A classification of a family of free fermionic orbifold models

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1. Free fermion models

$\mathcal{H}_0^{(a)}$: Herm. $\mathbb{C}$-vector sp. w. onb $\left\{ \begin{matrix} i_a, j_b \in \mathbb{N},\ 0<i_1<\dots<i_k, \\ 0<j_1<\dots<j_\ell \end{matrix} \right\}$ $\quad \psi^{(2a-1)}_{i_1-1/2} \cdots \psi^{(2a-1)}_{i_k-1/2}\, \psi^{(2a)}_{j_1-1/2} \cdots \psi^{(2a)}_{j_\ell-1/2} |0\rangle$

$\mathcal{H}_1^{(a)}$: ——— " ——— $\quad \psi^{(2a-1)}_{i_1} \cdots \psi^{(2a-1)}_{i_k}\, \psi^{(2a)}_{j_1} \cdots \psi^{(2a)}_{j_\ell} \begin{cases} |+\rangle \\ |-\rangle \end{cases}$ or

representations of 2 copies of a Virasoro algebra $L_n^{(b)}$, $n \in \mathbb{Z}$, $b \in \{2a-1, 2a\}$,

$C = \frac{1}{2} \cdot id$: $\quad [L_m^{(b)}, L_n^{(b)}] = (n-m) L_{m+n}^{(b)} + \frac{c}{12} n(n^2-1)\delta_{m+n,0}$

extended by $(-1)^F$:

$$v = \psi^{(2a-1)}_{\mu_1} \cdots \psi^{(2a-1)}_{\mu_k}\, \psi^{(2a)}_{\nu_1} \cdots \psi^{(2a)}_{\nu_\ell} \ |*\rangle$$

$$L_0^{(b)} v = \begin{cases} \left(\sum \mu_i + h(*)\right) v & \text{if } b = 2a-1 \\ \left(\sum \nu_i + h(*)\right) v & \text{if } b = 2a \end{cases}, \quad \left(L_n^{(b)}\right)^\dagger = L_{-n}^{(b)},$$

$$(-1)^F v = (-1)^{k+\ell+f(*)} v$$

$$h(0) = 0, \quad h(+) = h(-) = \tfrac{1}{8}, \quad f(0) = f(+) = 0, \quad f(-) = 1$$

Constructing CFTs from free fermions

$N \in \mathbb{N}$, $\forall \alpha \in \mathbb{F}_2^N$: $\quad \mathcal{H}_\alpha := \bigotimes_{a=1}^{N} \mathcal{H}_{\alpha_a}^{(a)}$;

choose: $\mathcal{F} \subset \mathbb{F}_2^N$, $\quad \mathcal{H} \subset \bigoplus_{\alpha \in \mathcal{F}} \mathcal{H}_\alpha$ a subspace,

$\{1, \dots, 2N\} = \mathcal{L} \dot\cup \mathcal{R}$, an involution $(-1)^{F_S}$ on each $\mathcal{H}_\alpha$

such that (1) $L_n := \sum_{b \in \mathcal{L}} L_n^{(b)} \quad \left(c := \frac{|\mathcal{L}|}{2}\right)$,

$\overline{L}_n := \sum_{b \in \mathcal{R}} L_n^{(b)} \quad \left(\overline{c} := \frac{|\mathcal{R}|}{2}\right)$

obey $\operatorname{Spec}(L_0 - \overline{L}_0) \subset \frac{1}{2} \cdot \mathbb{Z}$, $\ker L_0 \cap \ker \overline{L}_0 \cong \mathbb{C}$

(2) for $\tau \in \{z \in \mathbb{C} \mid \operatorname{Im}(z) > 0\}$, $q = \exp(2\pi i \tau)$

the partition function

$$Z(\tau) := \operatorname{tr}_{\mathcal{H}} \left\{ \left(\tfrac{1}{2}(\mathbb{1} + (-1)^{F_S}) \right) q^{L_0 - c/24} \, \overline{q}^{\overline{L}_0 - \overline{c}/24} \right\}$$

is modular covariant.

Heterotic free fermion models

set
- $2N = \cancel{64},$ [illegible] $\quad |\mathcal{L}| = \cancel{20},$ [illegible] $\quad |\mathcal{R}| = \cancel{44}$ [illegible]
- $(-1)^{F_S} = (-1)^{\alpha_0}$ on $\mathcal{H}_\alpha \longrightarrow (-1)^{F_S} \equiv 1$
- $\mathcal{F} \subset \mathbb{F}_2^N$ a subspace, $\mathcal{H} = P^{\mathcal{F}}\left(\bigoplus_{\alpha\in\mathcal{F}} \mathcal{H}_\alpha\right),$

$$P^{\mathcal{F}}_{|\mathcal{H}_\alpha} = \frac{1}{|\mathcal{F}|}\sum_{\beta\in\mathcal{F}} (-1)^{\mathcal{F}\cdot\beta}\,\chi\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

$\chi : \mathcal{F}\times\mathcal{F} \longrightarrow \{\pm 1\}$ s.th. $\forall \alpha\in\mathcal{F}: \beta \mapsto \chi\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$ is a character

$$\boxed{\text{necessary and sufficient conditions on } \mathcal{F} \text{ and } \chi\text{: [Mueller/Witten '86]}}$$

- for $\mathcal{F}\subset\tilde{\mathcal{F}}$: $\tilde{\mathcal{H}} = P^{\tilde{\mathcal{F}}}\left(\bigoplus_{\alpha\in\tilde{\mathcal{F}}} \mathcal{H}_\alpha\right)$ gives an orbifold of $\mathcal{H}$ by $(\mathbb{Z}_2)^r$, $r = \dim\tilde{\mathcal{F}} - \dim\mathcal{F}$

2. Semi-realistic free fermion models and geometry

contributions by: Antoniadis, Bachas, Ellis, Hagelin, Faraggi, Förste, Kounnas, Mueller, Nanopoulos, Nilles, Nooij, Timirgaziu, Vaudrevange, Windey, Witten, Yuan, ...

heterotic free fermion models

→ # generations, gauge algebra

→ geometric interpretations on $\mathbb{C}^3/\text{lattice}$ + B-field + gauge bdl. in special cases

Facts:

(1) [Mueller/Witten '86]

- $\exists$ free fermion model = boson compactified on $T = \mathbb{R}^6/(x \sim x + \frac{1}{\sqrt{2}}u \;\forall u \in \mathbb{Z}^6 : \sum u_i \equiv 0\,(2))$ with maximal gauge symmetry

- for appropriate groups G inducing symmetries of the associated CFT, orbifolding by G can be implemented at the level of free fermion constructions

(2) $\exists$ free fermion model = boson compactified on $T_0 = \mathbb{R}^6/\frac{1}{\sqrt{2}}\mathbb{Z}^6$ with maximal gauge symmetry

(3) $\exists$ semi-realistic free fermion models obtained as G-orbifolds from (2), G = Abelian extension of $(\mathbb{Z}_2)^2$
[Ferrara/Girardello/Kounnas/Porrati '87; Faraggi/Nanopoulos '93, ...]

3. A classification of orbifolds

$X = E_1 \times E_2 \times E_3, \qquad E_i = \mathbb{C}/_{z_i \sim z_i + 1 \sim z_i + \tau}$

Classify G in $\quad 0 \longrightarrow G_S \longrightarrow G \xrightarrow{\pi} T_0 \longrightarrow 0$

where
- $G_S \subset G$ subgroup of shifts $s_x \in G$: $s_x(z) = z + x$, $x, z \in X$
- $\pi(g) = t$ iff $g = s \circ t$, s a shift by $g(0)$, $t(0) = 0$
- $T_0 = \langle t_1, t_2, t_3 \rangle$,

$$\left.\begin{array}{l} t_1: (z_1, z_2, z_3) \mapsto (z_1, -z_2, -z_3) \\ t_2: (z_1, z_2, z_3) \mapsto (-z_1, z_2, -z_3) \\ t_3: (z_1, z_2, z_3) \mapsto (-z_1, -z_2, z_3) \end{array}\right\} \quad t_i(z) = -z + 2z_i \cdot e_i$$

$$\boxed{\text{without loss of generality: } \mathbb{Z}_2^{r+2} \cong G = G_S \times G_T, \quad r \leq 4, \quad G_T \cong T_0 \quad \text{[Donagi/W'08]}}$$

Hodge numbers of all possible $\widetilde{X/G}$ with X, G as above

$(h^{(1,1)}, h^{(1,2)})$	models (rank-number), $G \cong (\mathbb{Z}_2)^{rank+2}$
(51, 3)	(0-1)
(31, 7)	(1-6)
(27, 3)	(1-1), (2-9)
(21, 9)	(2-13)
(19, 19)	$\boxed{(0-2)}$
(19, 7)	(2-6)
(17, 5)	(2-3), (3-3)
(15, 15)	(1-2), (1-8)
(15, 3)	(2-1), (3-5), (4-1)
(12, 6)	(3-1), (3-2)
(11, 11)	(0-3) ≡ (1-10), $\boxed{(1-3), (1-7)}$, (2-4), (2-10)
(9, 9)	(2-2), (2-7), (3-6)
(7, 7)	(1-4), (1-9), (2-11), $\boxed{(2-5), (2-14)}$, (3-4)
(5, 5)	(2-8)
(3, 3)	(0-4), (1-5), (1-11), (2-12)

4. Results [Donagi/W '02-'08]

- no CY_3 X/G with G as above has Hodge numbers $(h^{1,1}, h^{1,2})$ with $|h^{1,1} - h^{1,2}| = 3$
- several Borcea-Voisin 3-folds allow free fermion constructions, all are exceptional: $h^{1,1} = h^{1,2}$ or no Borcea-Voisin mirror exists
- Schoen's 3-fold and some of its quotients allow free fermion constructions