Consistency of

$\mathbb{Z}_M$ orbifold conformal field theories on K3

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$\boxed{\mathcal{M}: \text{moduli space of } N=(4,4) \text{ SCFTs with } c=6}$

local structure

[Narain '86; Seiberg '88; Cecotti '91]

global structure

[Narain '86; Aspinwall / Morrison '94; Nahm / W. '99]

special points and subvarieties

- Gepner models
- nonlinear σ-models
  - toroidal theories
  - orbifold CFTs

Local description of the moduli space

[Narain '86; Seiberg '88, Cecotti '91]

$$\boxed{\begin{array}{l} \mathcal{T}^{4,4+\delta} = O^+(4,4+\delta;\mathbb{R})/SO(4)\times O(4+\delta) \\ \text{Grassmannian of oriented positive definite} \\ \text{fourplanes in } \mathbb{R}^{4,4+\delta} \end{array}}$$

$O^+(a,b;\mathbb{R}) \subset O(a,b;\mathbb{R})$ components containing $SO(a)\times O(b)$

modular properties of

conformal field theoretic elliptic genus $\mathcal{E}^\delta$

[Schellekens/Warner '86]

[Nahm/W. '99] $\Downarrow$

$$\boxed{\left.\begin{array}{c}\delta=0\\ \text{torus } X\end{array}\right\} \quad \text{or} \quad \left\{\begin{array}{l}\delta=16\\ \text{K3 surface } X\end{array}\right.} \qquad \boxed{\mathcal{M} = \mathcal{M}^{\text{torus}} \cup \mathcal{M}^{K3}}$$

$\mathcal{E}^\delta = \mathcal{E}_X$ elliptic genus of X

[Eguchi/Ooguri/Taormina/Yang '89]

local structure

[Aspinwall/Morrison '94]

$$\boxed{O^+(H^{even}(X,\mathbb{R}))/SO(4)\times O(4+\delta) = \mathcal{T}^{4,4+\delta} \cong \mathcal{T}^{3,3+\delta} \times \mathbb{R}^+ \times H^2(X,\mathbb{R})}$$

$$x \longmapsto (\underbrace{\Sigma,\ V,\ B}_{\text{geometric interpretation}})$$

$$\boxed{x = \mathrm{span}_{\mathbb{R}}\left(\xi(\Sigma),\ v^0 + B + \left(V - \frac{B^2}{2}\right)v\right)}$$

$\xi(\sigma) = \sigma - \langle \sigma, B\rangle v \quad$ for $\sigma \perp v, v^0$

v, v^0 : primitive null vectors, $\langle v, v^0\rangle = 1$
in the even self-dual lattice $H^{even}(X,\mathbb{Z})$
v, v^0 generate $H^4(X,\mathbb{Z})$, $H^0(X,\mathbb{Z})$

global structure

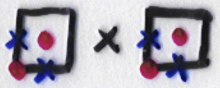
[Narain '86; Aspinwall/Morrison '94]

$$\boxed{\mathcal{M} = O^+(H^{even}(X,\mathbb{Z}))\backslash O^+(H^{even}(X,\mathbb{R}))/SO(4)\times O(4+\delta)}$$

- „classical symmetries" which fix v, v^0 : $O^+(H^2(X,\mathbb{Z}))$
- shifts of B by $\lambda \in H^2(X,\mathbb{Z})$: $(v, v^0) \mapsto (v, v^0 + \lambda - \frac{\lambda^2}{2} v)$

* Fourier-Mukai transform $v \leftrightarrow v^0$ [Nahm/W. '99]

$\mathbb{Z}_M$ orbifold constructions of K3

	fixed pts.	exc. div.	$h^{1,1}(\widetilde{T^4/\mathbb{Z}_M})$	Kummer type lattice
$\mathbb{Z}_2$	16 $\mathbb{Z}_2$ type 	$\{E_i \mid i\in\mathbb{F}_2^4\}$ $=\mathcal{E}_2$	$4+16\cdot 1=20$	$\Pi=\mathrm{span}_{\mathbb{Z}}\{\mathcal{E}_2;\ \frac{1}{2}\sum_{i\in H}E_i,$ $H\subset\mathbb{F}_2^4$ hyperplane$\}$
$\mathbb{Z}_3$	9 $\mathbb{Z}_3$ type 	$\{E_t^{(\pm)} \mid t\in\mathbb{F}_3^2\}$ $=\mathcal{E}_3$	$2+9\cdot 2=20$	$E_t:=E_t^{(+)}-E_t^{(-)},\ t\in\mathbb{F}_3^2$ $\Pi=\mathrm{span}_{\mathbb{Z}}\{\mathcal{E}_3;\ \frac{1}{3}(\sum_{t\in L_1}E_t-\sum_{s\in L_2}E_s),$ $L_1,L_2\subset\mathbb{F}_3^2$ parallel planes$\}$
$\mathbb{Z}_4$	4 $\mathbb{Z}_4$ type 6 $\mathbb{Z}_2$ type 	$\{E_i^{(\pm,0)} \mid i\in I^{(4)};$ $E_i \mid i\in I^{(2)}\}$ $=\mathcal{E}_4$	$2+4\cdot 3+6\cdot 1=20$	$E_i:=3E_i^{(+)}+2E_i^{(0)}+E_i^{(-)},\ i\in I^{(4)}$ $\Pi\mathrm{I}=\mathrm{span}_{\mathbb{Z}}\{\mathcal{E}_4;$ $\frac{1}{4}(E_{0000}+E_{1111}-E_{0011}-E_{1100})+\frac{1}{2}(E_{0101}+E_{0110})$ $\frac{1}{2}(E_{0000}+E_{0011}+E_{0100}+E_{0111}+E_{0101}+E_{0110}),$ $\frac{1}{2}(E_{1100}+E_{0011}+E_{0001}+E_{0100}+E_{1101}+E_{0111})\}$
$\mathbb{Z}_6$	1 $\mathbb{Z}_6$ type 4 $\mathbb{Z}_3$ type 5 $\mathbb{Z}_2$ type 	$\{E_0^{(k)} \mid k\in\{1,\dots,5\};$ $E_t^{(\pm)} \mid t\in I^{(3)};$ $E_i \mid i\in\tilde{I}^{(2)}\}$ $=\mathcal{E}_6$	$2+1\cdot 5+4\cdot 2+5\cdot 1=20$	$E_0:=E_0^{(1)}+2E_0^{(2)}+\dots+5E_0^{(5)}$ $\overline{\mathrm{IV}}=\mathrm{span}_{\mathbb{Z}}\{\mathcal{E}_6;$ $\frac{1}{6}E_0+\frac{1}{3}(E_{1,-1}+E_{1,0}+E_{0,1}+E_{1,1})$ $+\frac{1}{2}(E_{0011}+E_{1100}+E_{1001}+E_{0110}+E_{0101})\}$

Result

Consistency of $\mathbb{Z}_M$ orbifold conformal field theories enforces

$$x_T = \mathrm{span}_{\mathbb{R}}\left(\xi(\Sigma_T),\ v^0 + B_T + \left(V_T - \tfrac{B_T^2}{2}\right)v\right)$$

$$\downarrow$$

$$\pi_* x_T = \mathrm{span}_{\mathbb{R}}\left(\hat{\xi}(\Sigma),\ \hat{v}^0 + B + \left(V - \tfrac{B^2}{2}\right)\hat{v}\right)$$

where $\Sigma = \pi_* \Sigma_T$

$B = \frac{1}{\sqrt{M}} B_T + \frac{1}{M} B_{\mathbb{Z}}^{(M)}$,

$\langle B_{\mathbb{Z}}^{(M)}, E\rangle = \frac{M}{m}$ for E corresponding to a $\mathbb{Z}_m$ type fixed point

$V = \frac{V_T}{M}$

see also: $\mathbb{Z}_2$: [Aspinwall '95]

[Douglas '96; Blum/Intriligator '97]