A Brief Introduction to Error Analysis

Error Analysis is a crucial part of a scientific study. Unfortunately, when first learning about error analysis, it doesn't help that it is poorly named. *Error* implies a mistake made by the experimenter that was completely in their control (like looking at one star when you were supposed to look at another), but this is not what is meant. Scientists don't knowingly do an experiment wrong and then write a paper on it. A better term to replace *error* is *uncertainty* (I'll try and always use *uncertainty*, but *error* is so common that I'll probably slip back to it now and then). Now that we're past the semantics, let's actually define uncertainty.

A source of uncertainty is something that interferes with an experiment and is totally out of the experimenter's control. The effect is that it makes you *less sure* of your result, so we use uncertainty analysis to evaluate just how much confidence we have in our answer. Scientists are never 100% certain of anything, even the things we take for granted. For example, if you've taken a basic physics or astronomy course, you probably have seen than the universal gravitational constant (G) is $6.67\text{x}10^{-11}$ $m^3/kg/s^2$. However, we only know its value is between $6.67361\text{x}10^{-11}$ and $6.67495\text{x}10^{-11}$ (more compactly written $6.67428\text{x}10^{-11}\pm0.00067\text{x}10^{-11}$). Even though we've been measuring this value for hundreds of years, we don't know it to 100% accuracy. In astronomy especially, the uncertainty tends to be a lot larger due to the fact that it's a completely observational science. We have almost no control over the stuff that lies between us and our source of interest, and unfortunately that stuff tends to get in the way.

There are two types of errors, *systematic* and *random*. A *systematic* uncertainty is something that causes a measurement to always be consistently different from the actual value. This could happen if you are timing something and you consistently start your timer late by a couple seconds. In astronomy, a big systematic uncertainty is caused by interstellar dust that constantly blocks a star's light. The problem with these types of systematic uncertainties is that since they are constant, they are hard to find. A simpler systematic uncertainty is that caused by the limitations of your measuring device. For example, a ruler only has tick marks down to 1/16 of an inch, and that's as precise a measurement as you can get. The same applies to a stopwatch that only reports time to 1/100 of a second, or a telescope that reports astronomical coordinates–called right ascension and declination–to only half a degree. The nice thing about these uncertainties is that it is usually easy to predict how large they are.

There are also *random* uncertainties. These are things that cause a value to change unpredictably each time your measure it. These can be caused by fluctuations in your measuring device, different interpretations of a measurement (like with an analog scale), or interference by something in the environment. A common example is measuring the length of a room with a meter stick. If I gave everyone a meter stick and asked them to find the length of the room, they would all get different answers. It's not that anyone did anything wrong, but the act of having to pick up the meter stick and move it constantly is a random source of error. In astronomy, the most obvious source of random error is the atmosphere, whose constant motion causes stars to brighten and dim.

A common way to remove random uncertainty is to take many measurements and average them, but this doesn't always work. Sometimes you have to change how you do your actual measurement.

For This Class

In real physics, uncertainty analysis involves calculus and statistics, neither of which I will put you through. The only uncertainty calculation you will have to often do is percent error:

$$PercentError = \frac{MeauredValue - AcceptedValue}{AcceptedValue} \times 100$$

This is only for when we have determined a specific value (the distance to Jupiter for example). Our main goal is to get you to be able to identify major sources of uncertainty, explain how they might affect the result, and then think of ways to correct or remove them. If I see people recycling the same uncertainties over and over, I might ask you to think of something else.

You should note that the planetarium lab requirements are slightly different. In those reports, you need two sources of uncertainty and they must be random uncertainties. If you choose to report a systematic uncertainty, you have to *quantify* it to get all the points. That is, you must give the specific numerical effect it has on your answer.