Non-Commutative Geometry & Yukawa Couplings in F-Theory

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based on work with S. Cecotti, J Heckaman, C. Vafa 0910.0447

Outline

- Motivation
- F-Theory
- Yukawa in Commutative Geometry
- More F-Theory
- Yukawa in Non-Commutative Geometry
- Conclusions and Applications

Motivation

- Structures of Yukawa's in F-theory: what types are allowed?
- Standard Model/MSSM $\subset$ String Theory? Flavor Hierarchy from??

In This Talk

- Yakawa Coupling in 7-brane Gauge Theory
- Possible Relevance for Hierarchy in Quark Sector

Flavor Hierarchy

$$W_{MSSM} = m_u^{ij} Q^i \bar{U}^j H_u + m_d^{ij} Q^i \bar{D}^j H_d + \cdots$$

i, j: Flavor indices

Diagonalize: $V_L \cdot m \cdot V_R^\dagger = \mathrm{diag}\big(m_1, m_2, m_3\big) \sim \mathrm{diag}\big(m_1, 0, 0\big)$

CKM Matrix: $|V_{CKM}| = V_{L,u} \cdot V_{L,d}^\dagger \sim \mathbf{I}$

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hierarchical !

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we explain WHY naturally

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$$m^{ij} : \text{rank} \;\; 1 \quad + \;\; \text{corrections}$$

F-theory

Structure	Dimensions
gravity	10
gauge fields (7-brane)	8
matter fields ($7 \cap 7'$)	6
Yuk. couplings ($7 \cap 7' \cap 7''$)	4

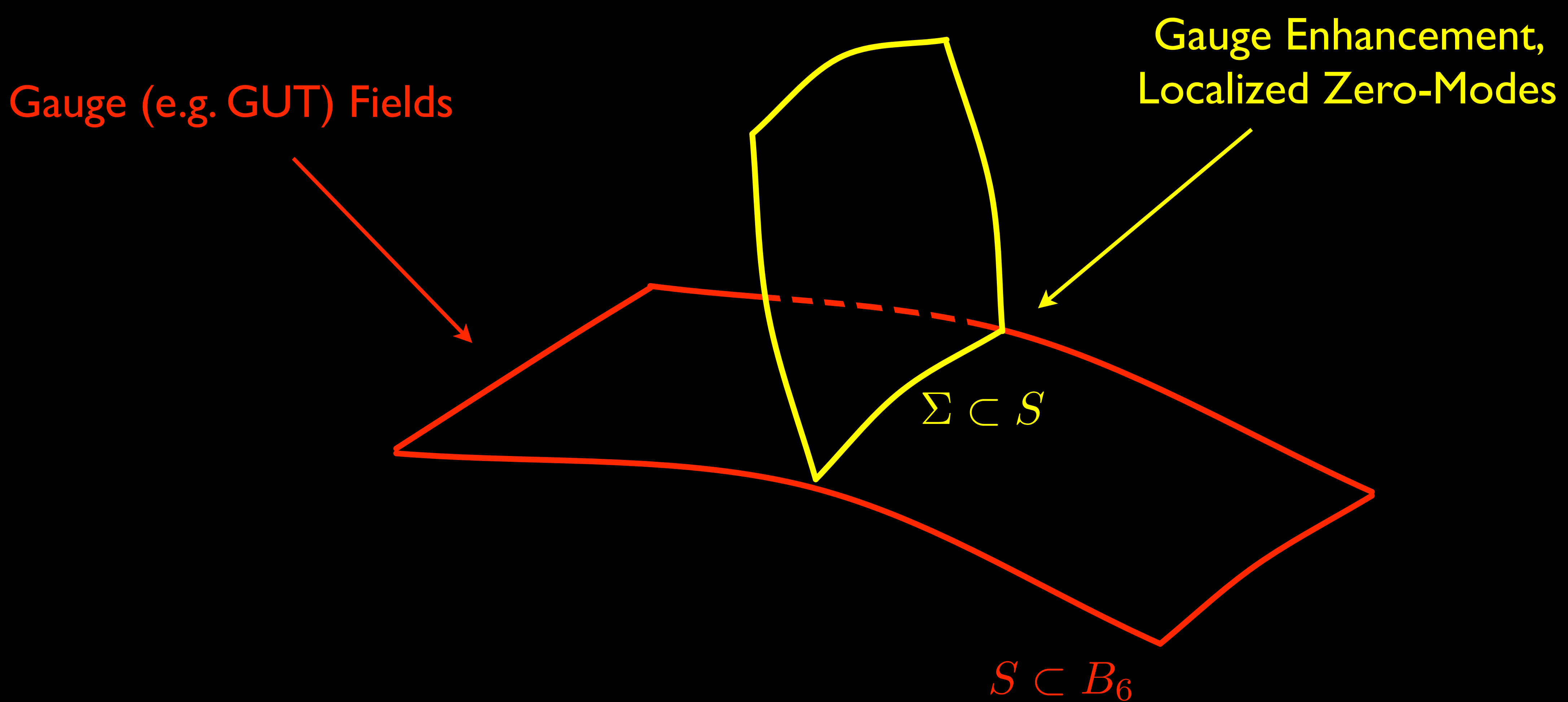
[Beasley-Heckman-Vafa, Donagi-Wijnholt]

F-theory

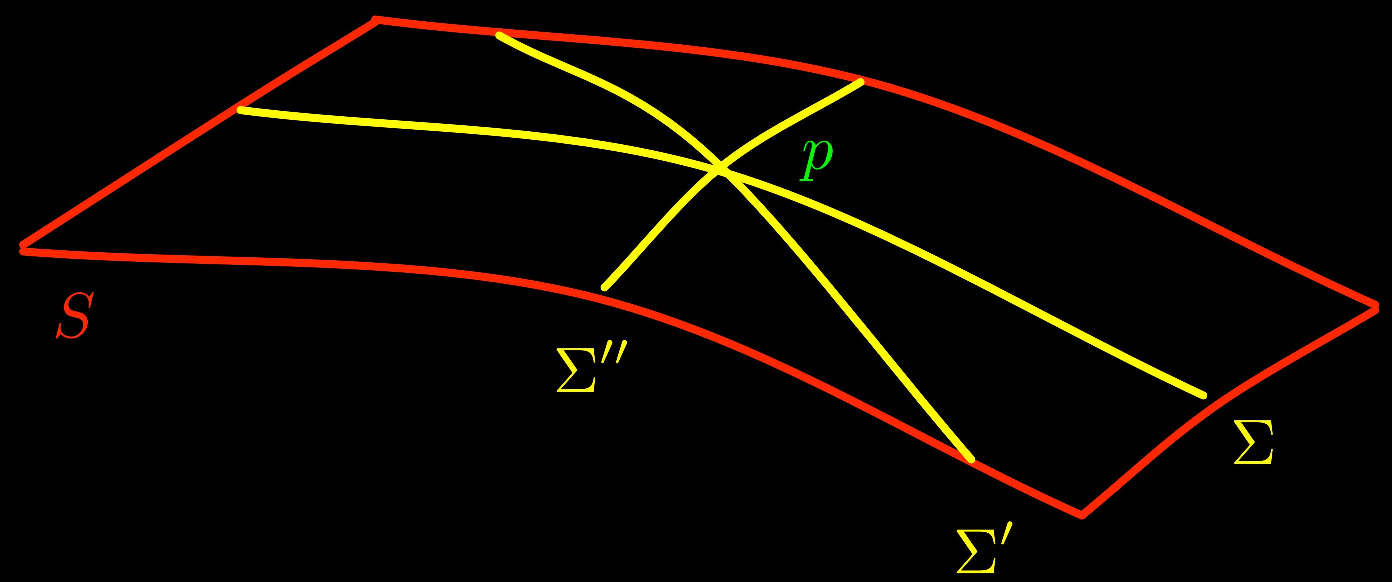
Structure	Dimensions	Dimensions in $B_6 \subset CY_4$
gravity	10	6
gauge fields (7-brane)	8	4 (S)
matter fields $(7 \cap 7')$	6	2 (Σ)
Yuk. couplings $(7 \cap 7' \cap 7'')$	4	0 (p)

[Beasley-Heckman-Vafa, Donagi-Wijnholt]

F-theory


Gauge (e.g. GUT) Fields
Gauge Enhancement,
Localized Zero-Modes
$\Sigma \subset S$
$S \subset B_6$

F-theory



$G_p \supset G_\Sigma \supset G_S$

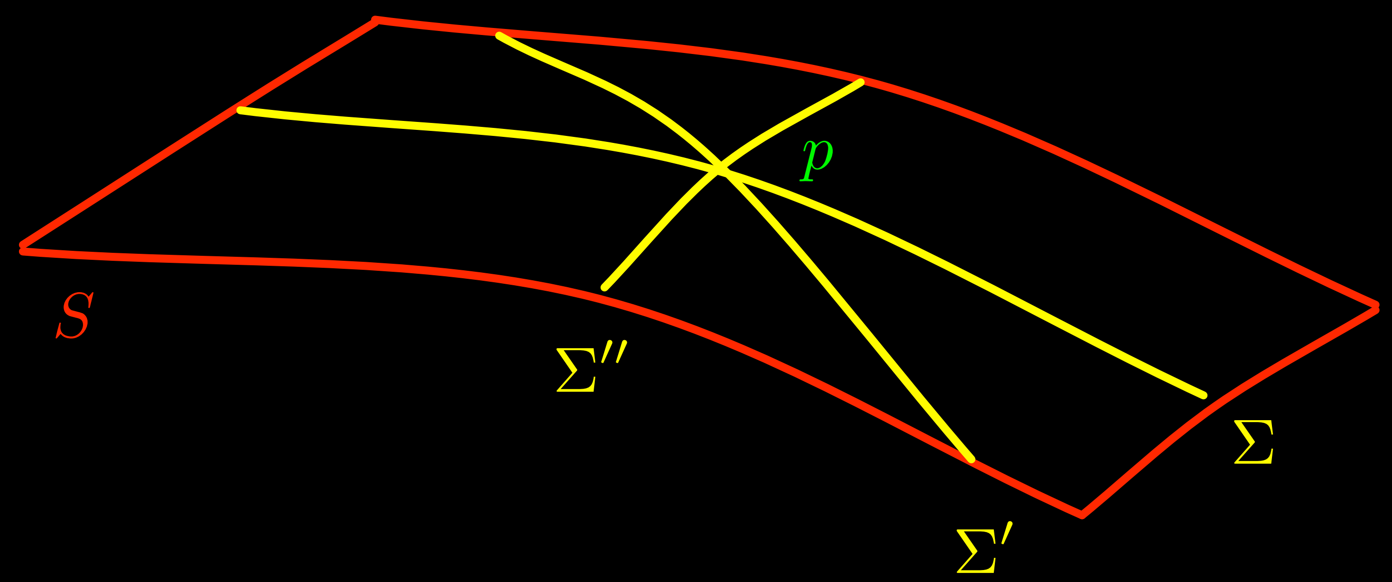
F-theory

Decoupling Assumption:

non-compact

compact

$$\frac{M_{GUT}}{M_{Pl}} \sim 10^{-3} \ll 1$$



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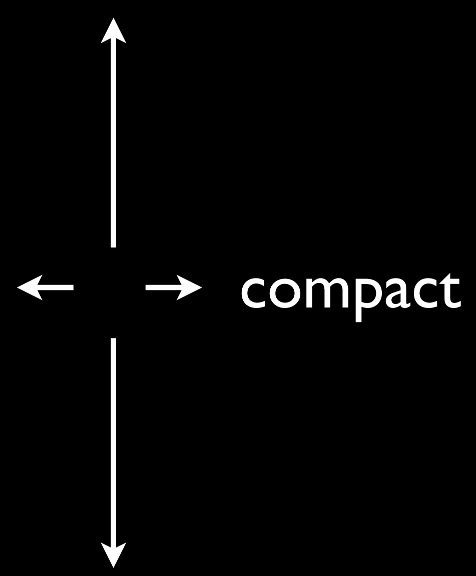

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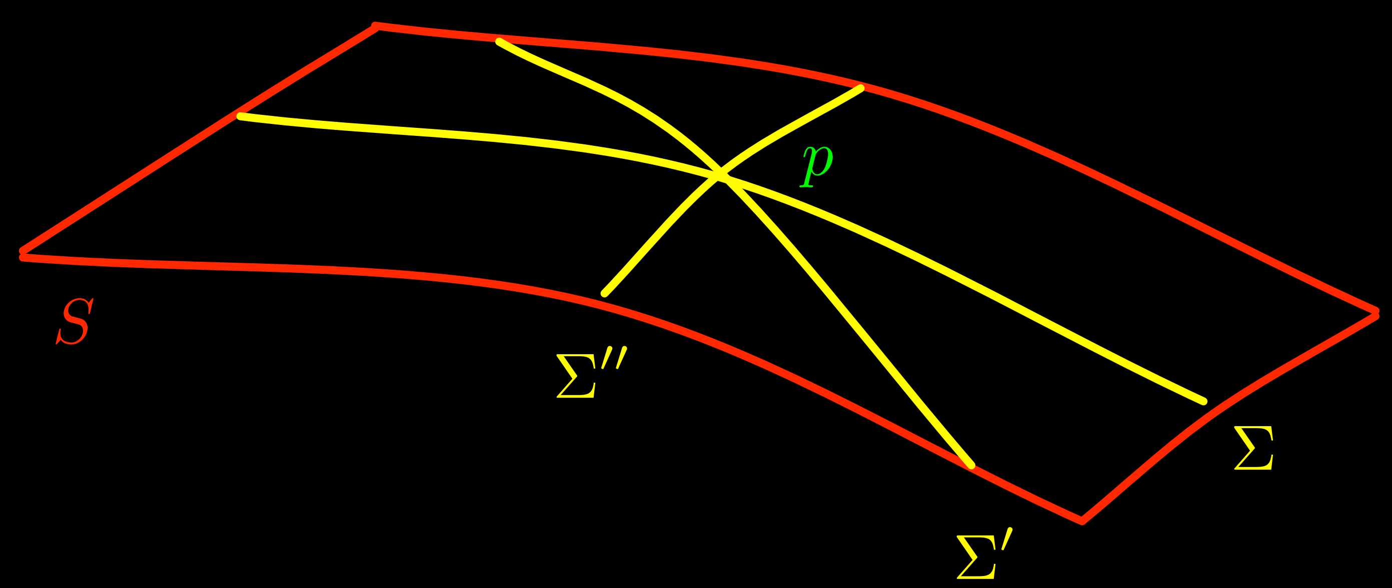

$$G_p \supset G_\Sigma \supset G_S$$

$\Rightarrow$ Yukawa coupling described by gauge theory living on S of gauge group G_p Higgsed down to G_S.

III. Commutative 7-Brane Theory

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$$\Rightarrow \quad \text{Rank } 1 \text{ Yukawa}$$

Superpotential

$$W = \int_S \mathrm{Tr}\left(\varphi \wedge F^{(0,2)}\right)$$

[Beasley-Heckman-Vafa, Donagi-Wijnholt]

$$F^{(0,2)} = \bar\partial \bar A + \bar A \wedge \bar A \quad , \quad \bar A = A^{(0,1)}$$

$$\varphi^{(2,0)} : \ \mathrm{adj}(G)$$

cf. D9 in pert. IIB $\rightarrow$ hol. C-S

[Witten]

$$W = \int_{CY_3} \mathrm{Tr}\left(\Omega \wedge (\bar A \bar\partial \bar A + \frac{2}{3} \bar A \wedge \bar A \wedge \bar A)\right)$$

dim. reduce $\quad \Omega \bar A_\perp \rightarrow \varphi^{(0,2)}$

F-term Eqn

$$
\begin{aligned}
F^{(0,2)} &= \bar{\partial}\bar{A} + \bar{A} \wedge \bar{A} = 0 \\
\bar{\partial}_A \varphi &= 0
\end{aligned}
$$

F-term Eqn

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Complex Gauge Equivalence

$$\bar{A} \to g^{-1}\bar{A}g + g^{-1}\bar{\partial}g \quad , \quad \varphi \to g^{-1}\varphi g$$

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~~D-term Eqn~~

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W independent of $g_{i\bar{j}},\ A^{(1,0)},\ \bar{\varphi}$

Complex Gauge Equivalence

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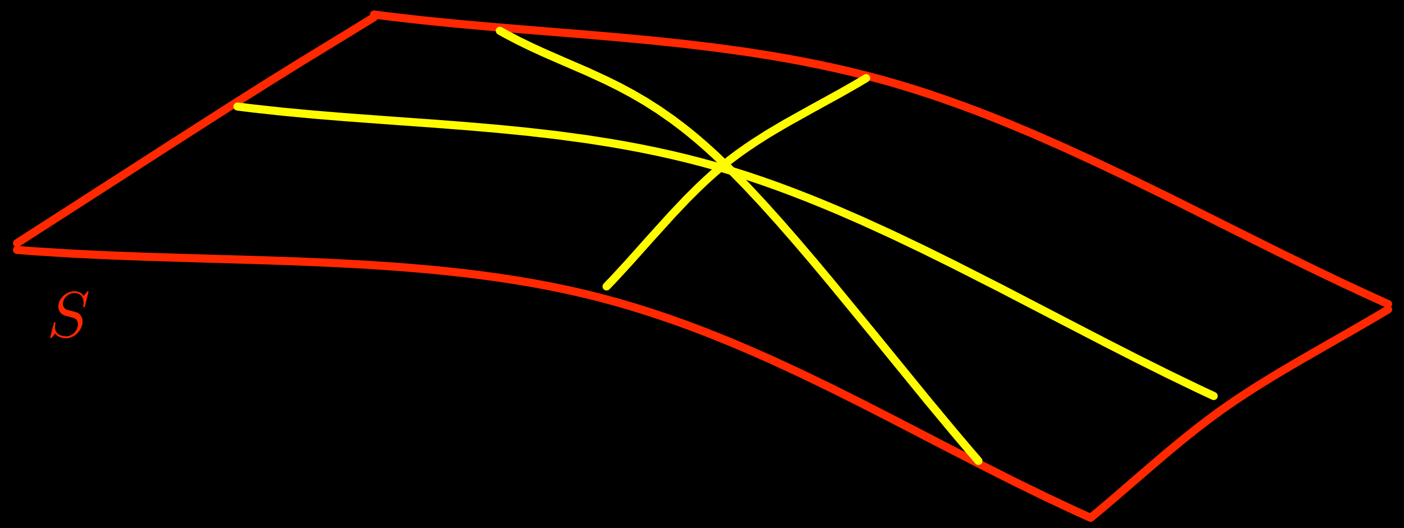
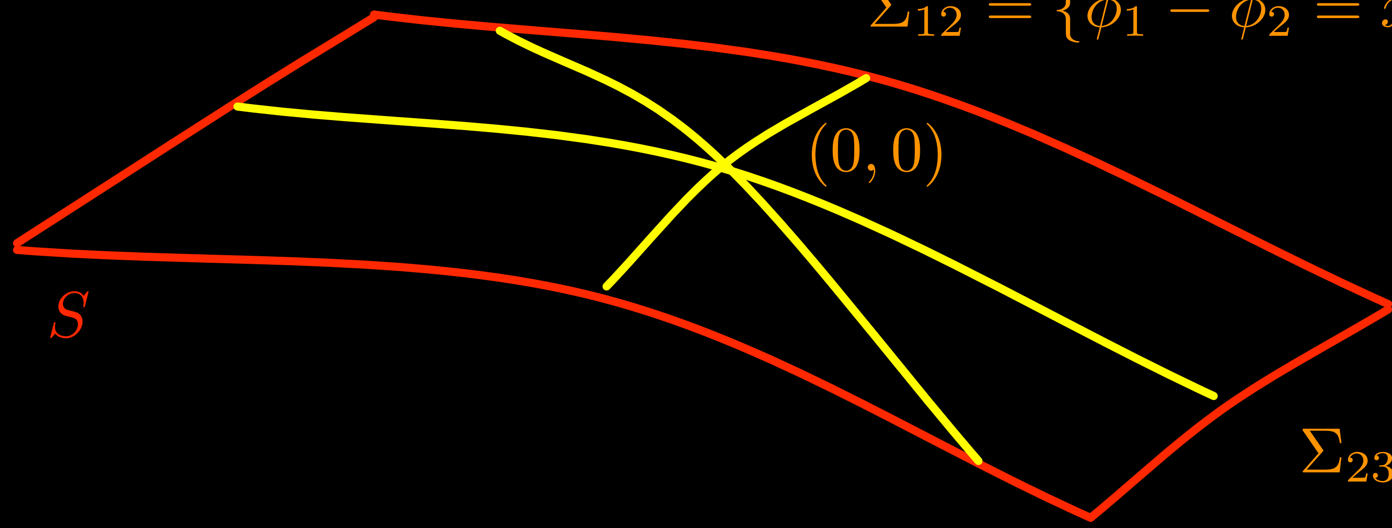
$$W = \int \mathrm{Tr}\left(\varphi \wedge F^{(0,2)}\right) = \int \mathrm{Tr}\left(\varphi^{(1)} \wedge \bar{A}^{(1)} \wedge \bar{A}^{(1)}\right) \neq 0$$

$\Leftrightarrow$ obstruction to deformation



assume transverse intersection $\Rightarrow$ Yukawa integral localized at one point $\Rightarrow$ local patch $\mathbb{C}^2$

(x, y)



$$\Sigma_{12} = \{\phi_1 - \phi_2 = x = 0\}$$

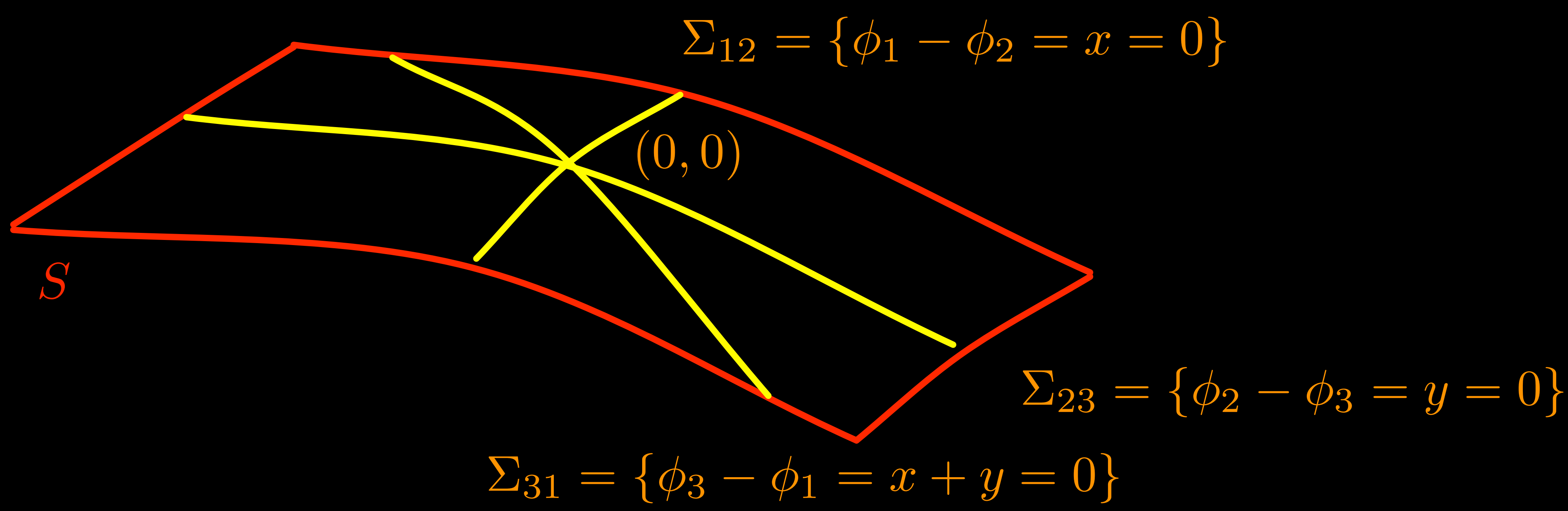
$$\Sigma_{23} = \{\phi_2 - \phi_3 = y = 0\}$$

$$\Sigma_{31} = \{\phi_3 - \phi_1 = x + y = 0\}$$

assume transverse intersection $\Rightarrow$ Yukawa integral localized at one point $\Rightarrow$ local patch $\mathbb{C}^2$ (x,y)

eg. G = U(3)

$$\bar{A}^{(0)} = 0 \quad , \quad \varphi^{(0)} = \begin{pmatrix} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & \phi_3 \end{pmatrix} dx \wedge dy = \begin{pmatrix} x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -y \end{pmatrix} dx \wedge dy$$



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Note: turning on background $F^{(1,1)}$ won't change W!

F-term zero-mode eqn $\Rightarrow$

$$\bar{A}^{(1)}_{ab} = \bar{\partial}\left(\frac{\phi^{(1)}_{ab} - h_{ab}}{\phi_a - \phi_b}\right) \quad , \quad \phi^{(1)}_{ab}\Big|_{\Sigma_{ab}} = h_{ab}\Big|_{\Sigma_{ab}} \quad \text{hol.}$$

Gauge-inequivalent zero-modes given by

$\mathcal{O}/\{\phi_a - \phi_b\}$, namely $h_{ab} \sim h_{ab} + \tilde{h}_{ab}\,(\phi_a - \phi_b)$

captured by a <u>cohomology</u> defined for given background!

Yukawa = pairing between cohomology classes

Yukawa as Residues

for example, take $h_{31} = 1$ (Higgs field)

$$W^{(ij)} = \oint dx \oint dy \, \frac{h_{12}^{(i)}\, h_{23}^{(j)}}{x\, y}$$

$$= 0 \quad \text{unless} \quad h_{12}^{(i)},\ h_{23}^{(j)} \quad \text{both constants}$$

$$\Rightarrow \text{Rank } 1$$

in general $\text{Rank} = \sharp\left(\Sigma_{12} \cap \Sigma_{23}\right)$

NEED BETTER!!

Need to consider $H = H_R + \tau H_{NS}$

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$\Rightarrow$ Non-Commutativity in Geometry

$$\begin{aligned} [X^i, X^j] &= \theta^{ij}(X) \longleftarrow \text{holomorphic, Poisson} \\ \nabla_k \theta^{ij} &= H^{ij}{}_k \end{aligned}$$

How to understand this Non-Commutativity in Open Sector?

(pert. IIB, flat space, R-flux)

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Myer's Effect

$$V_{D0}^{flux} = \mathrm{Tr}(X^I X^J X^K) G_{0IJK} + \cdots$$

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T-dual

$$\begin{aligned} V_{D3}^{flux} &= \mathrm{Tr}(X^j X^{\bar{j}} X^{\bar{k}}) H_{i\bar{j}\bar{k}} + \cdots \\ \Rightarrow \nabla_k \theta^{ij} &= H^{ij}{}_k + \cdots \end{aligned}$$

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deforming B-model by elements of $H^0(\Lambda^2 T^{1,0})$ [Kapustin, K-Li]

$$\Omega \rightarrow \Omega + \theta' \quad , \quad \theta'_k = \theta^{ij}\Omega_{ijk}$$

integrability of the generalized complex structure

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open B-model amplitude $\Rightarrow$ N-C with θ [Pestun, Cattaneo-Felder]

relation between generalized Kähler geometry and world-sheet B-field fluxes [Gates,Hull, Rocek]

$$\Rightarrow \nabla\theta = H + \cdots$$

(pert. IIB)

Bulk H-fluxes

$\Rightarrow$ N-C deformation of 7-brane Gauge Theory

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Q: How does Yuk get deformed?

non-commutativity of surface S

$$[x, y] = \theta(x, y) \quad \text{on local patch} \quad \mathbb{C}^2$$

constant $\theta \longrightarrow$ Moyal $\star$ product

$$\begin{aligned} f \star g &= e^{\theta^{ij}\partial_i \wedge \partial_j} f \otimes g \\ &= fg + \theta^{ij}\partial_i f \partial_j g + \mathcal{O}(\theta^2) \end{aligned}$$

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For general $\theta^{ij}\partial_i \wedge \partial_j = V_1 \wedge V_2 \quad , \quad [V_1, V_2] = 0$
covariantize to $\circledast$ for general $\theta(x,y)$ and
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pert. series in θ

deformed Superpotential: $\wedge \longrightarrow \circledast$

$$W = \int_S \mathrm{Tr}\Big(\varphi \circledast (\bar\partial \bar{A} + \bar{A} \circledast \bar{A})\Big)$$

complex gauge transformation

$$\bar{A} \rightarrow g^{-1\circledast} \circledast \bar{A} \circledast g + g^{-1\circledast} \circledast \bar\partial g \quad , \quad \varphi \rightarrow g^{-1\circledast} \circledast \varphi \circledast g$$

$$g^{-1\circledast} \circledast g = g \circledast g^{-1\circledast} = 1 \quad , \quad g^{-1\circledast} = g^{-1} + \mathcal{O}(\theta)$$

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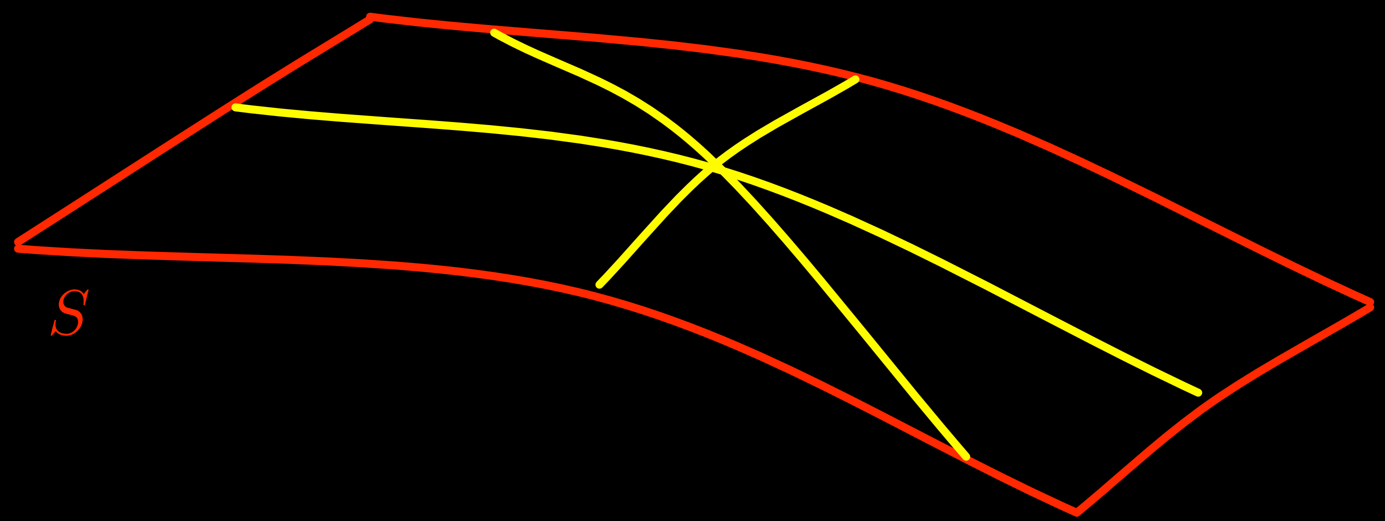
$\bar{A}, \varphi$ take value now in Universal Enveloping Algebra

Matter Curves $\Rightarrow$ Fuzzy Matter Curves



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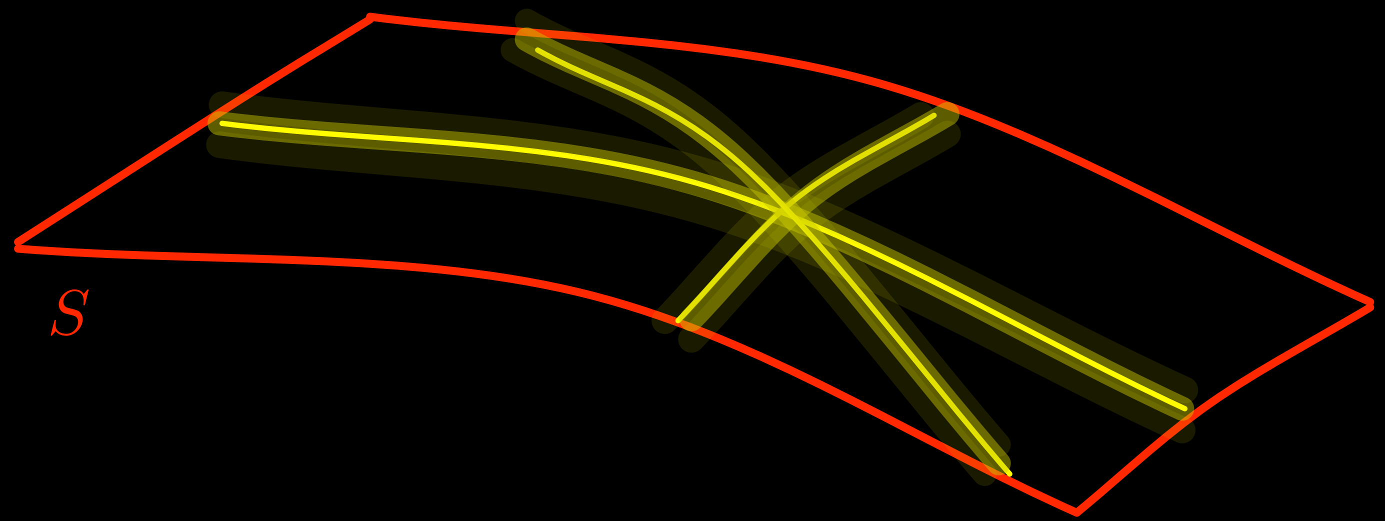

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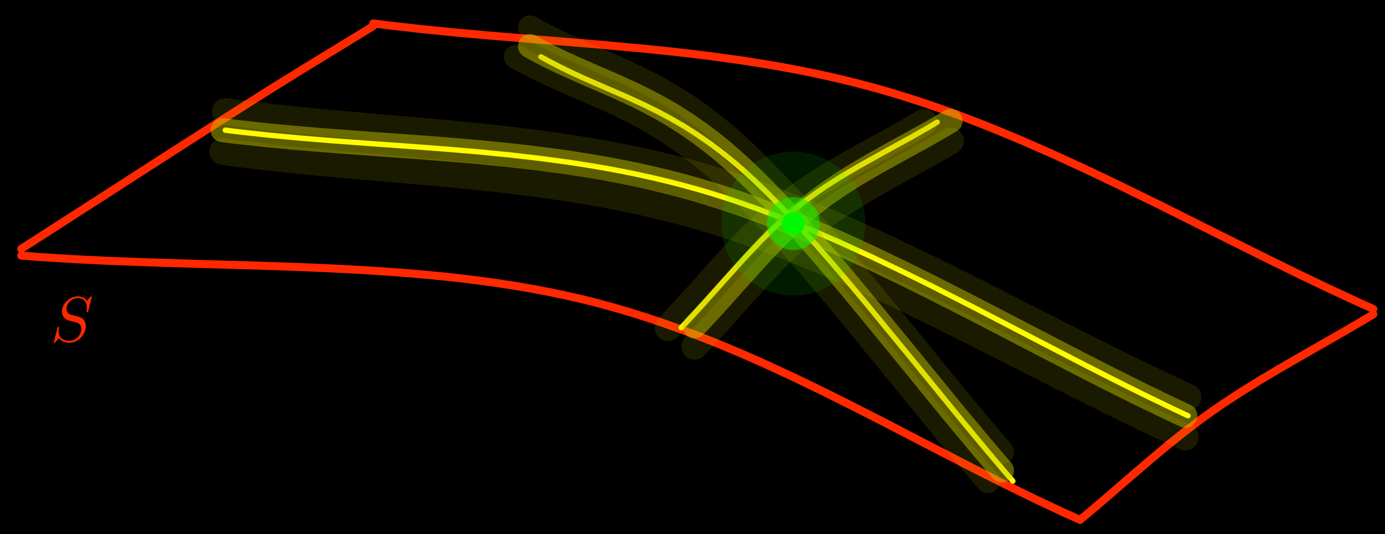

Yukawa no longer localized at one point

$\Rightarrow$ Higher Rank!

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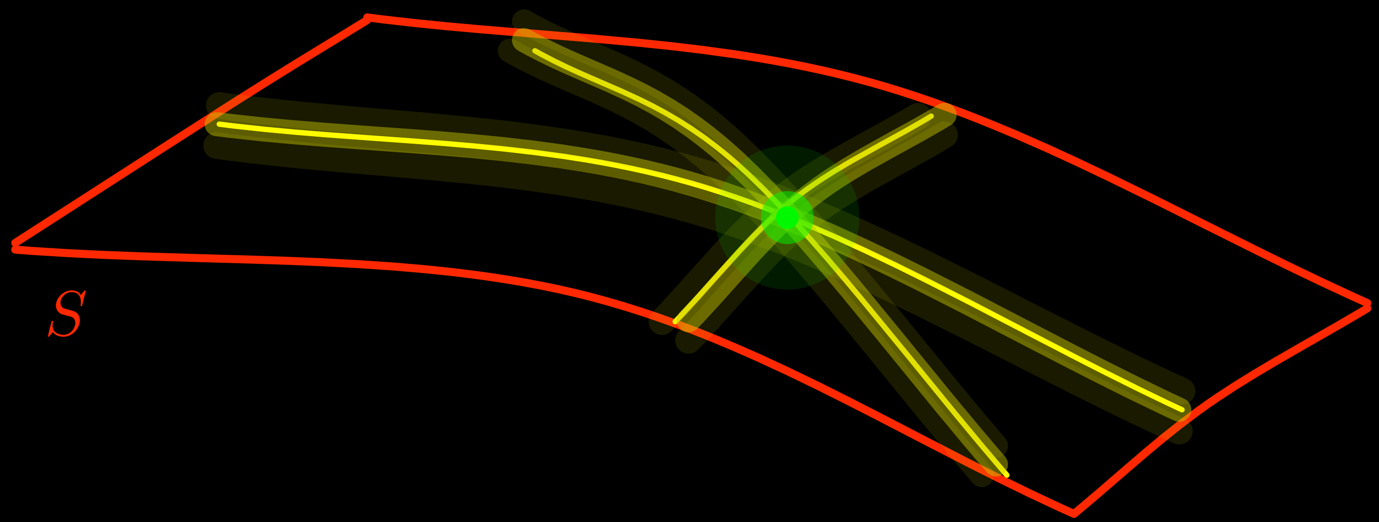

Yukawa no longer localized at one point
$\Rightarrow$ Higher Rank!

Still topological: pairing of a <u>quantum cohomology</u>

Yukawa as Quantum Residues

$$\begin{aligned} \theta &= 0 \\ W^{(ij)} &= \oint dx \oint dy \, \frac{h_{12}^{(i)} \, h_{23}^{(j)}}{x \, y} \end{aligned}$$

Yukawa as Quantum Residues

$$\theta \neq 0$$

$$W^{(ij)} = \oint dx \oint dy \, \frac{h_{12}^{(i)} \, h_{23}^{(j)}}{x \, y} + \mathcal{O}(\theta)$$

$$= \oint \oint \frac{h_{12}^{(i)}}{[\varphi^{(0)},]_{12,*}} \circledast \frac{h_{23}^{(j)}}{[\varphi^{(0)},]_{23,*}} \circledast (dx \wedge dy)$$

$$\varphi_1^{(0)} \circledast \frac{h_{12}^{(i)}}{[\varphi^{(0)},]_{12,*}} - \frac{h_{12}^{(i)}}{[\varphi^{(0)},]_{12,*}} \circledast \varphi_2^{(0)} = h_{12}^{(i)} \quad , \quad \frac{h_{12}^{(i)}}{[\varphi^{(0)},]_{12,*}} = \frac{h_{12}^{(i)}}{x} + \mathcal{O}(\theta)$$

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Maximal Rank, Hierarchy Given by Fluxes!

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Natural way to get rank one+corrections. Might be relevant for F-theory phenomenology.

In particular, if N-C vanishes at intersecting point, numerology for CKM matrix very satisfactory.

But this should be taken with a grain of salt.

Open Question

- Exceptional group?
- What structure of N-C is imposed on us?
- $SL(2,\mathbb{Z})$ monodromy
- Non-transversal intersection?