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Two-loop Integrability for Superconformal Chern-Simons

Joseph Minahan

Uppsala University

arXiv:0806.3951 K. Zarembo, JM

arXiv:0901.1142 W. Schulgin, K. Zarembo and JM

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University of North Carolina

Outline

Introduction:

Integrability in Planar $\mathcal{N} = 4$ Super Yang-Mills

Field Content

Adjoint fields: X^I, $I = 1 \dots 6$; ψ^α, $\psi^\dagger_\alpha$, $\alpha = 1 \dots 4$; D_μ

$SO(6)$ R-symmetry

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Single trace operators

Planar limit (Large N):

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$$\langle\mathcal{O}(x)\mathcal{O}(0)\rangle=|x|^{-2\Delta},\quad \Delta=\Delta_0+\gamma$$

$$\mathcal{O}_{\mathrm{ren}}=Z(\Lambda)\mathcal{O}_{\mathrm{bare}}\Rightarrow\gamma=Z^{-1}\frac{\partial}{\partial\ln\Lambda}Z$$

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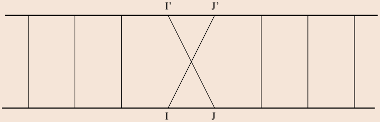
Mixing

$$\mathcal{O}^A_{\mathrm{ren}}=Z^{AB}(\Lambda)\mathcal{O}^B_{\mathrm{bare}}\Rightarrow\Gamma^{AB}=(Z^{-1}\frac{\partial}{\partial\ln\Lambda}Z)^{AB}$$

Spin chains K. Zarembo, JM (2002)

Scalar sector: Planar one-loop level



$$L_{int} = -\frac{1}{2} g_{\rm YM}^2 \, {\rm tr}([X_I, X_J]^2)$$

$$\Gamma = \frac{\lambda}{8\pi^2} \sum_{\ell=1}^{L} \left(1 - P_{\ell,\ell+1} + \frac{1}{2} K_{\ell,\ell+1}\right)$$

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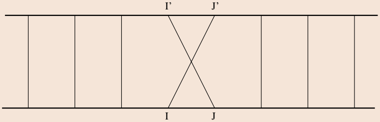

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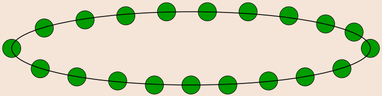

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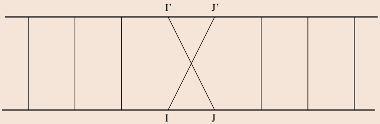

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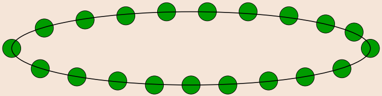

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Extended to all sectors: Beisert & Staudacher (2003)
Extended to higher loops: Beisert, Kristjansen & Staudacher (2003)

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- $SO(4)\simeq SU(2)\times SU(2)$ Van Raamsdonk
- Gauge fields: A_μ^{ab}, $\hat{A}_\mu^{\dot a\dot b}$;
  Bifundamental matter (real): $X^{a\dot a\, I}$, $\psi^{a\dot a\,\alpha}$, $I=1\ldots8$, $\alpha=1\ldots8$
- Chern-Simons action

$$\frac{k}{4\pi}\int d^3x\,\epsilon^{\mu\nu\lambda}\,\mathrm{tr}(A_\mu\partial_\nu A_\lambda+\tfrac{2}{3}A_\mu A_\nu A_\lambda)-(A_\mu\to\hat{A}_\mu)$$

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- But $\ldots N = 2$ is not large N

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  - $\mathbf{8_v} = \mathbf{4} + \bar{\mathbf{4}}$; $\mathbf{8_s} = \mathbf{4} + \bar{\mathbf{4}}$
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$\mathcal{N}=6$ Superconformal Chern-Simons : Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{k}{4\pi}\,\mathrm{tr}\Big[\varepsilon^{\mu\nu\lambda}\Big(A_\mu\partial_\nu A_\lambda+\frac{2}{3}A_\mu A_\nu A_\lambda-\hat{A}_\mu\partial_\nu\hat{A}_\lambda-\frac{2}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\lambda\Big)\\
&+D_\mu Y^\dagger_A D^\mu Y^A+i\bar{\psi}^A \not{D}\psi_A\\
&+\frac{1}{12}Y^AY^\dagger_AY^BY^\dagger_BY^CY^\dagger_C+\frac{1}{12}Y^AY^\dagger_BY^BY^\dagger_CY^CY^\dagger_A\\
&-\frac{1}{2}Y^AY^\dagger_AY^BY^\dagger_CY^CY^\dagger_B+\frac{1}{3}Y^AY^\dagger_BY^CY^\dagger_AY^BY^\dagger_C\\
&-\frac{1}{2}Y^\dagger_AY^A\bar{\psi}^B\psi_B+Y^\dagger_AY^B\bar{\psi}^A\psi_B+\frac{1}{2}\bar{\psi}^AY^BY^\dagger_B\psi_A-\bar{\psi}^AY^BY^\dagger_A\psi_B\\
&+\frac{1}{2}\varepsilon^{ABCD}Y^\dagger_A\bar{\psi}_{cB}Y^\dagger_C\psi_D-\frac{1}{2}\varepsilon_{ABCD}Y^A\bar{\psi}^BY^CC\psi^D_c\Big]
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- Gauge inv. ops are traces alternating between † and un†ed terms.

$$\chi^{B_1 B_2 \dots B_L}_{A_1 A_2 \dots A_L} tr[Y^{A_1} Y^\dagger_{B_1} Y^{A_2} Y^\dagger_{B_2} \dots Y^{A_L} Y^\dagger_{B_L}]$$

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- Chiral primary operators: $tr[(Y^1 Y^\dagger_4)^L]$

$$\Delta - J = 0$$

J is one of the three $SU(4)$ charges

$$\begin{array}{lll} \mathbf{4}: & (\pm\frac{1}{2}, \pm\frac{1}{2}, \mp\frac{1}{2}) & \text{odd \# of } - \\ \bar{\mathbf{4}}: & (\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}) & \text{even \# of } - \end{array}$$

Y^1 and $Y^\dagger_4$ each have $J = \frac{1}{2}$. In 3d, bare dimension is also $\frac{1}{2}$.

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- For large k the string dual is type IIA on $AdS_4 \times CP_3$,
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- String dual is classically integrable (Arutyunov and Frolov; Stefanski)
- ABJ: Modify to $SU(N) \times SU(\hat{N})$, still has $\mathcal{N}=6$
  String dual: background $B \sim \frac{N-\hat{N}}{k} J$

Two loop anomalous dimensions

Scalar sector

Spin chain with sites alternating between **4** and $\bar{\mathbf{4}}$

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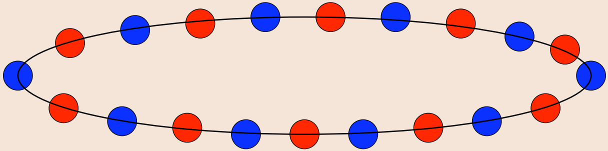

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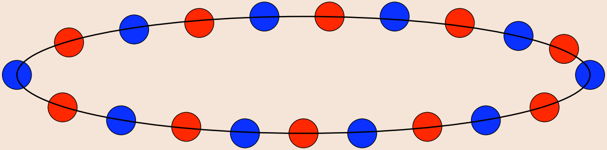
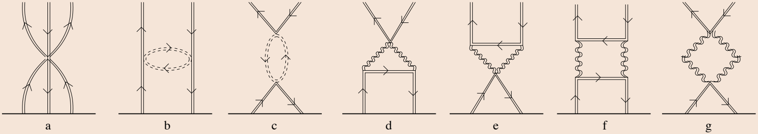
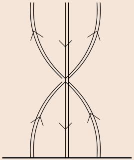


Operator mixing via two loop planar graphs



Scalar vertex



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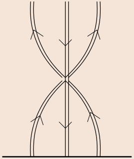

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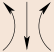
$+\frac{1}{2} Y^A Y^\dagger_A Y^B Y^\dagger_C Y^C Y^\dagger_B$ 

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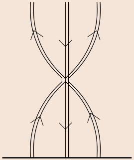

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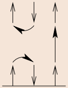
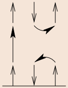
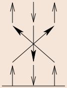
Insertions

   $\times 3$

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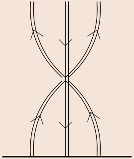

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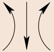
$+\frac{1}{2}Y^A Y^\dagger_A Y^B Y^\dagger_C Y^C Y^\dagger_B$ 

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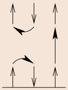

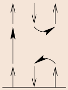

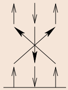

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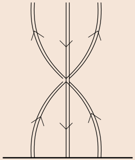

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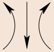
$$\int_{1/\Lambda} d^3x \left(\frac{1}{4\pi|x|}\right)^3 \simeq \frac{1}{16\pi^2}\ln\Lambda$$

Sewing Rules

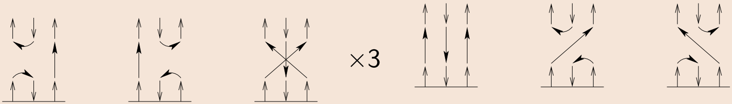
$-\frac{1}{12} Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger$ 

$-\frac{1}{12} Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C Y_A^\dagger$ 

$-\frac{1}{3} Y^A Y_C^\dagger Y^B Y_A^\dagger Y^C Y_B^\dagger$ 

$+\frac{1}{2} Y^A Y_A^\dagger Y^B Y_C^\dagger Y^C Y_B^\dagger$ 

Insertions



Anomalous dimension matrix

$$\Gamma = Z^{-1}\frac{\partial Z}{\partial \ln \Lambda} \qquad \mathcal{O}^I_{Ren} = Z_{IJ}(\Lambda)\mathcal{O}^J_{Bare}$$

$$= \frac{N^2(2\pi/k)^2}{16\pi^2}\sum_{\ell=1}^{2L} H^{(6v)}_{\ell,\ell+1,\ell+2}$$

$H^{(6v)} =$

$$\Gamma^{(6v)} = \frac{\lambda^2}{2}\sum_{\ell=1}^{2L}\left(-K_{\ell,\ell+1} + 1 - 2P_{\ell,\ell+2} + P_{\ell,\ell+2}K_{\ell,\ell+1} + K_{\ell,\ell+1}P_{\ell,\ell+2}\right)$$

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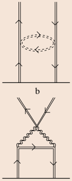
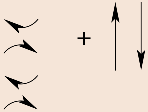
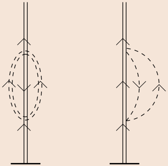
Other contributions



contributes 

Cancels n.n.

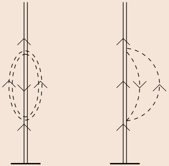
Self Energies



etc.

Self energy contributions only contribute to constant term

Self Energies



etc.

Self energy contributions only contribute to constant term

Constant determined from chiral primary

Γ acting on chiral primary is zero.

$$P_{\ell,\ell+2}\mathcal{O}_{cp} = \mathcal{O}_{cp} \qquad K_{\ell,\ell+1}\mathcal{O}_{cp} = 0$$

$$\Rightarrow \Gamma = \frac{\lambda^2}{2}\sum_{\ell=1}^{2L}\left(2 - 2P_{\ell,\ell+2} + P_{\ell,\ell+2}K_{\ell,\ell+1} + K_{\ell,\ell+1}P_{\ell,\ell+2}\right)$$

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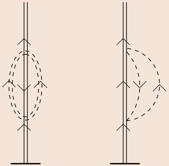

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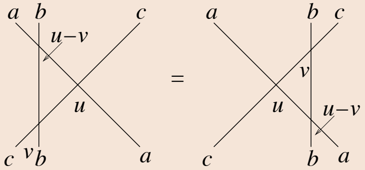
Is Γ integrable?

Yang-Baxter Equation

Integrability checked by Yang-Baxter eq.

$$
\begin{aligned}
R_{ab}(u) \quad &: \quad V_a \otimes V_b \rightarrow V_a \otimes V_b \\
R_{a\bar{b}}(u) \quad &: \quad V_a \otimes V_{\bar{b}} \rightarrow V_a \otimes V_{\bar{b}}
\end{aligned}
$$



$$
\begin{aligned}
R_{ab}(u-v)R_{ac}(u)R_{bc}(v) \quad &= \quad R_{bc}(v)R_{ac}(u)R_{ab}(u-v) \\
R_{a\bar{b}}(u-v)R_{ac}(u)R_{\bar{b}c}(v) \quad &= \quad R_{\bar{b}c}(v)R_{ac}(u)R_{a\bar{b}}(u-v)
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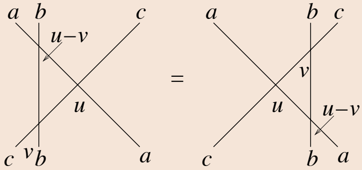

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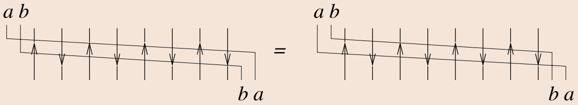
\begin{aligned}
R_{ab}(u-v)R_{ac}(u)R_{bc}(v) &= R_{bc}(v)R_{ac}(u)R_{ab}(u-v) \\
R_{a\bar{b}}(u-v)R_{ac}(u)R_{\bar{b}c}(v) &= R_{\bar{b}c}(v)R_{ac}(u)R_{a\bar{b}}(u-v)
\end{aligned}
$$

Solution: $SU(4)$

$$
R_{ab} = u - P_{ab} \qquad R_{\bar{a}\bar{b}} = u - P_{\bar{a}\bar{b}} \qquad R_{a\bar{b}} = u + K_{a\bar{b}} - 2
$$

Monodromy matrix

$$T_a(u) = R_{ac_1}(u)R_{a\bar{c}_1}(u)\dots R_{ac_L}(u)R_{a\bar{c}_L}(u)$$
$$T_{\bar{a}}(u) = R_{\bar{a}c_1}(u)R_{\bar{a}\bar{c}_1}(u)\dots R_{\bar{a}c_L}(u)R_{\bar{a}\bar{c}_L}(u)$$



$$R_{ab}(u-v)T_a(u)T_b(v) = T_b(v)T_a(u)R_{ab}(u-v)$$
$$R_{a\bar{b}}(u-v)T_a(u)T_{\bar{b}}(v) = T_{\bar{b}}(v)T_a(u)R_{a\bar{b}}(u-v)$$

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\begin{aligned}
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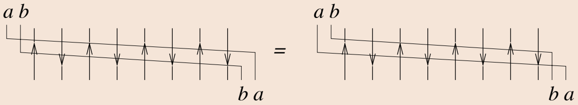
$$



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R_{ab}(u-v)T_a(u)T_b(v) &= T_b(v)T_a(u)R_{ab}(u-v)\\
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\end{aligned}
$$

Transfer matrices

$$
\tau(u) = \mathrm{tr}_a T_a(u) \qquad \bar{\tau}(u) = \mathrm{tr}_{\bar{a}} T_{\bar{a}}(u)
$$

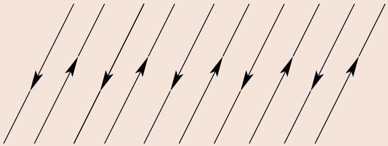
$$
\Rightarrow [\tau(u),\tau(v)] = [\bar{\tau}(u),\bar{\tau}(v)] = [\tau(u),\bar{\tau}(v)] = 0
$$

$\tau(u)$ and $\bar{\tau}(u)$ are polynomials of conserved charges

Conserved charges

Shift operator:

$$\Pi \equiv \tau(0)\bar{\tau}(0) = \prod_{\ell=1}^{2L} P_{\ell,\ell+2}$$



$$\Pi\mathcal{O} = \mathcal{O}$$

Trace condition

Hamiltonian:

$$\Gamma = \lambda^2(\tau(0)\bar{\tau}(0))^{-1}\frac{d}{du}(\tau(u)\bar{\tau}(u))\Big|_{u=0}$$

Alternating $SU(4)$ spin chain, ground state: $\mathrm{tr}[(Y^1 Y_4^\dagger)^L]$

$$\Gamma = \frac{\lambda^2}{2}\sum_{\ell=1}^{2L}\left(2 - 2P_{\ell,\ell+2} + P_{\ell,\ell+2}K_{\ell,\ell+1} + K_{\ell,\ell+1}P_{\ell,\ell+2}\right)$$

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$SU(2)\times SU(2)$ sector

- $Y^1 \to Y^2$, $Y_4^\dagger \to Y_3^\dagger$
- $K_{\ell,\ell+1}$ is always zero, only $P_{\ell,\ell+2}$ contributes
- Reduces to 2 ind. spin chains (Heisenberg ferromagnets)

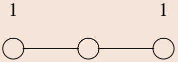
Bethe eqs:

$$\left(\frac{u_j+i/2}{u_j-i/2}\right)^L = \prod_{k=1,k\neq j}^{M_u}\frac{u_j-u_k+i}{u_j-u_k-i}$$

$$\left(\frac{v_j+i/2}{v_j-i/2}\right)^L = \prod_{k=1,k\neq j}^{M_v}\frac{v_j-v_k+i}{v_j-v_k-i}$$

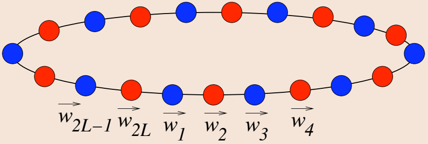
$$\prod_{j=1}^{M_u}\frac{u_j+i/2}{u_j-i/2}\prod_{j=1}^{M_v}\frac{v_j+i/2}{v_j-i/2}=1\,,\quad \gamma=\lambda^2\left(\sum_{j=1}^{M_u}\frac{1}{u_j^2+1/4}+\sum_{j=1}^{M_v}\frac{1}{v_j^2+1/4}\right)$$

General $SU(4)$ sector



Bethe eqs determined by Dynkin diag.

General chain for rank n group with reps labeled by highest wts. $\vec{w}_\ell$



Bethe roots $u_j^{(m)}$ assoc. w/ simple root $\vec{\alpha}_m$

$$\prod_{\ell=1}^{2L} \frac{u_j^{(m)} + i\vec{w}_\ell\cdot\vec{\alpha}_m/2}{u_j^{(m)} - i\vec{w}_\ell\cdot\vec{\alpha}_m/2} = -\prod_{m'=1}^{n}\prod_{k=1}^{M_{m'}} \frac{u_j^{(m)} - u_k^{(m')} + i\vec{\alpha}_m\cdot\vec{\alpha}_{m'}/2}{u_j^{(m)} - u_k^{(m')} - i\vec{\alpha}_m\cdot\vec{\alpha}_{m'}/2}$$

$SU(4)$: $u_j \equiv u_j^{(1)}$, $r_j \equiv u_j^{(2)}$, $v_j \equiv u_j^{(3)}$, $\vec{\alpha}_1\cdot\vec{w}_{2\ell-1} = 1$, $\vec{\alpha}_3\cdot\vec{w}_{2\ell} = 1$.

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k=1,k\neq j}^{M_u} \frac{u_j - u_k + i}{u_j - u_k - i} \prod_{k=1}^{M_r} \frac{u_j - r_k - i/2}{u_j - r_k + i/2}$$

$$1 = \prod_{k=1,k\neq j}^{M_r} \frac{r_j - r_k + i}{r_j - r_k - i} \prod_{k=1}^{M_u} \frac{r_j - u_k - i/2}{r_j - u_k + i/2} \prod_{k=1}^{M_v} \frac{r_j - v_k - i/2}{r_j - v_k + i/2}$$

$$\left(\frac{v_j + i/2}{v_j - i/2}\right)^L = \prod_{k=1,k\neq j}^{M_v} \frac{v_j - v_k + i}{v_j - v_k - i} \prod_{k=1}^{M_r} \frac{v_j - r_k - i/2}{v_j - r_k + i/2}$$

The $OSp(6|4)$ sector

$SO(6)$ R-symmetry $\qquad Sp(4,R) \simeq SO(2,3)$ 3d conf. sym.
24 ferm. gens. Q^I_α, S^I_α, $I = 1\ldots6$, $\alpha = 1, 2$

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Charges: $(-D-S, -D+S; J_1, J_2, J_3)$

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$$Y^1\sim(-\tfrac{1}{2},-\tfrac{1}{2};+\tfrac{1}{2},+\tfrac{1}{2},-\tfrac{1}{2}),\ Y^\dagger_4\sim(-\tfrac{1}{2},-\tfrac{1}{2};+\tfrac{1}{2},+\tfrac{1}{2},+\tfrac{1}{2}),$$
$$\mathrm{tr}[(Y_1Y_4^\dagger)^L]\Rightarrow(-L,-L;L,L,0)$$

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$$\vec{\alpha}_1=(0,0;0,1,-1),\ \vec{\alpha}_2=(0,0;1,-1,0),\ \vec{\alpha}_3=(0,0;0,1,1)$$
$$\vec{\alpha}_4=(0,1;-1,0,0),\ \vec{\alpha}_5=(1,-1;0,0,0)$$

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$$Y^1\sim(-\tfrac12,-\tfrac12;+\tfrac12,+\tfrac12,-\tfrac12),\ Y_4^\dagger\sim(-\tfrac12,-\tfrac12;+\tfrac12,+\tfrac12,+\tfrac12),$$
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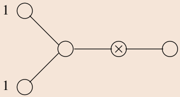
$$\vec\alpha_1=(0,0;0,1,-1),\ \vec\alpha_2=(0,0;1,-1,0),\ \vec\alpha_3=(0,0;0,1,1)$$
$$\vec\alpha_4=(0,1;-1,0,0),\ \vec\alpha_5=(1,-1;0,0,0)$$

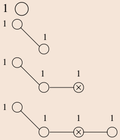
$$\vec\alpha_4\cdot\vec\alpha_4=0,\ \vec\alpha_5\cdot\vec\alpha_5=-2,$$
$$\vec\alpha_2\cdot\vec\alpha_4=-1,\ \vec\alpha_4\cdot\vec\alpha_5=+1$$



$$1\ =\ \prod_{k=1,k\neq j}^{M_r}\frac{r_j-r_k+i}{r_j-r_k-i}\prod_{k=1}^{M_u}\frac{r_j-u_k-i/2}{r_j-u_k+i/2}\prod_{k=1}^{M_v}\frac{r_j-v_k-i/2}{r_j-v_k+i/2}\prod_{k=1}^{M_s}\frac{r_j-s_k-i/2}{r_j-s_k+i/2}$$

$$1\ =\ \prod_{k=1}^{M_r}\frac{s_j-r_k-i/2}{s_j-r_k+i/2}\prod_{k=1}^{M_w}\frac{s_j-w_k+i/2}{s_j-w_k-i/2},\quad 1=\prod_{k=1,k\neq j}^{M_w}\frac{w_j-w_k-i}{w_j-w_k+i}\prod_{k=1}^{M_s}\frac{w_j-s_k+i/2}{w_j-s_k-i/2}$$

Magnons

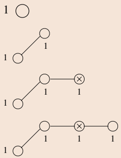


$$Y^1 \longrightarrow Y^2$$

$$Y^1 \longrightarrow Y^3$$

$$Y^1 \longrightarrow \psi_{-4}$$

$$Y^1 \longrightarrow \psi_{+4}$$



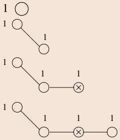
$$Y_4^\dagger \longrightarrow Y_3^\dagger$$

$$Y_4^\dagger \longrightarrow Y_2^\dagger$$

$$Y_4^\dagger \longrightarrow \psi_{c-}^{\dagger\,1}$$

$$Y_4^\dagger \longrightarrow \psi_{c+}^{\dagger\,1}$$

Magnons



$Y^1 \longrightarrow Y^2$

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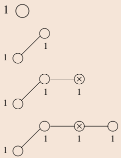


$Y_4^\dagger \longrightarrow Y_3^\dagger$

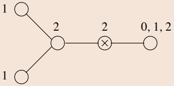
$Y_4^\dagger \longrightarrow Y_2^\dagger$

$Y_4^\dagger \longrightarrow \psi^{\dagger\,1}_{c-}$

$Y_4^\dagger \longrightarrow \psi^{\dagger\,1}_{c+}$

Where are D_μ modes?

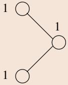
- In $\mathcal{N}=4$, there are magnons for D_μ.
- $\mathcal{N}=6$ CS, D_μ modes are:



$= D_-,\ D_0,\ D_+$

These are two unbound magnons

- Same is true for 5th transverse mode on CP_3:



Hamiltonian in full sector

- Full H constructed outside of $SU(4)$ sector (Zwiebel; Schulgin, Zarembo, JM)

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- Full H constructed outside of $SU(4)$ sector (Zwiebel; Schulgin, Zarembo, JM)
- Does not have a particularly nice form
- Example of one fermion interacting with bosons

$$
\begin{aligned}
H\circ Y_A^\dagger\psi_B Y_C^\dagger \;=\;& -2Y_A^\dagger\psi_C Y_B^\dagger - 2Y_B^\dagger\psi_A Y_C^\dagger - 2Y_C^\dagger\psi_B Y_A^\dagger \\
& - Y_B^\dagger\psi_C Y_A^\dagger - Y_C^\dagger\psi_A Y_B^\dagger \\
& -6\varepsilon_{BCDE}Y_A^\dagger Y^D\psi_c^E - 6\varepsilon_{ABDE}\psi_c^D Y^E Y_C^\dagger - \varepsilon_{ABCD}\psi_c^D Y^E Y_E^\dagger \\
& +3\varepsilon_{ACDE}Y_B^\dagger Y^D\psi_c^E + 3\varepsilon_{ACDE}\psi_c^D Y^E Y_B^\dagger + \varepsilon_{ABCD}Y_E^\dagger Y^E\psi_c^D \\
& -2\varepsilon_{ABCD}Y_E^\dagger Y^D\psi_c^E + 2\varepsilon_{ABCD}\psi_c^E Y^D Y_E^\dagger , \\
H\circ \psi_A Y_B^\dagger Y^C \;=\;& -3Y^C Y_B^\dagger\psi_A - 3Y^C Y_A^\dagger\psi_B \\
& +\delta_B^C Y^D Y_A^\dagger\psi_D + \delta_B^C Y^D Y_D^\dagger\psi_A + \delta_A^C Y^D Y_D^\dagger\psi_B + \delta_A^C Y^D Y_B^\dagger\psi_D \\
& +2\delta_B^C\psi_A Y_D^\dagger Y^D - \delta_B^C\psi_D Y_A^\dagger Y^D - \delta_A^C\psi_B Y_D^\dagger Y^D + 2\delta_A^C\psi_D Y_B^\dagger Y^D \\
& +3\varepsilon_{ABDE}Y^D\psi_c^C Y^E + \delta_B^C\varepsilon_{ADEF}Y^D\psi_c^E Y^F - 2\delta_A^C\varepsilon_{BDEF}Y^D\psi_c^E Y^F , \\
H\circ Y^A Y_B^\dagger\psi_C \;=\;& -3\psi_C Y_B^\dagger Y^A - 3\psi_B Y_C^\dagger Y^A \\
& +\delta_B^A\psi_D Y_C^\dagger Y^D + \delta_B^A\psi_C Y_D^\dagger Y^D + \delta_C^A\psi_B Y_D^\dagger Y^D + \delta_C^A\psi_D Y_B^\dagger Y^D \\
& +2\delta_B^A Y^D Y_D^\dagger\psi_C - \delta_B^A Y^D Y_C^\dagger\psi_D - \delta_C^A Y^D Y_D^\dagger\psi_B + 2\delta_C^A Y^D Y_B^\dagger\psi_D \\
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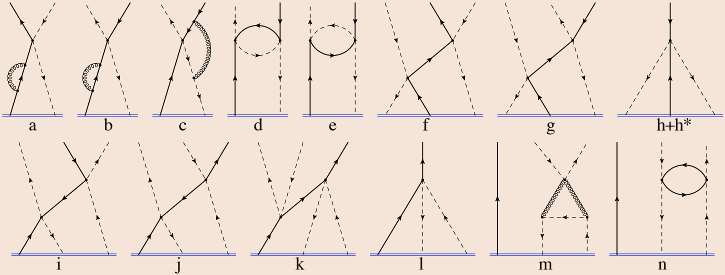
\end{aligned}
$$

Diagrams contributing to H with single fermion



ABJ: Extend to $SU(N) \times SU(\hat{N})$

- Still has $\mathcal{N} = 6$ supersymmetry
- $R^6 \sim (N\hat{N})^{1/2}k$, $\lambda = \frac{N}{k}$, $\hat{\lambda} = \frac{\hat{N}}{k}$
- Background $B_{\mu\nu}$: no longer clear if string theory is integrable $B_{\mu\nu}$ effectively adds a θ term to sigma model.
- CP invariance is broken. World-sheet parity is also broken $\Rightarrow$ At two loops expect dependence on $\lambda\hat{\lambda}$ (parity preserving) and λ^2 and $\hat{\lambda}^2$ (parity violating)

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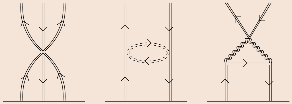
Parity is unbroken in scalar sector Bak, Gang, Rey



a b d

Scalars and ferms are bifundamentals, $\Rightarrow$ graphs $\sim N\hat{N}$

$$\Gamma = \frac{\lambda\hat{\lambda}}{2}\sum_{\ell=1}^{2L}(2 - 2P_{\ell,\ell+2} + P_{\ell,\ell+2}K_{\ell,\ell+1} + K_{\ell,\ell+1}P_{\ell,\ell+2})$$

$$\gamma = \lambda\hat{\lambda}\left(\sum_{j=1}^{M_u}\frac{1}{u_j^2+1/4} + \sum_{j=1}^{M_v}\frac{1}{v_j^2+1/4}\right)$$

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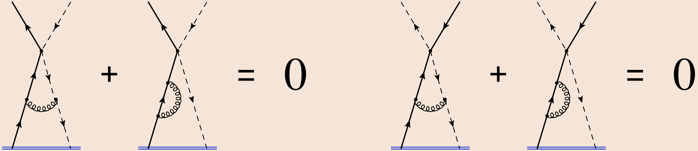

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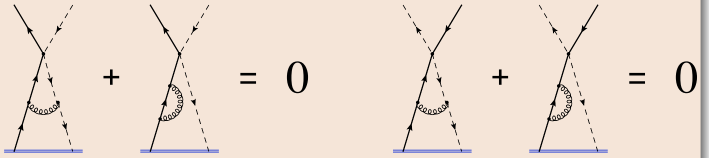

- $\mathcal{N} = 6$ Susy seems to guarantee 2-loop integrability (Zwiebel).

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- This nonconstant function should first appear at 4-loop order (O. Ohlsson Sax, C. Sieg, JM work in progress)