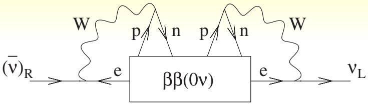
$\beta\beta$ Matrix Elements

J. Engel

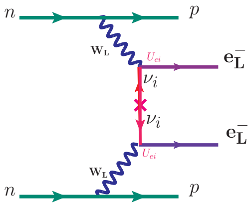
March 5, 2018

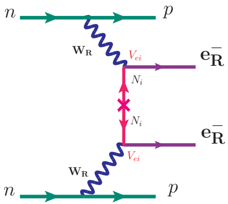
Intro: New Physics Can Contribute Directly to $\beta\beta$ Decay

If neutrinoless decay occurs then ν's are Majorana, no matter what:



but high-scale physics can contribute directly alongside light-neutrino exchange:





Exchange of heavy right-handed neutrino in left-right symmetric model.

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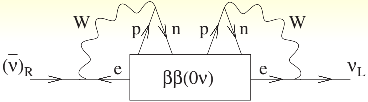

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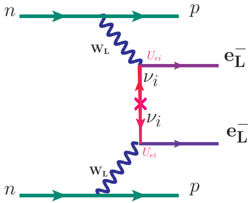

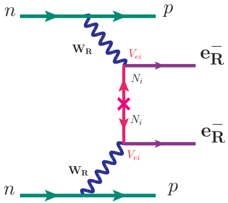

Exchange of heavy right-handed neutrino in left-right symmetric model.

Amplitude of "exotic" mechanism:

$$\frac{Z_{0\nu}^{\text{heavy}}}{Z_{0\nu}^{\text{light}}} \approx \left(\frac{M_{W_L}}{M_{W_R}}\right)^4 \left(\frac{\langle q^2\rangle_\nu}{m_{\beta\beta}\, m_N}\right) \qquad \langle q^2\rangle_\nu \approx 10^4\ \text{MeV}^2$$

$$\approx \mathbf{1} \quad \text{if} \quad m_N \approx m_{W_R} \approx 1\,\text{TeV} \quad \text{and} \quad m_{\beta\beta} \approx \sqrt{\Delta m^2_{\text{atm}}}$$

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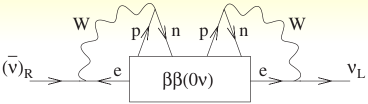

So exotic stuff can occur with roughly the same rate as inverted-hierarchy light-ν exchange would, even if the hierarchy is normal and light-ν exchange is unobservable.

If scale of new physics is far above a TeV, however, then one might not expect to see a signal.

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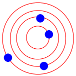
Nuclear Structure: Contrasting the Approaches

Starting point is always mean field(s)



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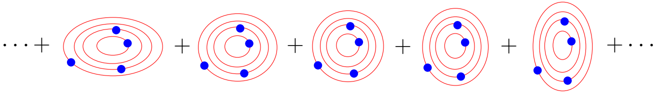

"Energy-Density Functional Theory" employs Generator-Coordinate Method (GCM), which mixes many such states with different collective properties.

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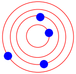

Other methods build on single independent-particle state.

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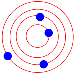

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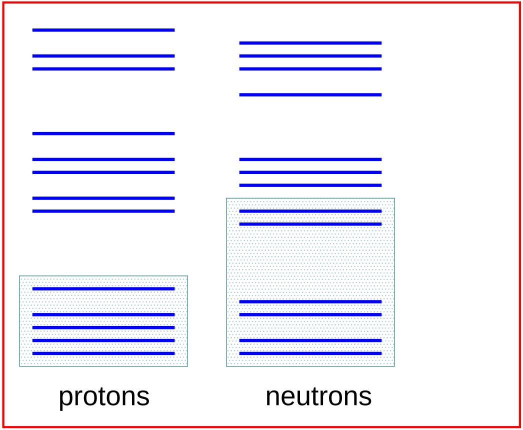

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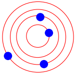

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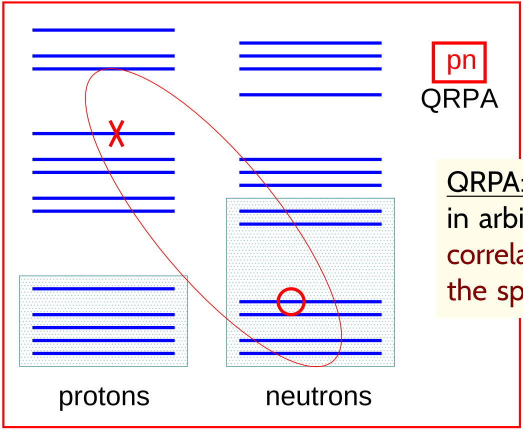

QRPA: Large single-particle spaces in arbitrary single mean field; simple correlations and excitations within the space.

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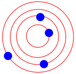

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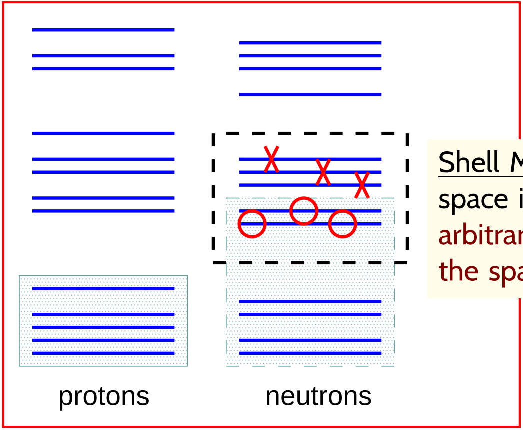

Shell Model: Small single-particle space in simple spherical mean field; arbitrarily complex correlations within the space.

Nuclear Structure: Contrasting the Approaches

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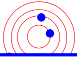
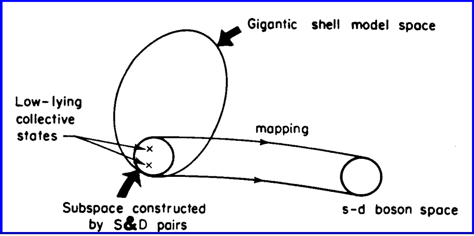
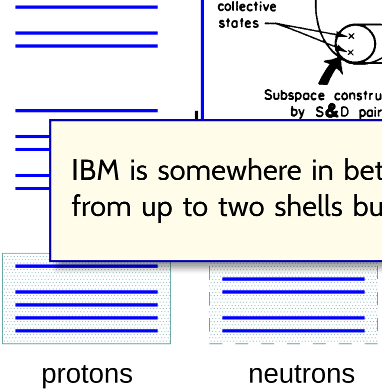


Other me… …ticle state.





…e
…field;
…within

IBM is somewhere in between, mapping matrix elements from up to two shells but truncating to collective pairs.

Nuclear Structure: Contrasting the Approaches

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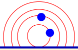
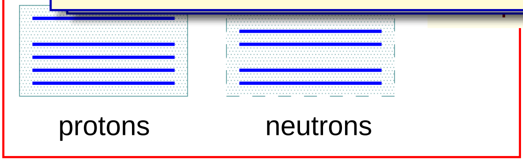


Goals of Nuclear Theory Topical Collaboration:

- Improve and combine these methods.
- Avoid fitting parameters to data directly in heavy nuclei.

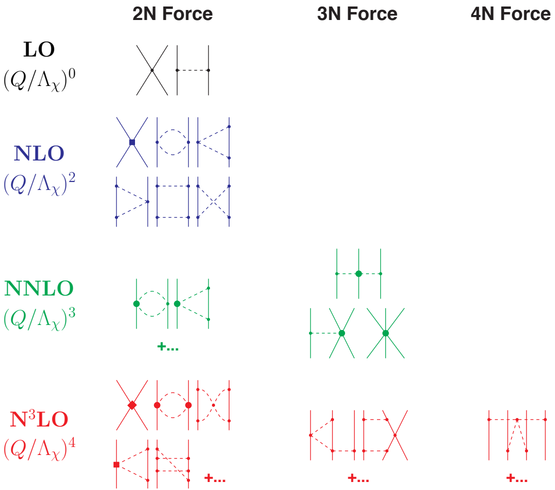
  Such fitting is not a bad thing, but makes it hard to estimate uncertainty when calculating something that's never been measured.



The Way Forward: Ab Initio Nuclear Structure

Often starts with chiral effective field theory.

Nucleons, pions sufficient below chiral-symmetry breaking scale.



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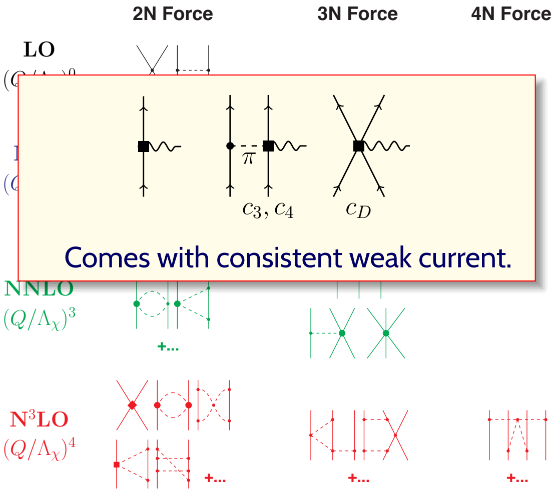
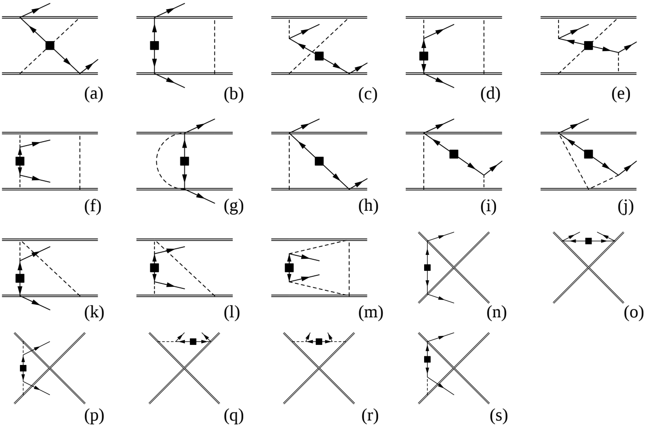


Brief Aside: χEFT Treatment of Light-ν Exchange

New work by Cirigliano et al.

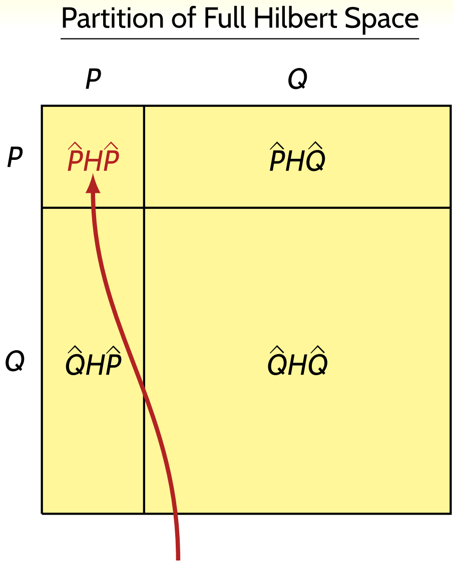
Corrections of order 10% to standard diagram:



Just starting to look at these.

Anyway..., back to the nuclear many-body problem.

Ab Initio Shell Model

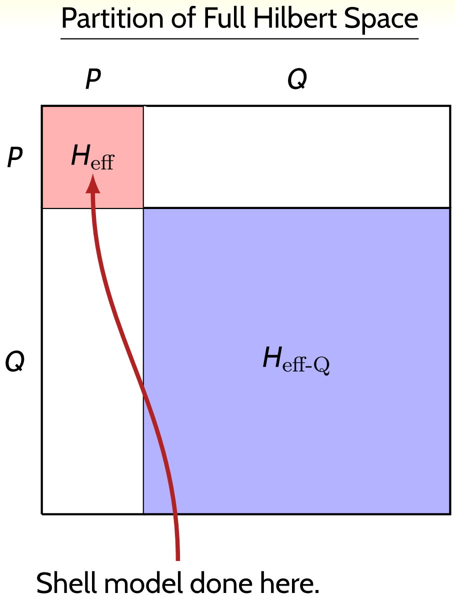


Shell model done here.

P = valence space
Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q, with H_{eff} in P reproducing d most important eigenvalues.

Ab Initio Shell Model



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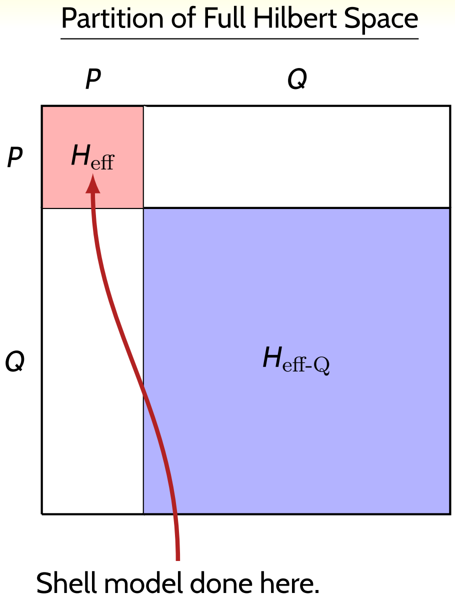

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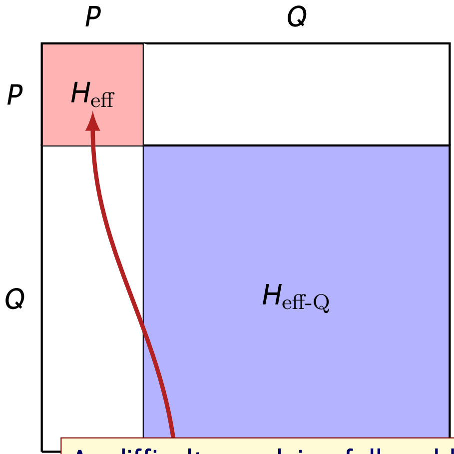
For transition operator $\widehat{M}$, must apply same transformation to get $\widehat{M}_{\mathrm{eff}}$.

Ab Initio Shell Model

Partition of Full Hilbert Space



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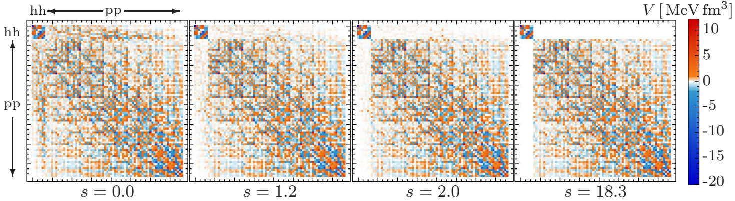
As difficult as solving full problem. But idea is that N-body effective operators may not be important for N $>$2 or 3.

In-Medium Similarity Renormalization Group

One way to determine the transfomation

Flow equation for effective Hamiltonian.
Asymptotically decouples shell-model space.

$$\frac{d}{ds}H(s) = [\eta(s), H(s)]\,, \qquad \eta(s) = [H_d(s), H_{od}(s)]\,, \quad H(\infty) = H_{\rm eff}$$



Hergert et al.

Trick is to keep all 1- and 2-body terms in H at each step *after normal ordering*.

If shell-model space contains just a single state, approach yields ground-state energy. If it is a typical valence space, result is effective interaction and operators.

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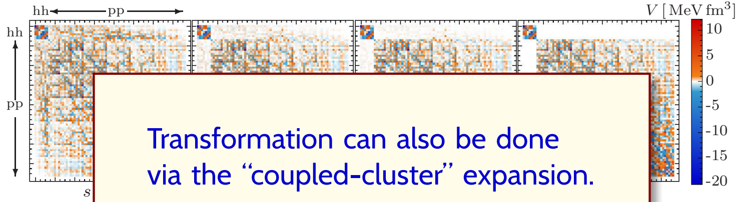

Transformation can also be done via the “coupled-cluster” expansion.

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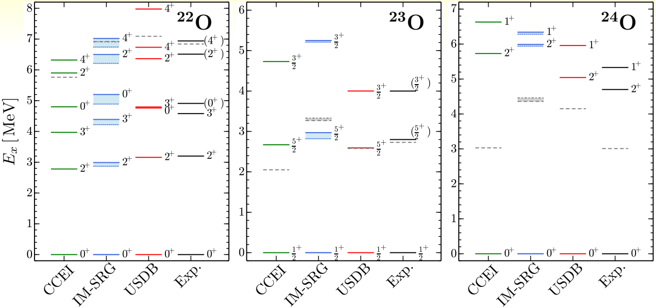
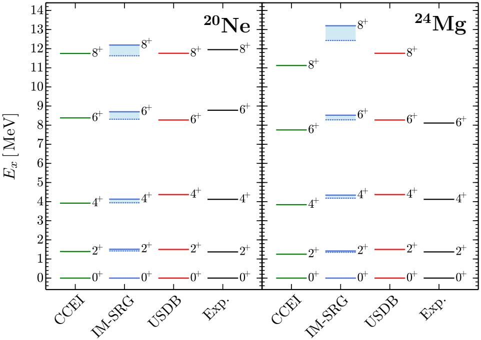
Ab Initio Calculations of Spectra



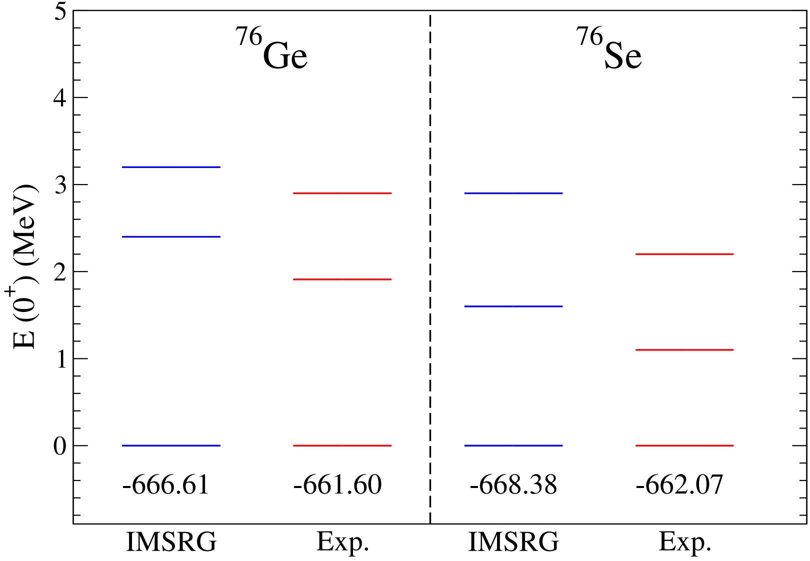
Neutron-rich oxygen isotopes

Deformed nuclei



Ab Initio ^{76}Ge and ^{76}Se

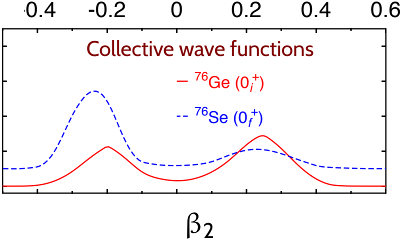
Stroberg, Holt, et al.



Complementary Density-Functional-Like Approach

Generator Coordinate Method (GCM)

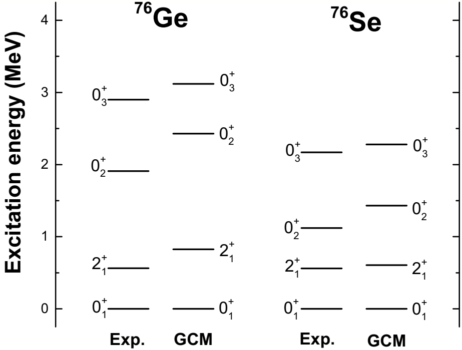
Construct set of mean fields by constraining coordinate(s), e.g. quadrupole moment $\langle Q_0 \rangle$. Then diagonalize H in space of states with different $\langle Q_0 \rangle$.



Rodriguez and Martinez-Pinedo:
Wave functions peaked at $\beta_2 \approx \pm .2$

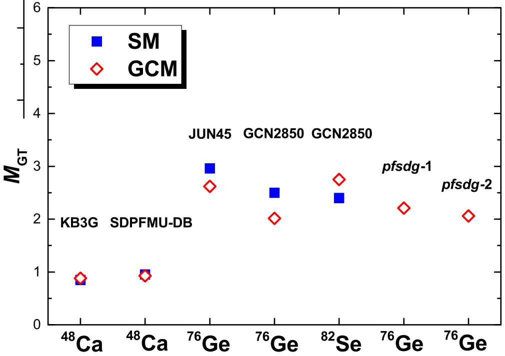
Can be improved by including crucial neutron-proton pairing amplitude as collective coordinate.

DFT Approach in Large Shell-Model Spaces



GCM Spectrum in 2 Shells

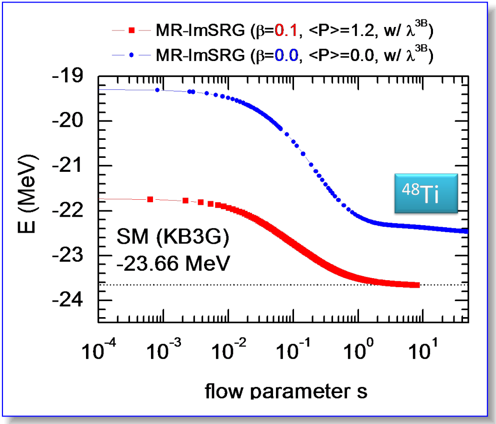
$0\nu\beta\beta$ Matrix Elements in 1 and 2 Shells



Combining GCM and Ab Initio Methods

GCM incorporates some correlations that are hard to capture automatically (e.g. shape coexistence). So use it to construct initial "reference" state, let IMSRG, do the rest.

Test in single shell



From J. Yao

Ab initio versions in progress.

Nuclear Matrix Element (Simplified)

$$\boxed{M_{0\nu} = g_A^2\, M_{0\nu}^{GT} - g_V^2\, M_{0\nu}^{F} + \ldots}$$

with

$$M_{0\nu}^{GT} = \langle f| \sum_{a,b} H_{GT}(r_{ab})\vec{\sigma}_a \cdot \vec{\sigma}_b \tau_a^+ \tau_b^+ \,|i\rangle$$

$$M_{0\nu}^{F} = \langle f|\,| \sum_{a,b} H_F(r_{ab}) \tau_a^+ \tau_b^+ \,|i\rangle$$

$$H_{GT}(r) \approx H_F(r) \approx \frac{R_{\text{nucl.}}}{r}$$

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$$M_{0\nu}^{F} = \langle f | \, | \sum_{a,b} H_F(r_{ab}) \tau_a^+ \tau_b^+ \, | i \rangle$$

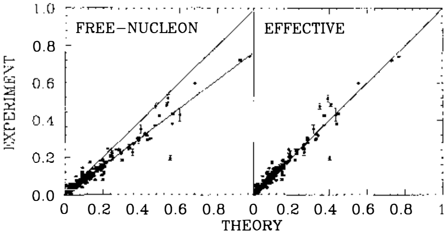
$$H_{GT}(r) \approx H_F(r) \approx \frac{R_{\text{nucl.}}}{r}$$

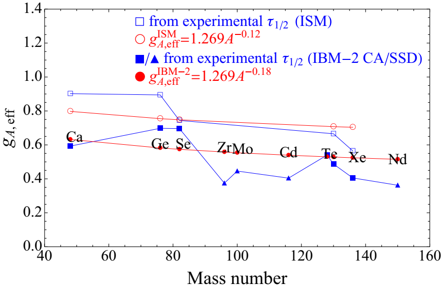
Also:

$$M_{2\nu} = g_A^2 \sum_m \frac{\langle f | \sum_a \vec{\sigma}_a \tau_a^+ \, | m \rangle \cdot \langle m | \sum_b \vec{\sigma}_b \tau_b^+ \, | i \rangle}{E_m - \frac{E_f + E_i}{2}}$$

Issue Facing All Models: "g_A"

40-Year-Old Problem: Effective g_A needed for single-beta and two-neutrino double-beta decay in shell model and QRPA.





If 0ν matrix elements quenched by same amount as 2ν matrix elements, experiments will be much less sensitive; rates go like fourth power of g_A.

Early Arguments For and Against Major 0ν Quenching

For:

- β and $2\nu\beta\beta$ decay are quenched by consistent amounts (corresponding roughly to similar effective values for g_A).
- Quenching comes mainly from correlations that escape model space. No reason why this should affect 0ν less than 2ν.

Against:

- Quenching that comes from two-body currents should be less at high momentum transfer, at which 0ν decay occurs (because of the virtual neutrino).
- 0ν decay can be evaluated in "closure." Only one matrix element rather than product of two.

We Are Making Progress on the Issue

In modern terms, quenching must be due to some combination of:

1. Truncation of model space.

   Should be fixable in ab-initio shell model, which compensates effects of truncation via effective operators.

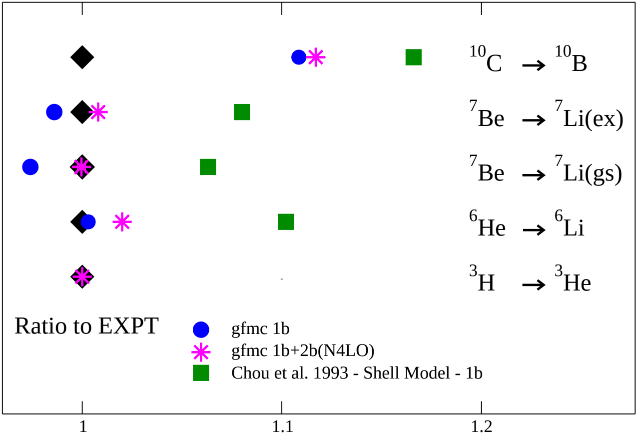
2. Many-body weak currents.

   Size still not clear, particularly for $0\nu\beta\beta$ decay, where current is needed at finite momentum transfer q.

   Leading terms in chiral EFT for finite q recently worked out. Careful fits and use in decay computations underway.

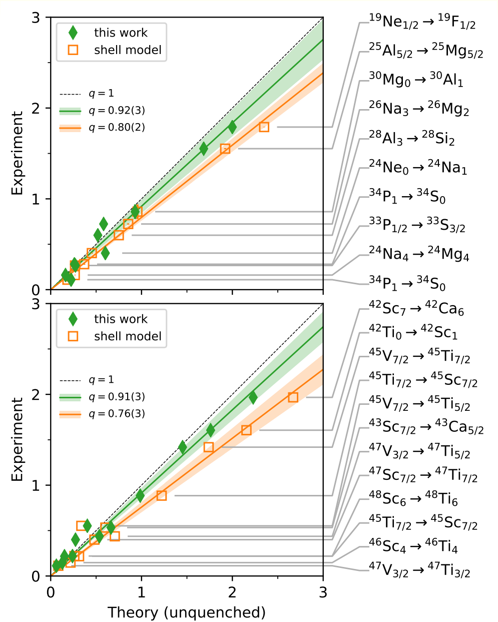
Quenching in Light Nuclei

Pastore et al: β Decay with Quantum Monte Carlo



Most of the effects are from correlations outside the valence shell.
Two-body currents don't do much.

Quenching in the *sd*-Shell…



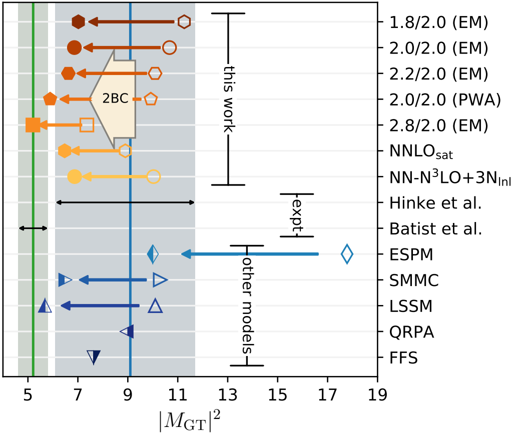
IMSRG calculation, Holt et al, preliminary

Shell model seems to include most correlations. Bulk of quenching comes from two-body current.

…And in ^{100}Sn

Coupled-Cluster Calculation of β Decay

Hagen et al, unpublished



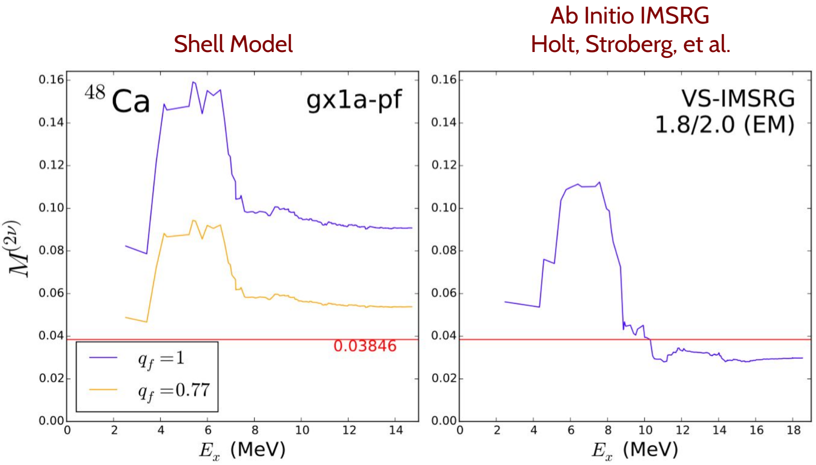
Again, most of the quenching accounted for by two-body current, and

Quenching increases with mass.

Spectator nucleons contribute coherently to two-body current.

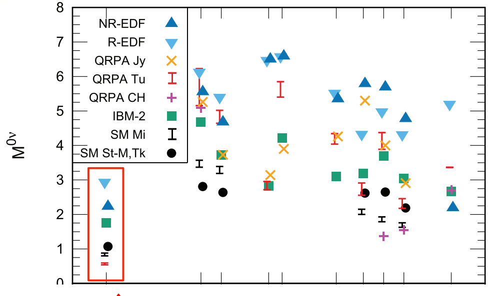
And $\beta\beta$ Decay? First Ab Initio Calculations

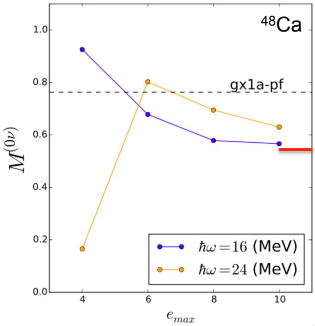
$2\nu\beta\beta$ in ^{48}Ca



Two-body currents not yet included.

And 0ν…





Coupled-clusters result is red bar at bottom.

Phenom. shell model

IMSRG, $\hbar\omega = 16$

IMSRG, $\hbar\omega = 24$

Coupled Clusters

Two-body currents again not yet included.

So, to Sum Up…

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We're getting there.