# A STORY OF K3

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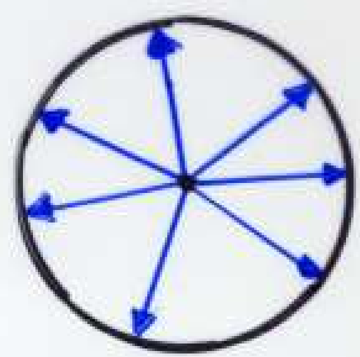
## Malus-Dupin rule (~1800)

A system of light rays emerging from a point source, which is reflected or refracted an arbitrary number of times in isotropic media, remains the system of normals to the wave surface.



# The discovery of K3 in geometric optics I

~1800 Malus-Dupin rule on systems of light rays in isotropic media

1832 Sir W.R. Hamilton „The Theory of Rays"
Transactions of the Royal Irish Academy

Oct 22, 1832 Third Supplement: Conical Refraction in Biaxal Crystals

Dec 14, 1832 experimental confirmation by Humphrey Lloyd

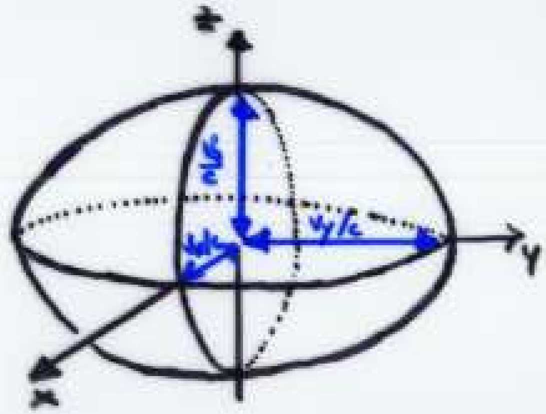
1862 Sir W.R. Hamilton:
„On some quaternionic equations connected with Fresnel's wave surface for biaxal crystals"
Proc. of the Royal Academy (1862), pp 122-124, 163

# Fresnel Wave Equation I

electromagnetic displacement: $D = \varphi E = E + 4\pi P$

$E$: electric intensity
$\varphi$: dielectric constant
$P$: polarization

## Fresnel index ellipsoid: $1 = E \cdot D = E \cdot \varphi E$




$v_x, v_y, v_z$:
principal velocities of light

energy flow per second and unit area:

$\Pi = \frac{c}{4\pi} E \times H$ Poynting vector

$H$: magnetic intensity

# Fresnel Wave Equation II

Wave emitted from a point source in a crystal:

$$D = \varphi E = \left( \frac{c^2}{v_x^2} E_x , \frac{c^2}{v_y^2} E_y , \frac{c^2}{v_z^2} E_z \right)$$

Maxwell equations:

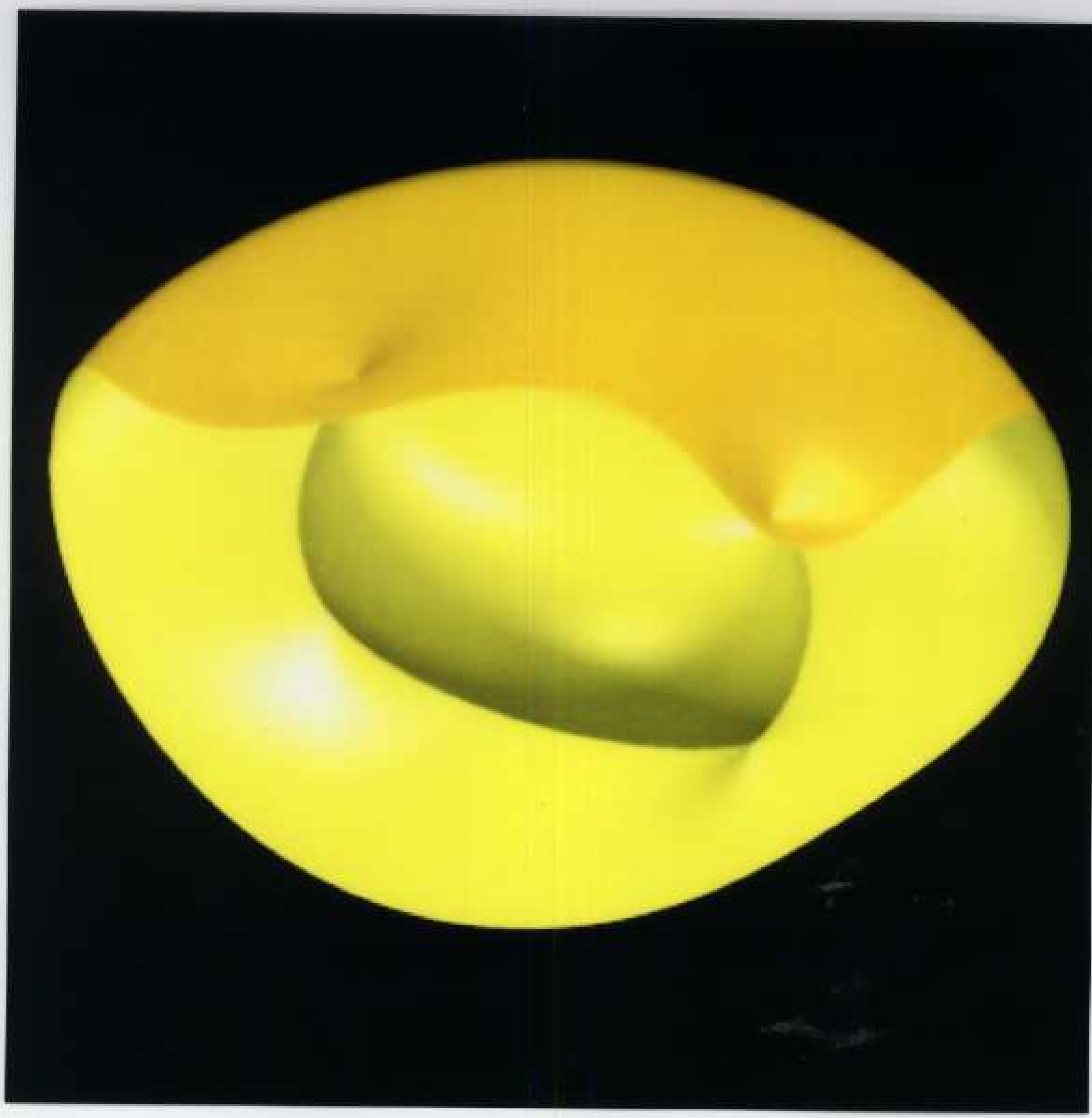
$$\operatorname{div} D = 0 \qquad \Big| \qquad \operatorname{curl} H = \frac{1}{c} \frac{\partial D}{\partial t}$$

$$\operatorname{div} H = 0 \qquad \Big| \qquad \operatorname{curl} E = -\frac{1}{c} \frac{\partial H}{\partial t}$$

energy flow $\Pi = \frac{c}{4\pi} E \times H = (x, y, z)$

$$E \cdot \Pi = 0 \quad \Longleftrightarrow \quad \boxed{\begin{aligned} &(v_x^2 x^2 + v_y^2 y^2 + v_z^2 z^2)(x^2 + y^2 + z^2) \\ &- v_x^2 (v_y^2 + v_z^2) x^2 - v_y^2 (v_x^2 + v_z^2) y^2 - v_z^2 (v_x^2 + v_y^2) z^2 \\ &+ v_x^2 v_y^2 v_z^2 = 0 \end{aligned}}$$

# Fresnel Wave Surface



taken from: http://www.oliverlabs.net/algebraicsurface/no_10.html, by Oliver Labs, University of Mainz

# The discovery of K3 in geometric optics II

1860 E. Kummer: „Allgemeine Theorie der gradlinigen Strahlensysteme"
[General Theory of systems of straight rays]
J. für reine und angewandte Mathematik 57 (1860), 189 - 320

1864 E. Kummer: „Über die Flächen vierten Grades mit sechzehn singulären Punkten"
[On the surface of degree four with sixteen singular points]
Monatsberichte der Königlichen Preußischen Akademie der Wissenschaften zu Berlin aus dem Jahre 1864, 246 - 260
18. April, Sitzung der physikalisch-mathematischen Klasse

1906-2000 E. Kähler

1915-1997 K. Kodaira

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K3 - surfaces

E. Kummer, 1864

„The Fresnel Wave Surface is a surface of degree four, with 16 singular points, four of which lie in one of the principal planes and are real, eight of which lie in the other principal planes and are imaginary, and the remaining four lie in a plane at infinity.

„Über die Flächen vierten Grades mit sechzehn singulären Punkten", Monatsberichte der Königlichen Preußischen Akademie der Wissenschaften zu Berlin aus dem Jahre 1864, 246–260.

[R.W.H.T. HUDSON: Kummer's Quartic Surface; Cambridge: at the University Press, 1905]

# Properties of K3-surfaces

think of $\quad x^4 + y^4 + z^4 + w^4 + Ax^2y^2 + \cdots \quad = 0$

with $\quad w=1$ or $\left\{\begin{matrix} z=1 \\ w=0 \end{matrix}\right\}$ or $\left\{\begin{matrix} y=1 \\ w=z=0 \end{matrix}\right\}$ or $\left\{\begin{matrix} x=1 \\ w=z=y=0 \end{matrix}\right\}$

$\longrightarrow$ object $X$ of complex dimension $4-1-1=\underline{2}$

$\boxed{\text{„K3-surface"}}$

can show: $X$ carries a metric $d^2s$ which

- is Euclidean to $2^{nd}$ order
  $d^2s = dx^2 + dy^2 + dz^2 + dw^2 + O(|x|^2+|y|^2+|z|^2+|w|^2)$
- solves Einstein's equation in the vacuum

$\Rightarrow$ CONSISTENT BACKGROUND FOR STRING PROPAGATION

☹ This metric is not explicitly known (unless $X$ has singularities).

# Why Strings?

? **unification**: standard model + general relativity ?

- existence of a graviton; at low energy: general relativity
- perturbatively: consistent theory of quantum gravity
- definite number of dimensions: space-time $\simeq \mathbb{R}^{1,9} = \mathbb{R}^{1,3} \times M$
- existence of gauge groups, e.g. $G_{SM} = SU(3) \times SU(2) \times U(1)$
- supersymmetry cures divergences
- parity asymmetry modelled
- no free parameters
- uniqueness

## Why no strings? (but CFT?)

- no experimental evidence
- what is string theory?
- too many classical vacua

space-time: $\mathbb{R}^{1,5} \times K3$

large volume

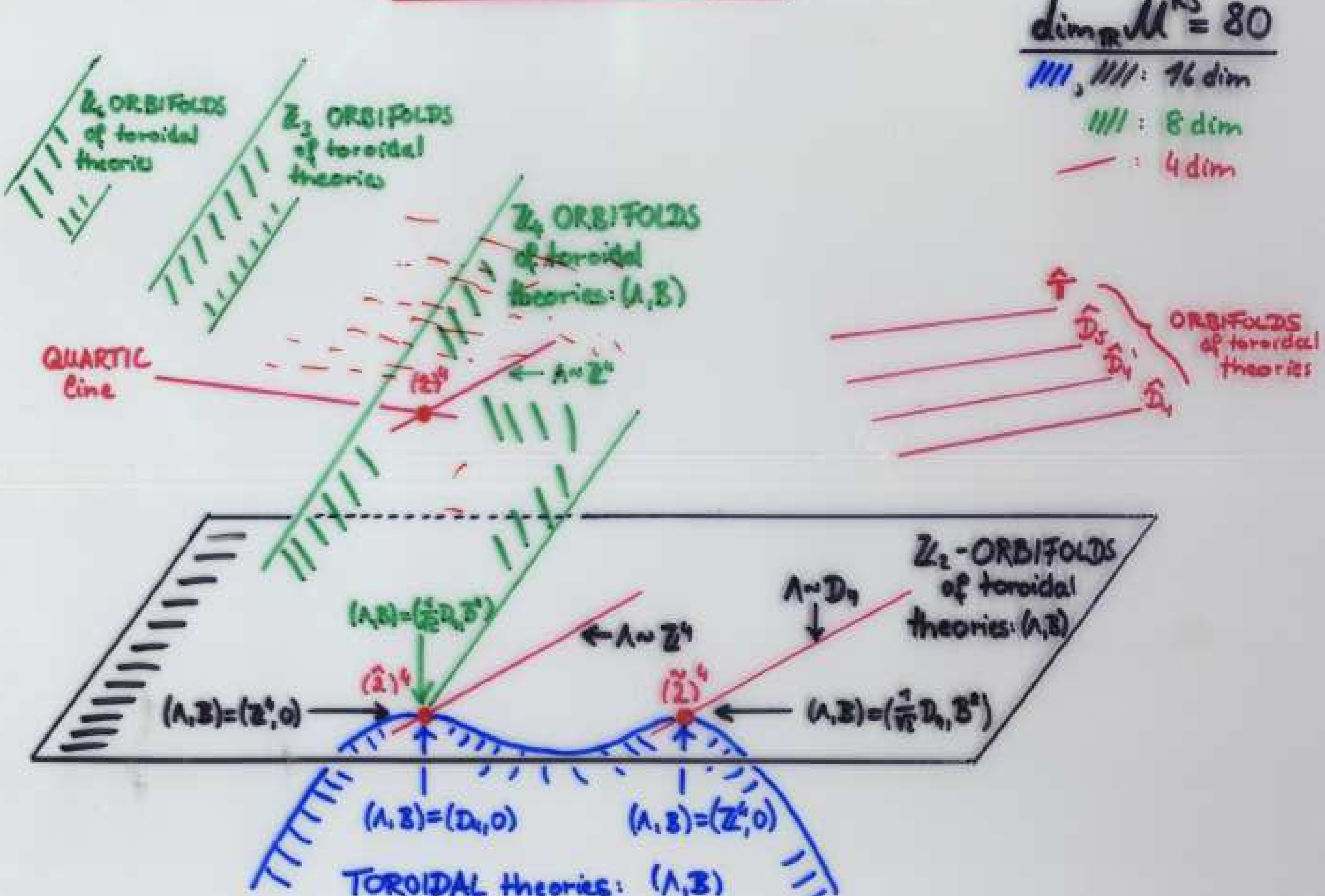
low energy

GEOMETRY

STATISTICAL MECHANICS

2-dimensional conformal field theory

# A Hiker's View of K3




[Nahm/W'99; Commun. Math. Phys. 216, 85-138 (2001);
W'00; Adv. Theor. Math. Phys. 5, 429-456 (2001)]