Black Hole Formation from High Energy Scattering in AdS/CFT

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Outline

I. How to create a Black Hole in AdS.
   - o The flat space picture.
   - o Graviton scattering in AdS and the 1/2 BPS geometries (LLM).

II. The gauge theory (initial) states.

III. The strong coupling description (a proposal)
   - o Matrix quantum mechanics

Main reference: arXiv:0709.3503

High Energy Scattering in Flat Space

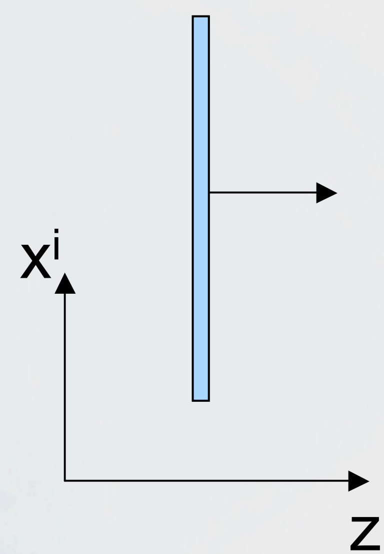
1. Consider two massless particles on a head-on collision.
   - To form a classical black hole we need $E_{cm} >> E_{planck}$.

2. To study such collision (classicaly) one can use the Aichelburg-Sexl metric:

$$ds^2 = -du dv + dx^i dx^i + \Phi(x^i)\delta(u) du^2 ,$$

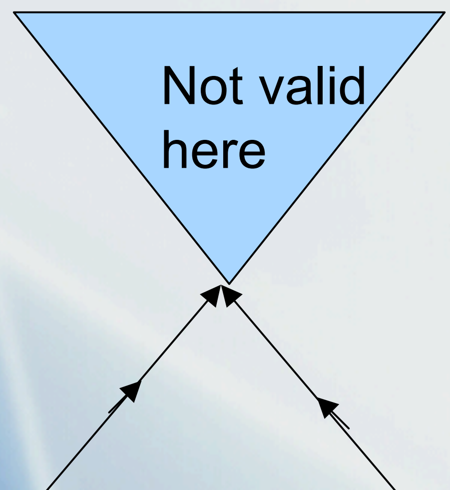
$$\Phi(x^i) = \frac{16\pi G\mu}{\mathrm{Vol}(S^{D-3})(D-4)|\vec{x}|^{D-4}} .$$

$$u = t - z , \quad v = t + z .$$



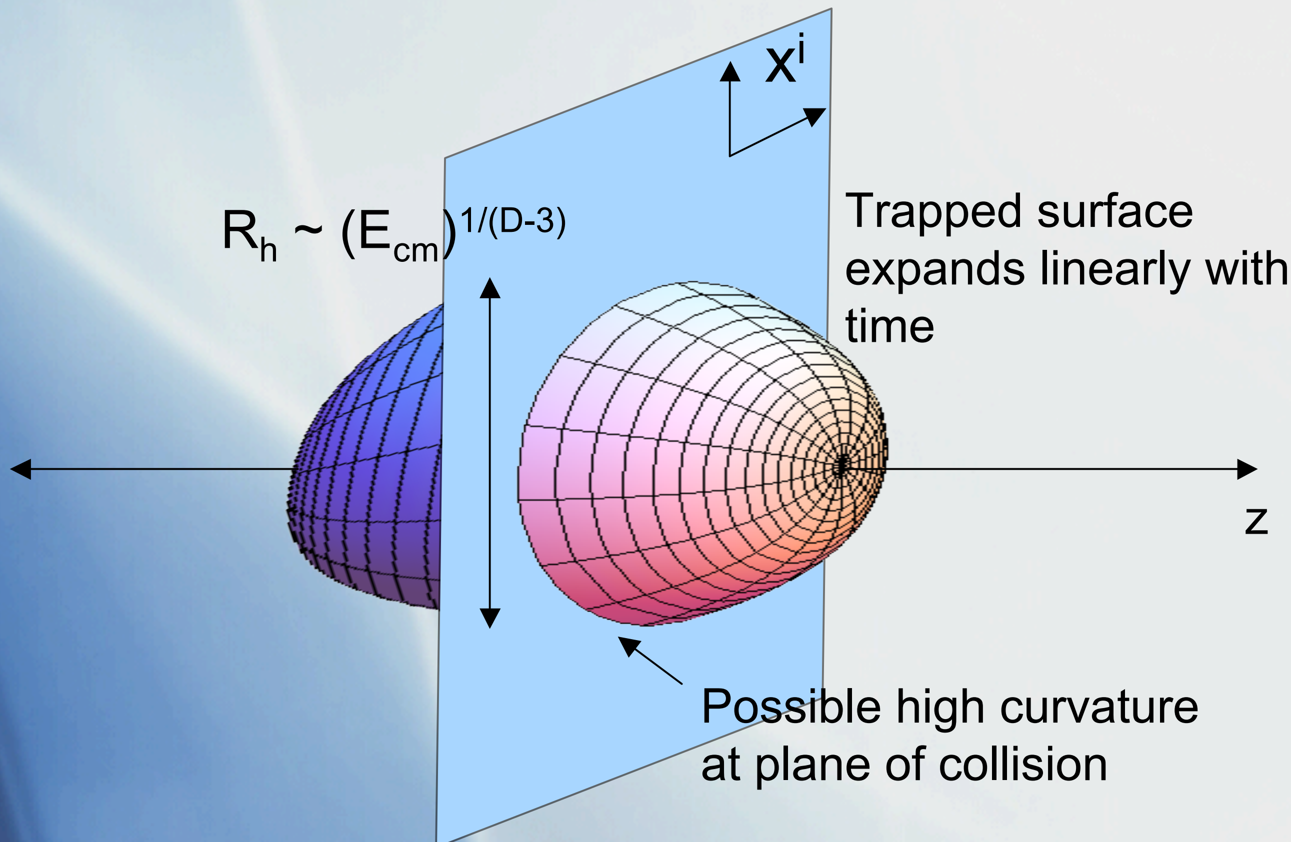
High Energy Scattering in Flat Space

1. This metric can be obtained by boosting the Schwarzchild metric and taking the mass to zero.
2. One can superpose two shock waves and form a solution to Einstein's equations outside the future light cone of the collision.



High Energy Scattering in Flat Space

1. This superposition leads to the formation of a Marginally Trapped Surface (Giddings, Eardley hep-qc/0201034)



High Energy Scattering in Flat Space

1. Danger with high curvatures: For a single shock,

$$R_{uivj} = -\frac{1}{2}\delta(u)\partial_i\partial_j\Phi(\vec{x})$$

but $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is still finite.

2. For two shocks we have divergence of the form

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \sim \frac{E^2}{|\vec{x}|^{2D-4}}\delta(u)\delta(v)$$

3. Need to regularize delta-function (by creating wave-packet)

$$\delta(u) \to \delta_\lambda(u)$$

High Energy Scattering in Flat Space

1. For low curvatures we need

$$(R_{\mu\nu\rho\sigma})^2 \ll 1/l_s^4 \qquad |\vec{r}_h| \gg \lambda \qquad |\vec{r}_h| \gg l_s$$

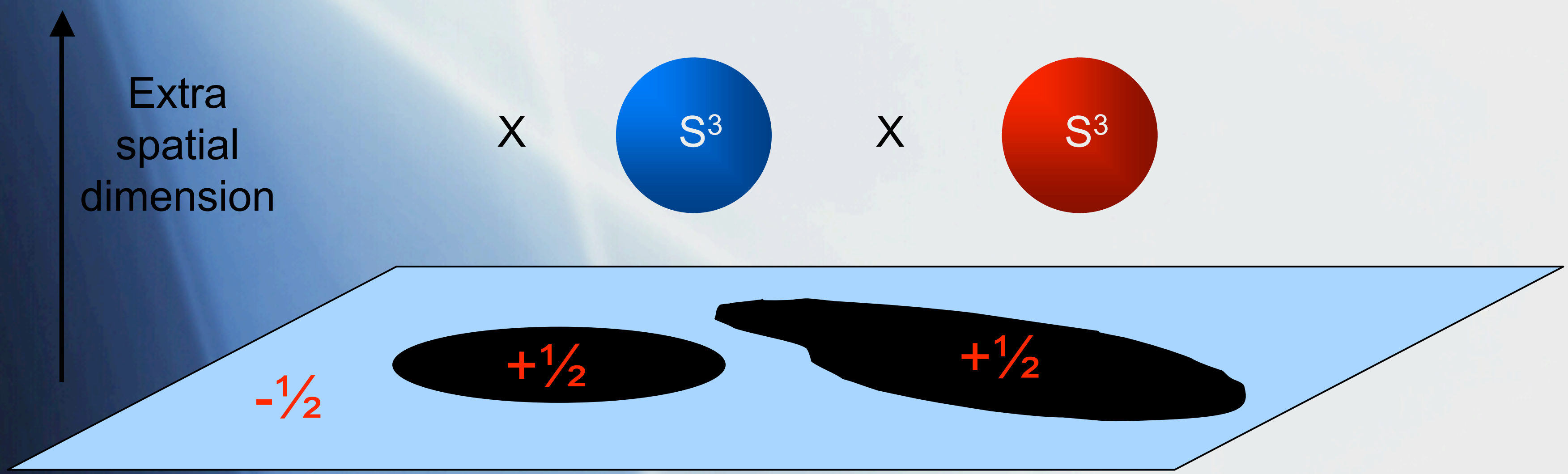
2. This gives

- $g^{-1/2}E^{-1/7} \ll \lambda \ll E^{1/7}$,
- $E^{1/7} \gg g^{-1/4}$.

Conventions. In this talk we will set the ten dimensional Planck's constant $l_p = 1$. In these units, the AdS radius is related to the rank of the SYM gauge group N by $R = (4\pi N)^{1/4}$. The string length is given by $l_s = \sqrt{\alpha'} = g^{-1/4}$, where g is the closed string coupling also related to the SYM coupling by $4\pi g = g_{YM}^2$. We will also introduce the parameter $\hbar = 1/N$.

Scattering Gravitons in AdS

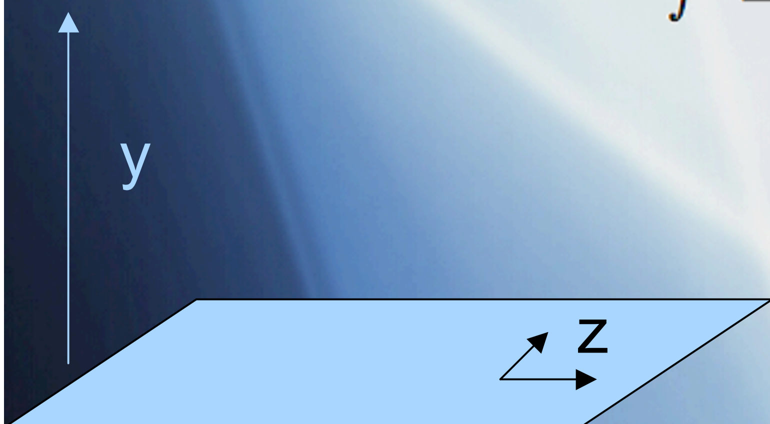
1. Here we want to explain how to get the smooth graviton wave-packets from the 1/2 BPS solutions of type IIB SUGRA.
   - N units of RR five-form flux
   - All solutions asymptotically $AdS_5 \times S^5$
   - Classified by a single function that takes values ± 1/2 on a two-dimensional plane. (Lin, Lunin, Maldacena, 2004)



Scattering Gravitons in AdS

1. Metric:

$$\begin{aligned} ds^2 &= -h^{-2}(Dt)^2 + h^2(dy^2 + dzd\bar{z}) + ye^{-G}d\Omega_3^2 \\ &\quad + ye^{G}d\widetilde{\Omega}_3^2, \\ h^{-2} &= 2y\cosh G, \\ f &= \frac{1}{2}\tanh G, \\ f(z,\bar{z},y) &= -\frac{y^2}{2}\int d^2z' \frac{\rho(z',\bar{z}')}{(|z-z'|^2+y^2)^2}. \end{aligned}$$



$$Dt = dt + V = dt + \frac{1}{2}i\bar{V}dz - \frac{1}{2}iVd\bar{z}$$

$$V(z,\bar{z},y) = \frac{1}{2}\int d^2z' \frac{\rho(z',\bar{z}')(z-z')}{(|z-z'|^2+y^2)^2}$$

Scattering Gravitons in AdS

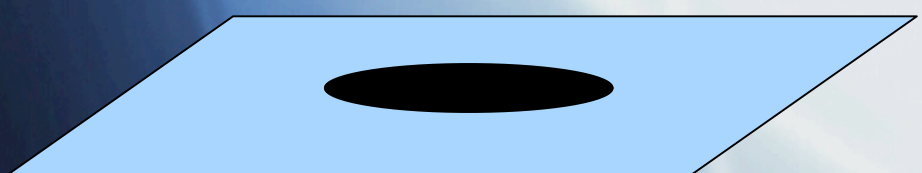
1. The area of the droplet is constrained by

$$\int_{\mathcal{D}} \frac{d^2 z}{\pi} = 1$$

2. The energy of the solution is

$$E = J = \frac{1}{\hbar^2 \pi} \int_{\mathcal{D}} d^2 z |z|^2 - \frac{1}{2\hbar^2}$$

3. Example: AdS_5 x S^5 = unit disk

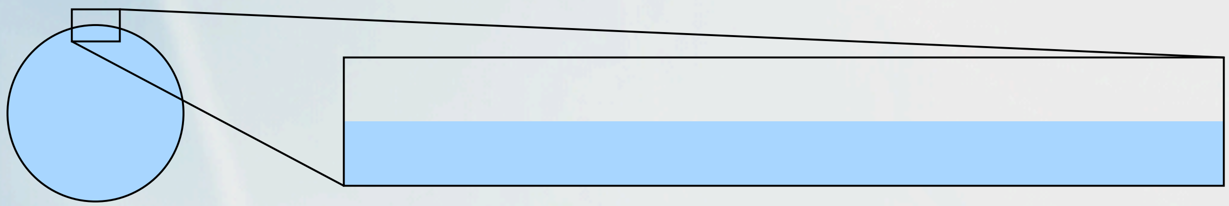


$$\begin{aligned} y &= \sinh\rho \sin\theta \ , \\ r &= \cosh\rho \cos\theta \ , \\ \tilde{\phi} &= \phi - t \ , \end{aligned}$$

$$ds^2 = R^2 \left[-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\tilde{\Omega}_3^2 + d\theta^2 + \cos^2 \theta d\tilde{\phi}^2 + \sin^2 \theta d\Omega_3^2 \right]$$

Scattering Gravitons in AdS

1. To get flat space we zoom in into the edge of the droplet (the equator of the S^5 and the origin of AdS_5).



2. Do this by rescaling coordinates and taking R-> ∞,

$$z \to \left(1 + \frac{x_2}{R^2}\right) \exp\left(\frac{\pi}{2} - \frac{x_1}{R}\right)\ ,\ \ y \to \frac{1}{R^2} y\ ,\ \ t \to \frac{1}{R} t$$

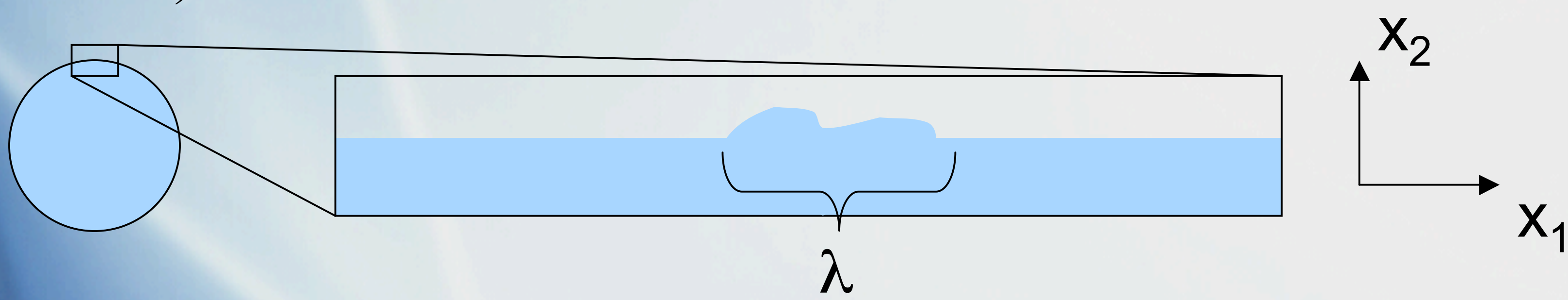
We get, ($y = r_1 r_2,\ x_2 = (r_1^2 - r_2^2)/2$)

$$\boxed{ds^2 = -dx_1(2dt - dx_1) + d\vec{r}^2}$$

Note that to get the flat space limit, we keep l_s fixed, so this is taking large N limit with g fixed but small. (not ‘t Hooft limit)

Scattering Gravitons in AdS

1. To create a graviton, we make a small ripple on the edge of the droplet: (note the two geodesics x_1 = const., $x_1 = 2\,t$ + const.)



2. Parametrize deformation of edge as,

$$r_{\text{boundary}} = 1 + \delta r(\phi)$$

3. The energy is,

$$E \approx \frac{1}{\pi \hbar^2} \int d\phi \, [\delta r(\phi)]^2$$

Scattering Gravitons in AdS

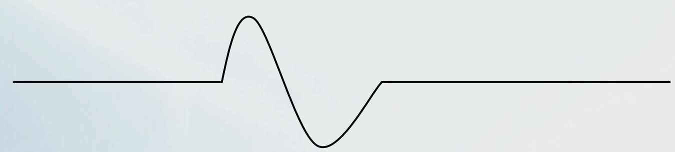
1. To keep the energy fixed as we take R to infinity, the perturbation must take form

$$\delta r(\phi) \; = \; \frac{4\pi^{3/2}\sqrt{E}}{R^3\sqrt{\lambda}} g(x_1/\lambda) \; .$$

2. Without lost of generality, g(x) is zero outside the interval [-1/2,1/2], and it's normalized as

$$\int_{-1/2}^{1/2} dx[g(x)]^2 = 1 \qquad \int_{-1/2}^{1/2} dx g(x) = -\frac{2\pi^{3/2}\sqrt{E}}{R^3\sqrt{\lambda}} \to 0$$

(Area conservation)



Scattering Gravitons in AdS

1. One can show that, after taking R to infinity, we get a regularized Aichelburg-Sexl metric in 10 dim.

$$\boxed{ds^2 = -dx_1(2dt - dx_1) + d\vec{r}^2 + \frac{(4\pi)^3 E}{|\vec{r}|^6}\delta_\lambda(x_1)dx_1^2}$$

where

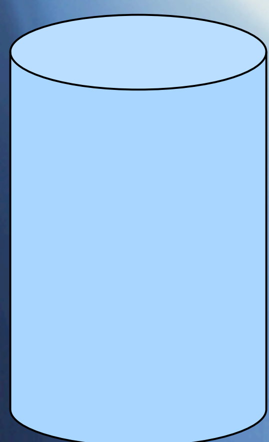
$$\delta_\lambda(x) = \int_{-1/2}^{1/2} dx'[g(x')]^2 \delta(x - \lambda x')$$

2. We can put now an anti-1/2 BPS particle traveling in the opposite direction by replacing $x_1 \rightarrow 2t - x_1$.

Gauge Theory Interpretation

1. Now we want to find the gauge theory dual of the regularized Aichelburg-Sexl metric and learn how to set up the initial states for the graviton scattering.

2. **1/2 BPS states of SYM theory: zero modes of one complex scalar**



SYM on R x S^3

- Expand in spherical harmonics

$$Z(t,\Omega) = \sum_A Z_A(t) Y_A(\Omega)$$

- Effective tree level action for zero mode (A = 0)

$$S = \int dt \ \mathrm{Tr}\left(|\dot{Z}|^2 - |Z|^2\right)$$

Gauge Theory Interpretation

1. Define the operators

$$A^\dagger = \frac{1}{\sqrt{2}}(Z - i\bar{\Pi}) \ , \qquad \bar{A}^\dagger = \frac{1}{\sqrt{2}}(\bar{Z} - i\Pi)$$

with

$$[A_i^j, (A^\dagger)_k^l] = \delta_i^l \delta_k^j \ , \quad [\bar{A}_i^j, (\bar{A}^\dagger)_k^l] = \delta_i^l \delta_k^j \ ,$$

Note that $\Pi^\dagger = \bar{\Pi}$.

2. The Hamiltonian and R-charge operator are

$$H = \mathrm{Tr}\left(A^\dagger A + \bar{A}^\dagger \bar{A}\right) \qquad J = \mathrm{Tr}\left(A^\dagger A - \bar{A}^\dagger \bar{A}\right)$$

3. Since [H,J] = 0, we can define

$$H' = H - J$$

Identify with global time in AdS

Gauge Theory Interpretation

1. The (anti) 1/2 BPS operators have the form,

$$|\psi_{1/2\mathrm{BPS}}\rangle = \psi(A^\dagger)|0\rangle \quad |\psi_{-1/2\mathrm{BPS}}\rangle = \psi(\bar{A}^\dagger)|0\rangle$$

where $(H \pm J)|\psi\rangle = 0$

Relation to Graviton Scattering:

1. An initial state for a (head-on) scattering process will take the form:

$$|\psi\rangle \propto e^{\,\mathrm{Tr}\Omega_1(\bar{A}^\dagger)} e^{\,\mathrm{Tr}\Omega_2(A^\dagger)}|0\rangle$$

Dictionary

1. To find the geometric interpretation of the 1/2 BPS states introduce a coherent state:

$$A_i^j|Z\rangle \;=\; Z_i^j|Z\rangle$$

2. The 1/2 BPS state becomes

$$\langle Z|\psi\rangle = e^{\,\mathrm{Tr}\,\Omega(Z)/\hbar}e^{-\,\mathrm{Tr}|Z|^2/(2\hbar)}$$

3. The normalization is given by a complex random-matrix integral

$$\langle\psi|\psi\rangle \;=\; \int [d^2Z]|\langle\psi|Z\rangle|^2$$

Dictionary

1. We can now go to an eigenvalue basis

(I have rescaled $Z \to Z/\sqrt{\hbar}$):

From Jacobian

$$\langle\psi|\psi\rangle \propto \prod_{i=1}^{N} \int d^2 z_i e^{\sum_j W(z_j,\bar{z}_j)/\hbar + \sum_{i<j} \log|z_i - z_j|^2}$$

where

$$W(z,\bar{z}) = -|z|^2 + \Omega(z) + \overline{\Omega(z)}$$

2. Taking the large N limit ($\hbar \to 0$) we can use the saddle point approximation and replace sums by integrals over eigenvalue distributions

Dictionary

1. Saddle point equations:

$$\delta F[\rho] = 0$$

where

$$\begin{aligned} F[\rho] &= -\frac{1}{\hbar}\int d^2z\rho(z)W(z,\bar{z}) \\ &\quad -\frac{1}{2}\int d^2z\int d^2z'\rho(z)\rho(z')\log|z-z'|^2 \end{aligned}$$

and subject to the constraint $\int d^2z\rho = \hbar^{-1}$

2. This gives constant density domains ("droplets") of eigenvalues:

$\rho = 1/(\hbar\pi)$

Dictionary

1. These are the 1/2 BPS geometries!
2. A precise dictionary for a single droplet was developed in (Vazquez hep-th/0612014)
   - o Write potential as,

$$\Omega(z) = \sum_{k>0} t_k z^k$$

then

$$t_k = \frac{1}{k} \oint_{\partial\mathcal{D}} \frac{dz}{2\pi i} \bar{z} z^{-k} \quad \Longleftrightarrow$$

3. The energy of the state coincides with SUGRA result:

$$E = \frac{1}{\hbar} \langle \, \mathrm{Tr}|Z|^2 \rangle - \frac{1}{2\hbar^2} \approx \frac{1}{\hbar^2 \pi} \int_{\mathcal{D}} d^2z |z|^2 - \frac{1}{2\hbar^2}$$

Dictionary

1. For a linearized perturbation around the circular droplet

$$t_k \approx \frac{1}{\pi k} \int_0^{2\pi} d\phi \delta r(\phi) e^{-ik\phi}$$

2. So we can now write a dual to the Aichelburg-Sexl geometry:

$$|\psi\rangle \;=\; \exp\left[-\sqrt{\frac{E\lambda}{\pi}}\int_{-1/2}^{1/2} dx g(x)\ \mathrm{Tr}\log\left(ie^{-ix\lambda/R} - \sqrt{\hbar}A^{\dagger}\right)\right]|0\rangle$$

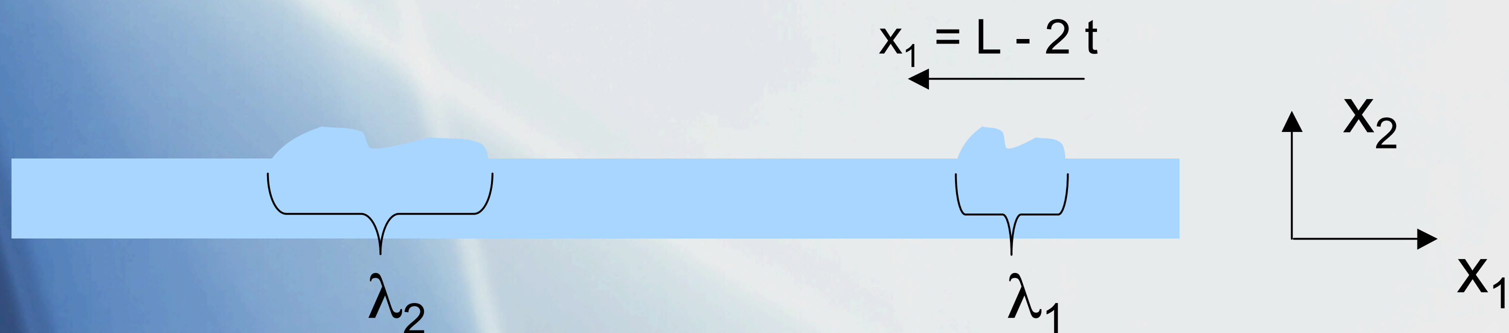
$$\Updownarrow$$

$$ds^2 = -dx_1(2dt - dx_1) + d\vec{r}^2 + \frac{(4\pi)^3 E}{|\vec{r}|^6}\delta_\lambda(x_1)dx_1^2$$

Dictionary

1. The dual of a shock wave at $x_1 = L$ (at $t = 0$) can be obtained by shifting $x \to x + L$ and $A^\dagger \to \bar{A}^\dagger$ in the previous state.
2. So long as the initial waves a sufficiently separated in space-time, one can write the initial state as a product of 1/2 and anti-1/2 BPS:

$$|\psi\rangle \propto e^{\,\mathrm{Tr}\Omega_1(\bar{A}^\dagger)}\, e^{\,\mathrm{Tr}\Omega_2(A^\dagger)}|0\rangle$$



Dictionary

1. To ensure that the resulting collision leads to the formation of a classical black hole, we need, again

$$\boxed{\begin{array}{l} \bullet\ g^{-1/2}E^{-1/7} \ll \lambda \ll E^{1/7}, \\ \bullet\ E^{1/7} \gg g^{-1/4}. \end{array}}$$

2. This is very easy to satisfy…

Note that to get the flat space limit, we keep l_s fixed. This is taking the large N limit with g fixed but small. (not 't Hooft limit)

Strong Coupling Description

1. So far, we have seen that we can set up initial states that, according to the dual semiclassical gravity, will result in the formation of classical black holes in the bulk.
2. Now we want to know how could we describe this process at strong coupling.
3. Proposal: **Matrix Quantum Mechanics**
    1. The low energy dynamics of SYM theory seems to be described by a reduced model of matrix quantum mechanics: Berenstein hep-th/0403110, hep-th/0507203.
    2. The lowest energy states of SYM are the susy or BPS states (vacua).
    3. Using the operator state correspondence, one can classify vacua according to which operator acquires a vev in flat space SYM.
    4. On the BPS states, it turns out that the only operators that acquire a v.e.v. are scalars

$$\mathcal{O} \sim \mathrm{Tr}(XYZ\cdots)$$

Strong Coupling Description

1. Moreover, the F-term conditions are zero in the chiral ring:

$$\langle \partial_{\Phi^i} W(\Phi) \mathcal{O}_1 \mathcal{O}_2 \cdots \rangle = 0$$

where the superpotential is

$$W = \mathrm{Tr}(X[Y,Z])$$

2. This means that all expectation values containing commutators are zero in the chiral ring

$$[X,Y] = 0\,, \quad [X,Z] = 0\,, \quad [Y,Z] = 0$$

   1. In particular, operators that differ by a commutator have same vev:

$$\langle\, \mathrm{Tr}(XYZ\cdots)\rangle = \langle\, \mathrm{Tr}(XZY\cdots)\rangle$$

Strong Coupling Description

1. Since the scalar operators are dual to the zero mode of the same scalar in S^3 (for SYM on R x S^3), this suggests that the dynamics of the BPS sector can be described by a reduced model including only these fields.
2. Effective tree-level Hamiltonian:

$$H_{eff} = \mathrm{Tr}\left(\sum_{a=1}^{6}\frac{1}{2}(D_t X^a)^2 + \frac{1}{2}(X^a)^2 + \sum_{a,b=1}^{6}\frac{g_{YM}^2}{8\pi^2}[X^a, X^b][X^a, X^b]\right)$$

3. We now want to impose the F-term conditions:

$$[X^a, X^b] = 0$$

Strong Coupling Description

1. Another way of seeing the meaning of F-term condition is that we are taking low energy limit:
   - o From AdS/CFT and general gauge theory considerations we know that the "size" of the ground state is order N^2

$$\frac{1}{N}\sum_{a=1}^{6}\langle 0|\ \mathrm{Tr}(X^a X^a)|0\rangle \sim N$$

   - o Rescaling the matrices as, $X^a \rightarrow \sqrt{N} X^a$ we see that

$$H_{eff} = N\ \mathrm{Tr}\left(\sum_{a=1}^{6}\frac{1}{2}(D_t X^a)^2 + \frac{1}{2}(X^a)^2 + \underbrace{\sum_{a,b=1}^{6}\frac{g_{YM}^2 N}{8\pi^2}[X^a, X^b][X^a, X^b]}_{\text{Will cost large energy in large N limit}}\right)$$

Strong Coupling Description

1. Therefore we can now consider a reduced model of commuting matrix quantum mechanics:

$$S = \int dt \ \mathrm{Tr}\left[(D_t X^a)^2 - \frac{1}{2}(X^a)^2\right] \qquad [X^a, X^b] = 0$$

2. To write Hamiltonian we need to take into account measure change in path integral:

$$H = \sum_i \left(-\frac{1}{2\mu^2}\nabla_i \mu^2 \nabla_i + \frac{1}{2}|\vec{x}_i|^2\right) \qquad \mu^2 = \prod_{i<j} |\vec{x}_i - \vec{x}_j|^2$$

$$\vec{x}_i = (X^1_{ii}, \ldots, X^6_{ii})$$

Strong Coupling Description

1. Since the inner product contains the measure,

$$\langle\psi|\psi\rangle = \int \prod_i d^6 x_i \mu^2 \psi^* \psi$$

We can re-scale the wavefunction as $\psi \to \psi/\mu$ and write

$$\boxed{\begin{aligned} H &= \sum_i \left(-\frac{1}{2}\nabla_i^2 + \frac{1}{2}|\vec{x}_i|^2 \right) + V_{\text{eff}} \\ V_{\text{eff}} &= -6 \sum_{i \neq j} \frac{1}{|\vec{x}_i - \vec{x}_j|^2} + \sum_i \sum_{j,k \neq i} \frac{(\vec{x}_i - \vec{x}_j)\cdot(\vec{x}_i - \vec{x}_k)}{|\vec{x}_i - \vec{x}_j|^2 |\vec{x}_i - \vec{x}_k|^2} \end{aligned}}$$

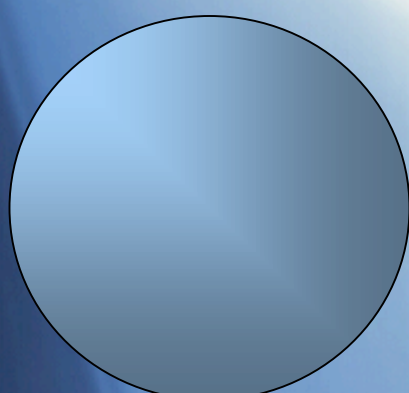
2. Ground state exactly known: $\psi_0 \sim \mu \exp\left(-\frac{1}{2} \sum_i |\vec{x}_i|^2 \right)$

Strong Coupling Description

1. Geometrical meaning of ground state (in large N limit)

$$\begin{aligned} |\psi_0|^2 &= \exp\left[-\sum_i |\vec{x}_i|^2 + \frac{1}{2}\sum_{i\neq j}\log|\vec{x}_i - \vec{x}_j|^2\right] \\ \text{Large N limit} \quad &\rightarrow \exp\left[-\int d^6x\rho(\vec{x})|\vec{x}|^2 + \frac{1}{2}\int d^6x d^6y \rho(\vec{x})\rho(\vec{y})\log|\vec{x}-\vec{y}|^2\right] \end{aligned}$$

2. Can show that saddle point approximation on the inner product gives an $S^5 \subset R^6$ (hep-th/0507203, hep-th/0509015)



$$\rho_0 = N\frac{\delta(|\vec{x}| - r_0)}{r_0^{2d-1}\mathrm{Vol}(S^{2d-1})}$$

$$r_0 = \sqrt{\frac{N}{2}}$$

Strong Coupling Description

1. The claim is that this model should also describe graviton scattering in the bulk. (Note that this is a non-trivial interacting N- body system)
2. The proposal is that one should take an initial state similar to the one we studied at weak coupling:

$$\psi \sim e^{\sum_i \Omega_1(\vec{x}_i)+\sum_j \Omega_2(\vec{x}_j)}\psi_0$$

where each wavefunction should be approximately BPS in the large N limit

$$\langle H - J \rangle \approx 0$$

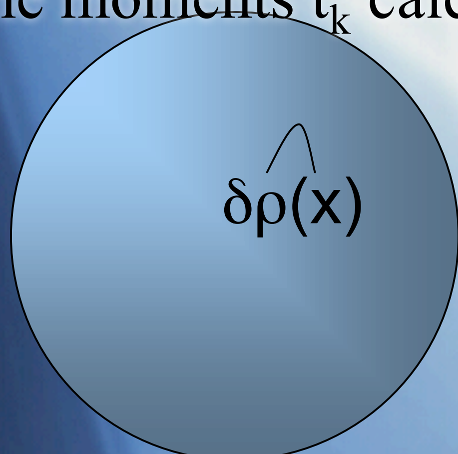
Strong Coupling Description

1. Berenstein, Cotta and Leonardi (hep-th/0605220, hep-th/0702090, 0801.2739) have studied holomorphic states with

$$\Omega \sim \sum_k t_k z^k$$

and showed numerically that they are indeed approximately BPS.

2. To study high energy gravitons in flat space, one should use the moments t_k calculated before



In units where radius of S^5 is one, the density perturbation has angular size

$$\Delta\phi \sim 1/R \sim N^{-1/4}$$

Strong Coupling Description

1. Studying such scattering processes will require new numerical techniques (difficult to evolve in time).
2. We can estimate how important are the off-diagonal excitations: (Berenstein, Correa, Vazquez hep-th/0509015)

$$H_{\text{off diag.}} \sim \sum_{i \neq j} \sum_{\alpha} w_{i,j}^{(\alpha)} (A_\alpha^\dagger)_i^j (A_\alpha)_j^i$$

$$w_{i,j}^{(\alpha)} = \sqrt{m_\alpha^2 + \frac{g_{YM}^2}{2\pi^2} |\vec{x}_i - \vec{x}_j|^2}$$

From conformal curvature coupling in action

From commutator in action

Strong Coupling Description

1. For two density perturbations separated a distance b in a flat space patch within the S^5, the energy of the off-diagonal modes connecting them is

$$E_{\text{off-diag}} > \sqrt{g}b = l_s^{-2}b$$

2. To be able to ignore such modes one needs $E_{\text{off-diag}} << E_{cm}$

$$b > l_s^2 E$$

This is the well known bound to create long strings.

3. Not good enough for Black Hole formation. Need (Giddings, Eardley)

$$b \sim E^{1/7} \ll l_s^2 E$$

Strong Coupling Description

1. However, recently Giddings, Gross and Maharanara (0705.181) have shown that long strings are not important for black hole formation in high energy scattering.
2. Therefore, it is still possible that the reduced matrix model can describe such process.

Outlook

1. **What have we learned?**
   1. Using the LLM dictionary, we learned to construct an initial state in SYM theory that is dual to a **regularized** Aichelburg-Sexl metric in flat space.
   2. We learned how to put two such shock waves in a head on collision so that they will produce a classical black hole.
   3. We made sure that the resulting trapped surface did not had any high curvatures.
   4. I gave a proposal on how to study such scattering process at strong coupling in terms of a reduced matrix model.
2. **What's next?**
   1. Understand how to compute scattering processes using the reduced matrix model (hard! But easier than full SYM…)
   2. Understand the role of the off-diagonal modes in the black hole formation