c_1$ can be introduced with weights $(3/2, -1/2)$ and stress tensor $T^{b_1 c_1}$.
- ▶ This gives rise to a fermionic primary current of weight 3/2:

$$G \equiv b_1 + ic'_1 b_1 c_1 - c_1 T^{\text{mat}} - \frac{5}{2} c_1''$$

- ▶ This closes on the stress tensor of the theory:

$$G(\sigma)G(\tau) \simeq \frac{10i}{(\tau^+ - \sigma^+)^3} + \frac{2i}{(\tau^+ - \sigma^+)} T^{\text{total}}(\tau)$$

where

$$T^{\text{total}} \equiv T^{\text{mat}} + T^{b_1 c_1}$$

Berkovits-Vafa construction

- ▶ This defines a **superconformal theory** of central charge 15.
- ▶ To construct physical states of the corresponding superstring theory, one starts with a Virasoro primary state $|\mathcal{U}\rangle$ of weight 1 in the theory defined by T^{mat} :

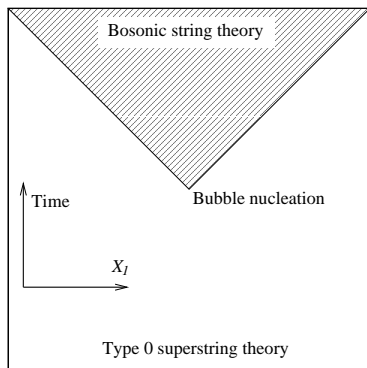
$$\begin{aligned}L_n^{\text{mat}} |\mathcal{U}\rangle &= 0 & n \geq 1 \\L_0^{\text{mat}} |\mathcal{U}\rangle &= 1\end{aligned}$$

We have an exact solution describing a **dynamical transition** between string theories that differ from one another in their **worldsheet gauge algebra**.

Transition to bosonic string theory

This transition follows an instability in an initial D -dimensional type 0 theory.

The dynamics then **spontaneously break worldsheet supersymmetry**, giving rise to a **bosonic string theory** in the same number of dimensions deep inside the tachyonic phase.



Outline

Overview of quintessent cosmology and linear dilaton backgrounds

Supercritical string theory: spacetime effective action

Stability in time-dependent backgrounds

Exact solutions with nonzero tachyon

Type 0 tachyon condensation

Conclusions

Conclusions

- ▶ Supercritical string theory has some surprising and interesting properties.
- ▶ We see that the [supercritical string](#) can be connected to the duality web of [critical string theory](#).
- ▶ We have found solutions that interpolate between [superstring theory](#) and purely [bosonic string theory](#).
- ▶ The surprising feature of these connections is the crucial role of [time dependence](#).
- ▶ There may be other interesting links between theories that we have yet to discover.

Thank you.