PHENOMENOLOGY

BEYOND

THE STANDARD MODEL

OUTLINE

1. Quiver gauge theories.

2. Conformality phenomenology.

3. 4 TeV Unification.

4. Quadratic divergences.

5. Anomaly Cancellation - Conformal U(1)s

6. Dark Matter Candidate.

SUMMARY.

CONCISE HISTORY OF STRING THEORY

- Began 1968 with Veneziano model.
- 1968-1974 dual resonance models for strong interactions. Replaced by QCD around 1973. DRM book 1974. Hiatus 1974-1984
- 1984 Cancellation of hexagon anomaly.
- 1985 $E(8) \times E(8)$ heterotic strong compactified on Calabi-Yau manifold gives temporary optimism of TOE.
- 1985-1997 Discovery of branes, dualities, M theory.
- 1997 Maldacena AdS/CFT correspondence relating 10 dimensional superstring to 4 dimensional gauge field theory.
- 1997-present Insights into gauge field theory including possible new states beyond standard model. String not only as quantum gravity but as powerful tool in nongravitational physics.

MORE ON STRING DUALITY:

Duality: Quite different looking descriptions of the same underlying theory.

The difference can be quite striking. For example, the AdS/CFT correspondence describes duality between a $d = 4$ SU(N) GFT and a $D = 10$ superstring. Nevertheless, a few non-trivial checks have confirmed this correspondence.

In its most popular version, one takes a Type IIB superstring (closed, chiral) in $d = 10$ and one compactifies on:

$$(AdS)_5 \qquad \times \qquad S^5$$

Perturbative finiteness of

$\mathcal{N} = 4$ SUSY Yang-Mills theory.

- Was proved by Mandelstam, Nucl. Phys. B213, 149 (1983).
- The Malcacena correspondence is primarily aimed at the $N \to \infty$ limit with the 't Hooft parameter of N times the squared gauge coupling held fixed.
- Conformal behavior valid here also for finite N.

1. QUIVER GAUGE THEORIES

BREAKING SUPERSYMMETRIES

To approach the real world, one needs less or no supersymmetry in the (conformal?) gauge theory.

By factoring out a discrete (abelian) group and composing an orbifold:

$$S^5/\Gamma$$

one may break $\mathcal{N} = 4$ supersymmetry to $\mathcal{N} = 2$, 1, or 0. Of special interest is the $\mathcal{N} = 0$ case.

We may take $\Gamma = Z_p$ which identifies p points in $\mathcal{C}_3$.

The rule for breaking the $\mathcal{N} = 4$ supersymmetry is:

$$\Gamma \subset SU(2) \quad \Rightarrow \quad \mathcal{N} = 2$$

$$\Gamma \subset SU(3) \quad \Rightarrow \quad \mathcal{N} = 1$$

$$\Gamma \not\subset SU(3) \quad \Rightarrow \quad \mathcal{N} = 0$$

In fact to specify the embedding of $\Gamma = Z_p$ we need to identify three integers (a_1, a_2, a_3):

$$\mathcal{C}_3 : \ (X_1, X_2, X_3) \ \xrightarrow{Z_p} \ (\alpha^{a_1} X_1, \alpha^{a_2} X_2, \alpha^{a_3} X_3)$$

with

$$\alpha = exp\left(\frac{2\pi i}{p}\right)$$

What is known to be true - proved both from string theory in

Bershadsky, Kakushadze and Vafa, hep-th/9803076,

and from field theory† in

Bershadsky and Johansen, hep-th/9803349,

- is that to leading order in $(1/N)$ such theories have all $\beta = 0$ $(\beta_g = \beta_Y = \beta_H = 0)$ to all orders of GFT perturbation theory.

This is remarkable from the field theory point of view.

† This proof involves Γ projections of the states and turns out to be disappointingly kinematic.

Without the stimulus of AdS/CFT it would be:

- Difficult to guess any $\mathcal{N} = 0$ theory with all $\beta-$functions vanishing to all orders of perturbation theory, even for leading order in 1/N.

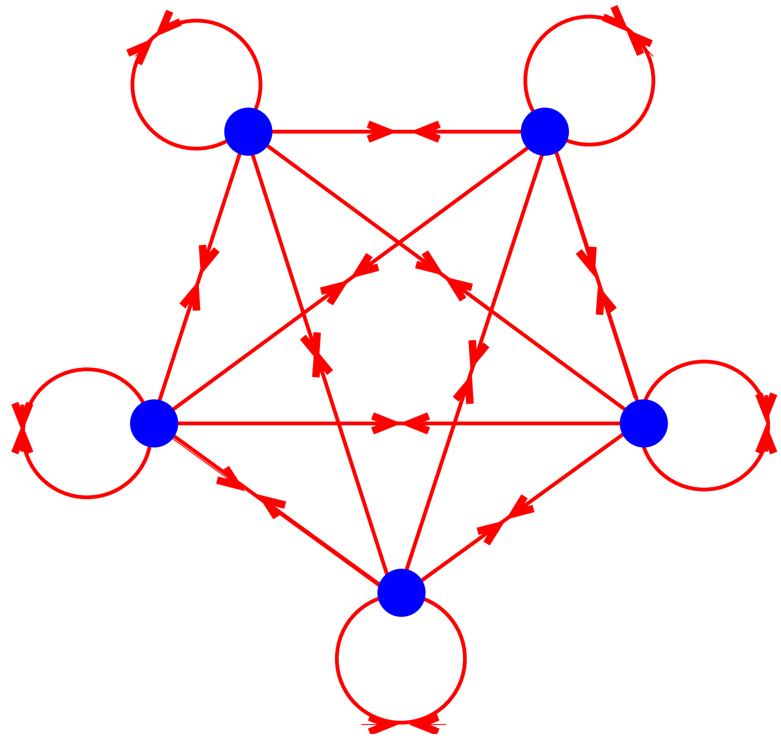
- Because without renormalizarion theorems ($\mathcal{N} = 0$) there is an infinite number of constraints on a finite number of representations.

MATTER REPRESENTATIONS

- The Z_p discrete group identifies p points in $\mathcal{C}_3$.

- The N converging D3-branes meet on all p copies, giving a gauge group: $SU(N) \times SU(N) \times ...... \times SU(N)$.

- The matter (spin-1/2 and spin-0) which survives is invariant under a product of a gauge transformation and a Z_p transformation.



One can draw p points and arrows for a_1, a_2, a_3.

$$e.g. \qquad Z_5 \;\; (1,3,0)$$

Quiver diagram (Douglas-Moore).

Scalar representation is:

$$\sum_{k=1}^{3} \sum_{i=1}^{p} (N_1, \bar{N}_{i \pm a_k})$$

For fermions, one must construct the **4** of R-parity $SU(4)$:

From the $a_k = (a_1, a_2, a_3)$ one constructs the 4-spinor $A_\mu = (A_1, A_2, A_3, A_4)$:

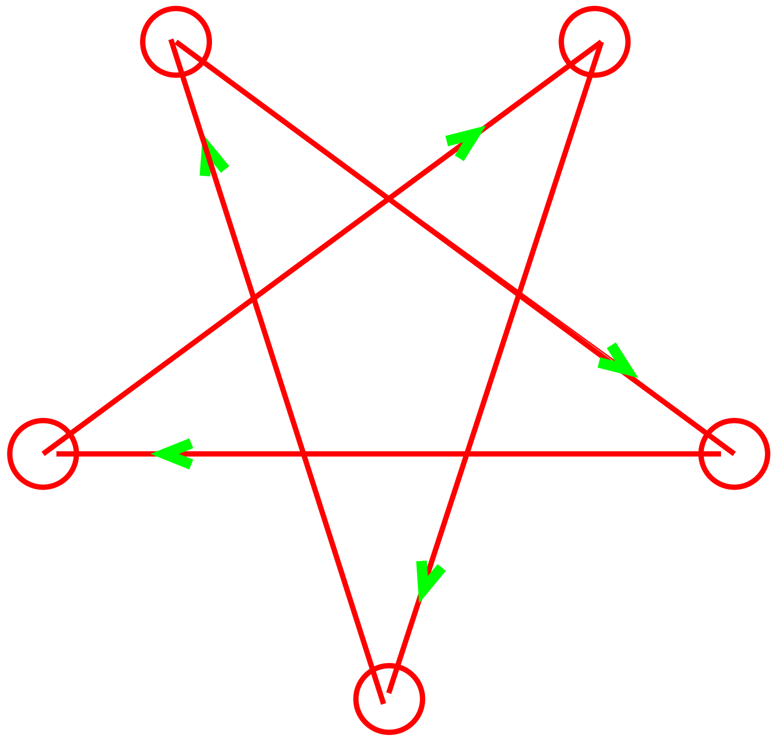
$$A_1 = \frac{1}{2}(a_1 + a_2 + a_3)$$

$$A_2 = \frac{1}{2}(a_1 - a_2 - a_3)$$

$$A_3 = \frac{1}{2}(-a_1 + a_2 - a_3)$$

$$A_4 = \frac{1}{2}(-a_1 - a_2 + a_3)$$

These transform as $exp\left(\frac{2\pi i}{p} A_\mu\right)$ and the invariants may again be derived (by a different diagram):



$$e.g. A_\mu = 2; \quad p = 5.$$

These lines are oriented.

One finds for the fermion representation

$$\sum_{\mu=1}^{4} \sum_{i=1}^{p} (N_i, \bar{N}_{i+A_\mu})$$

Nevertheless, since 4 global supersymmetries give conformality including for finite N (all $\beta-$functions vanish)

- To all orders of perturbation theory even for finite N of $SU(N)$ we can be more ambitious and ask for finiteness without any global supersymmetry and finite N of $SU(N)$.

References

Hierarchy and Naturalness

K.G. Wilson, Phys. Rev. **D3,** 1818 (1971).
(already discussed in the late 1960s)
hep-lat/0412043

What a difference 3 decades make.

J.E. Kim Trinification
Phys. Lett. **B591,** 119 (2004).

Triangle Anomalies.

J.S. Bell and R. Jackiw,
Nuovo Cimento A60, 47 (1969).

K. Dienes. Misaligned supersymmetry.
Nucl. Phys. **B429,** 533 (1994).

SUPERCONFORMAL SYMMETRY

$\mathcal{N} = 4$ SU(N) Yang-Mills 1983
String theory 1997 for infinite N.

lessened to SUPERSYMMETRY

Answered Wilson's objection 1974
$> 10^4$ papers.

or CONFORMALITY?

$\mathcal{N} = 4 \rightarrow \mathcal{N} = 0$ by orbifolding
$< 10^2$ papers. $3^3 \gg 422$
hidden misaligned supersymmetry

LHC (2009) SHOULD DISCRIMINATE

2. CONFORMALITY PHENOMENOLOGY

- Hierarchy between GUT scale and weak scale is 14 orders of magnitude. Why do these two very different scales exist?

- How is this hierarchy of scales stabilized under quantum corrections?

- Supersymmetry answers the second question but not the first.

Successes of

Supersymmetry

- Cancellations of UV infinities.
- technical naturalness of hierarchy.
- Unification of Gauge Couplings.
- Natural appearance in string theory.

Puzzles about

Supersymmetry

- The “mu” problem: why is the Higgs at the weak scale not the GUT scale (hierarchy).
- Breaking supersymmetry leads to too large a cosmological constant.
- Is supersymmetry fundamental for string theory?
- There are solutions of string theory without supersymmetry.

Supersymmetry and Grand Unification replaced by Conformality at TeV Scale.

- Will show idea is possible.
- Explicit examples containing standard model states.
- Conformality more rigid constraint than supersymmetry.
- Predicts additional states at TeV scale for conformality.
- Gauge coupling unification.
- Naturalness: cancellation of quadratic divergences.
- Anomaly cancellation: conformality of U(1) couplings.
- Dark Matter candidate.

Conformality as

hierarchy solution

- Quark and lepton masses, QCD and weak scales small compared to TeV scale.

- May be put to zero suggesting:

- Add degrees of freedom to yield GFT with conformal invariance.

- 't Hooft naturalness since zero mass limit increases symmetry to conformal symmetry.

The theory is assumed to be given by the action:

$$S = S_0 + \int d^4x \alpha_i O_i \tag{1}$$

where S_0 is the action for the conformal theory and the O_i are operators with dimension below four which break conformal invariance softly.

The mass parameters α_i have mass dimension $4 - \Delta_i$ where Δ_i is the dimension of O_i at the conformal point.

Let M be the scale set by the parameters α_i and hence the scale at which conformal invariance is broken. Then for $E >> M$ the couplings will not run while they start running for $E < M$. To solve the hierarchy problem we assume M is near the TeV scale.

Large class of d=4 CFTs

- each $SU(4)$ subgroup

- Choice of N.
- 1/N vanishing $\beta-$ functions.
- Finite N?
- Conformal invariance at infra-red fixed point.
- For $\mathcal{N} = 0$ there exists boson-fermion number equality.

Interactions. Gauge fields interact according to gauge coupling which, combined with corresponding theta angle for i th group, is writable as

$$\tau_i = \Theta_i + \frac{i}{4\pi g_i^2} = \frac{\tau d_i}{|\Gamma|}$$

where τ is complex parameter (independent i) and $|\Gamma|$ = order Γ.

Yukawa interactions. Triangles in quiver. Two directed fermion sides and an undirected scalar side.

$$S_{Yukawa} = \frac{1}{4\pi g^2} \Sigma\, d^{abc} Tr \Psi^a_{ij} \Phi^b_{jk} \Psi^c_{ki}$$

in which d^{abc} is ascertainable as Clebsch-Gordan coefficient from product of trivial representaions occurring respectively in $(4 \otimes R_i \otimes R_j^*)$, $(6 \otimes R_j \otimes R_k^*)$ and $(4 \otimes R_k \otimes R_i^*)$.

Quartic scalar interactions. Quadrilaterals in quiver. Four undirected sides. The coupling computable analagously to above.

Large class of d=4 CFTs

- Are they conformal for higher orders in 1/N?
- YES, for $\mathcal{N} = 2$: all such $\mathcal{N} = 2$ theories are obtainable.
- YES, for $\mathcal{N} = 1$: non-renormalization theorems ensure flat directions.
- For the case of $\mathcal{N} = 0$, general answer unknown but is under investigation.

Large class of d=4 CFTs

- Are they conformal for higher orders in 1/N?
- S-duality of underlying type IIB superstring implies $g \to 1/g$ symmetry.
- Assuming next-leading order in $1/N$ is asymptotically free IR flow at small g *increases* g.
- Consequently IR flow *decreases* g for large g and there must therefore be at least one zero $\beta = 0$ for some finite g. QED.

GENERAL PREDICTIONS.

Consider embedding the standard model gauge group according to:

$$SU(3) \times SU(2) \times U(1) \subset \bigotimes_i SU(Nd_i)$$

Each gauge group of the SM can lie entirely in a $SU(Nd_i)$ or in a diagonal subgroup of a number thereof.

Only bifundamentals (including adjoints) are possible. This implies no $(8, 2)$, etc. A conformality restriction which is new and satisfied in Nature!

No $U(1)$ factor can be conformal and so hypercharge is quantized through its incorporation in a non-abelian gauge group. This is the "conformality" equivalent to the GUT charge quantization condition in *e.g.* $SU(5)$!

Beyond these general consistencies, there are predictions of new particles necessary to render the theory conformal.

The minimal extra particle content comes from putting each SM gauge group in one $SU(Nd_i)$. Diagonal subgroup embedding *increases* number of additional states.

Number of fundamentals plus Nd_i times the adjoints is $4Nd_i$. Number N_3 of color triplets and N_8 of color octets satisfies:

$$N_3 + 3N_8 \geq 4 \times 3 = 12$$

Since the SM has $N_3 = 6$ we predict:

$$\Delta N_3 + 3N_8 \geq 6$$

The additional states are at TeV if conformality solves hierarchy. Similarly for color scalars:

$$M_3 + 3M_8 \geq 6 \times 3 = 18$$

The same exercise for $SU(2)$ gives $\Delta N_2 + 4N_3 \geq 4$ and $\Delta M_2 + 2M_3 \geq 11$ respectively.

3. 4 TeV UNIFICATION

- Above 4 TeV scale couplings will not run.
- Couplings of 3-2-1 related, not equal, at conformality scale.
- Embeddings in different numbers of the equal-coupling $SU(N)$ groups lead to the 4 TeV scale unification without logarithmic running over large desert.

Some illustrative examples of model building using conformality.

We need to specify an embedding $\Gamma \subset SU(4)$.

Consider Z_2. It embeds as $(-1, -1, -1, -1)$ which is real and so leads to a non-chiral model.

Z_3. One choice is $\mathbf{4} = (\alpha, \alpha, \alpha, 1)$ which maintains N=1 supersymmetry. Otherwise we may choose $\mathbf{4}$=$(\alpha, \alpha, \alpha^2, \alpha^2)$ but this is real.

Z_4. The only N = 0 complex embedding is $\mathbf{4}$=(i, i, i, i). The quiver is as shown on the next transparency with the $SU(N)^4$ gauge groups at the corners, the fermions on the edges and the scalars on the diagonals. The scalar content is too tight to break to the SM.

To obtain 3 chiral families in $\mathcal{N} = 0$ from abelian orbifolds, consider $\Gamma = Z_p$ with successively increasing $p = 2, 3, 4, 5, 6, 7....$ to access the simplest model.

p = 2 ± 1 real (require ($4 \neq 4^*$ for chirality)

p = 3 No chiral $\mathcal{N} = 0$

p = 4 (i, i, i, i).
Scalars † insufficient for SSB $SU(3)^4 \rightarrow (321)_{SM}$

p = 5 $(\alpha, \alpha, \alpha, \alpha^2)$ $(\alpha, \alpha^2, \alpha^3, \alpha^3)$
In both cases, scalars† insufficient for SSB.

p = 6 $(\alpha, \alpha, \alpha, \alpha^3)$ $(\alpha, \alpha, \alpha^2, \alpha^2)$ $(\alpha, \alpha^3, \alpha^3, \alpha^3)$
Scalars† insufficient for SSB $SU(3)^6 \rightarrow (321)_{SM}$

† scalars, unlike in GUTs, are in prescibed representations.

When we arrive at $p = 7$ there are viable models. Actually three different quiver diagrams can give:

1) 3 chiral families.

2)Adequate scalars to spontaneously break $SU(3)^7 \rightarrow SU(3) \times SU(2) \times U(1)$

and

3) $\sin^2\theta_W = 3/13 = 0.231$

The embeddings of $\Gamma = Z_7$ in SU(4) are:

7A. $(\alpha, \alpha, \alpha, \alpha^4)$

7B. $(\alpha, \alpha, \alpha^2, \alpha^3)^*$ C-H-H-H-W-H-W

7C. $(\alpha, \alpha^2, \alpha^2, \alpha^2)$

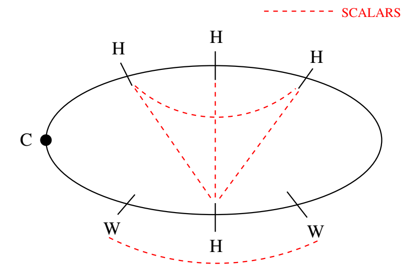
7D. $(\alpha, \alpha^3, \alpha^5, \alpha^6)^*$ C-H-W-H-H-H-W

7E. $(\alpha, \alpha^4, \alpha^4, \alpha^4)^*$ C-H-W-W-H-H-H

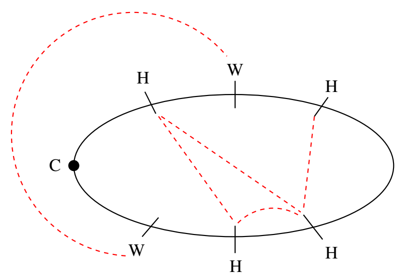
7F. $(\alpha^2, \alpha^4, \alpha^4, \alpha^4)$

* have properties 1), 2) and 3).

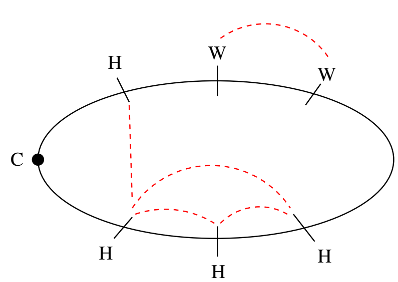
7B $\quad 4 = (1,\ 1,\ 2,\ 3) \quad 6 = (2,\ 3,\ 3,\ \text{-}3,\ \text{-}3,\ \text{-}2)$



7D $\quad 4 = (1,\ 3,\ 5,\ 5) \quad 6 = (1,\ 1,\ 3,\ \text{-}3,\ \text{-}1,\ \text{-}1)$



7E $\quad 4 = (1,\ 4,\ 4,\ 5) \quad 6 = (1,\ 2,\ 2,\ \text{-}2,\ \text{-}2,\ \text{-}1)$



The simplest abelian orbifold conformal extension of the standard model has $SU(3)^7 \rightarrow SU(3)^3$ trinification $\rightarrow (321)_{SM}$.

In this case we have α_2 and α_1 related correctly for low energy. But $\alpha_3(M) \simeq 0.07$ suggesting a conformal scale $M \geq 10$ TeV - too high for the L.H.C.

GAUGE COUPLING UNIFICATION (ABELIAN ORBIFOLD)

With the assumptions of grand unification and low-energy supersymmetry, one achieved a successful gauge unification. The LEP data gives the couplings at the Z-pole as $\alpha_3 = 0.118 \pm 0.003$, $\alpha_2 = 0.0338$ and $\alpha_1 = \frac{5}{3}\alpha_Y = 0.0169$ (where the errors on $\alpha_{1,2}$ are less than 1%.

Using the RG equations

$$\frac{1}{\alpha_i(M_G)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi} \ln\left(\frac{M_G}{M_Z}\right)$$

and, for the MSSM the values $b_i = (6\frac{3}{5}, 1, -3)$, inputting $\alpha_{2,3}(M_Z)$ leads to $M_G = 2.4 \times 10^{16}$ GeV and the *prediction* that

$$\sin^2\theta = 0.231$$

in excellent agreement with experiment.
Indeed this success is the main reason for belief in these two assumptions of low-energy supersymmetry and grand unification,

If we note that, at the Z-pole, the ratio $\alpha_2/\alpha_1 \simeq 2$ as pointed out first in
PHF, Phys. Rev. **D60,** 085004 (1999)
we can reproduce the correct gauge unification. Specifically for the abelian orbifold with $\Gamma = Z_7$ and $N = 3$ it is natural to accommodate one $SU(3)$ factor as $SU(3)_{color}$ and $SU(2)_L$ in a diagonal subgroup of two $SU(3)$ factors. Finally $U(1)$ is in a diagonal subgroup of four $SU(3)$ factors.
This gives the appropriate ratio betwen $\alpha_{1,2}$ and consequently

$$\sin^2\theta = \frac{\alpha_Y}{\alpha_2 + \alpha_Y} = \frac{3/5}{2 + 3/5} = 3/13 = 0.231$$

There is a small correction for the running between M_Z and the TeV scale, but this is compensated by the two-loop correction and the agreement remains as good as for SUSY-GUTs. This is strong encouragenment for the conformality approach.

The successful derivation of $\sin^2\theta_W \simeq 0.23$ from both the abelian orbifold (based on 333 unification) and the non-abelian manifold (based on 422 unification) is strong support for the conformality approach.

More detailed phenomenological study of the conformality idea is merited.

STRONG-ELECTROWEAK UNIFICATION

was proposed even before the firm establishment of the standard electroweak theory in the early 1970s. Minimal SU(5), both with and without supersymmetry, is ruled out. Such GUTs involve a scale $\sim 10^{16}$ GeV and a GUT hierarchy.

Compactifying IIB on $AdS_5 \times S^5/\Gamma$ leads to candidate semi-simple unification gauge groups.

The following theory has both a bottom-up and a top-down component and leads to several interesting features. Let us begin with bottom-up.

In the SM first consider the electroweak angle $sin^2\theta(\mu)$. At $\mu = M_Z$, its value is measured as 0.231 and with increasing μ it goes through 1/4 at $\mu \simeq 4$ TeV.

We may consider also the ratio $\alpha_3(\mu)/\alpha_2(\mu)$ which is above 3 at $\mu = M_Z$, decreases through 3 at $\mu \simeq 400$ GeV and 2 at $\mu \simeq 140$ TeV. It is 5/2 at exceptionally close to where $\sin^2\theta = 1/4$.

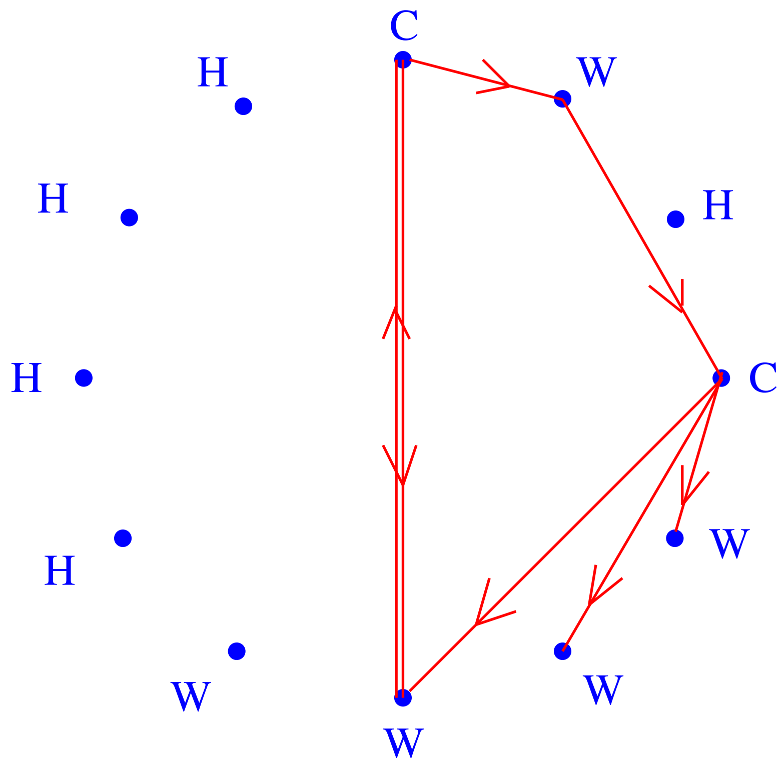
We take this numerology as a hint that in a trinification $SU(3)_C \times SU(3)_W \times SU(3)_H$ the couplings are in the ratio 5 :: 2 :: 2 at $\mu = 4$TeV

This can be achieved by embedding the 333-model in $SU(3)^{12}$ with the C, W, H groups diagonaly embedded in respectively 2, 5, 5 of the $SU(3)$'s. Let us now consider top-down.

Taking as orbifold S^5/Z_{12} with embedding of Z_{12} in the SU(4) R-parity specified by $\mathbf{4}$ $\equiv \alpha^{A_1}, \alpha^{A_2}, \alpha^{A_3}, \alpha^{A_4})$ and $A_\mu = (1, 2, 3, 6)$.

This accommodates the scalars necessary to spontaneously break to the SM. This theory thus predicts TWO numbers: $sin^2\theta(M_Z)$ and $\alpha_C(M_Z)$ whereas usual GUTs predict only ONE of these.

As a bonus, the dodecagonal quiver predicts three chiral families (see next transparency). Also there is no GUT hierarchy.

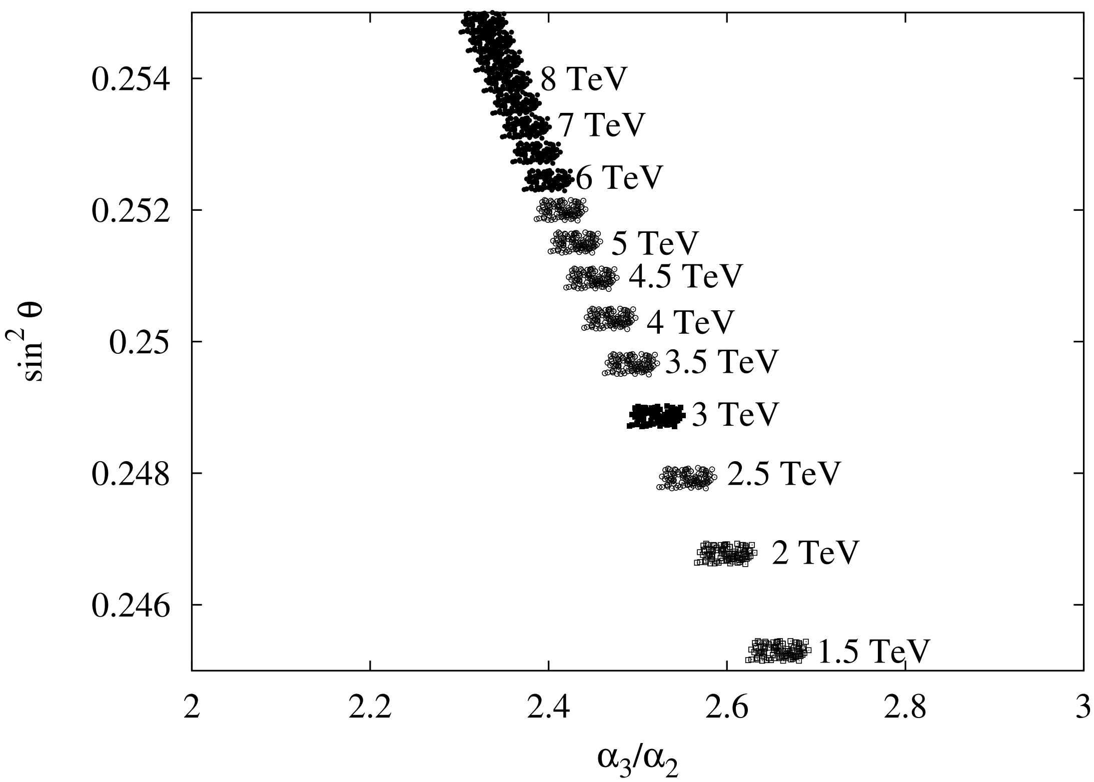


.

$$A_\mu = (1, 2, 3, 6)$$

$$SU(3)_C \times SU(3)_H \times SU(3)_H$$

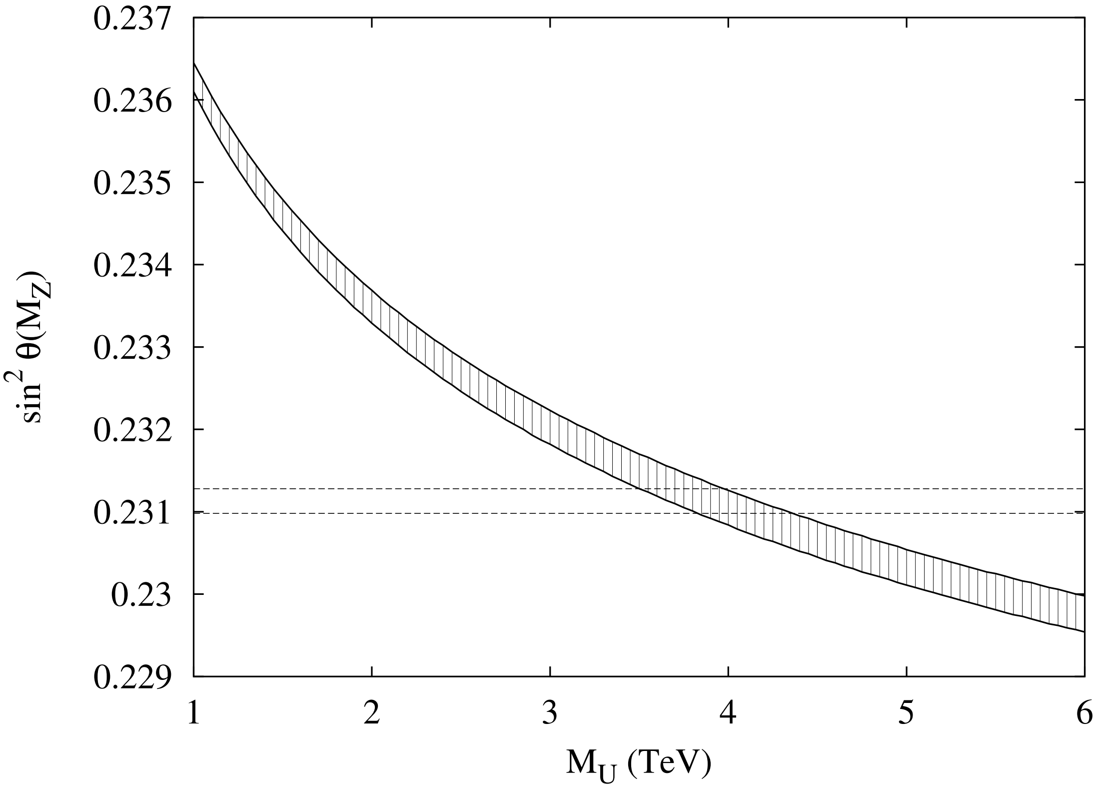
$$5(3, \bar{3}, 1) + 2(\bar{3}, 3, 1)$$



.

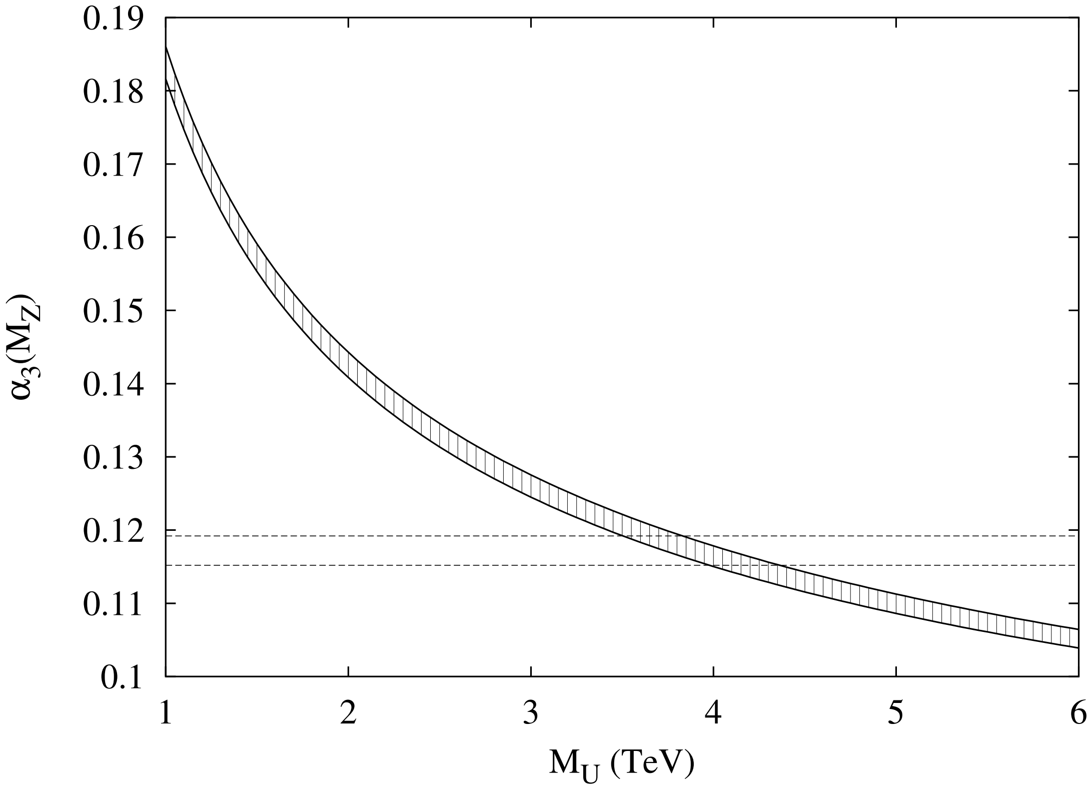
robustness of unification

hep-ph/0302074. PHF+Ryan Rohm + Tomo Takahashi



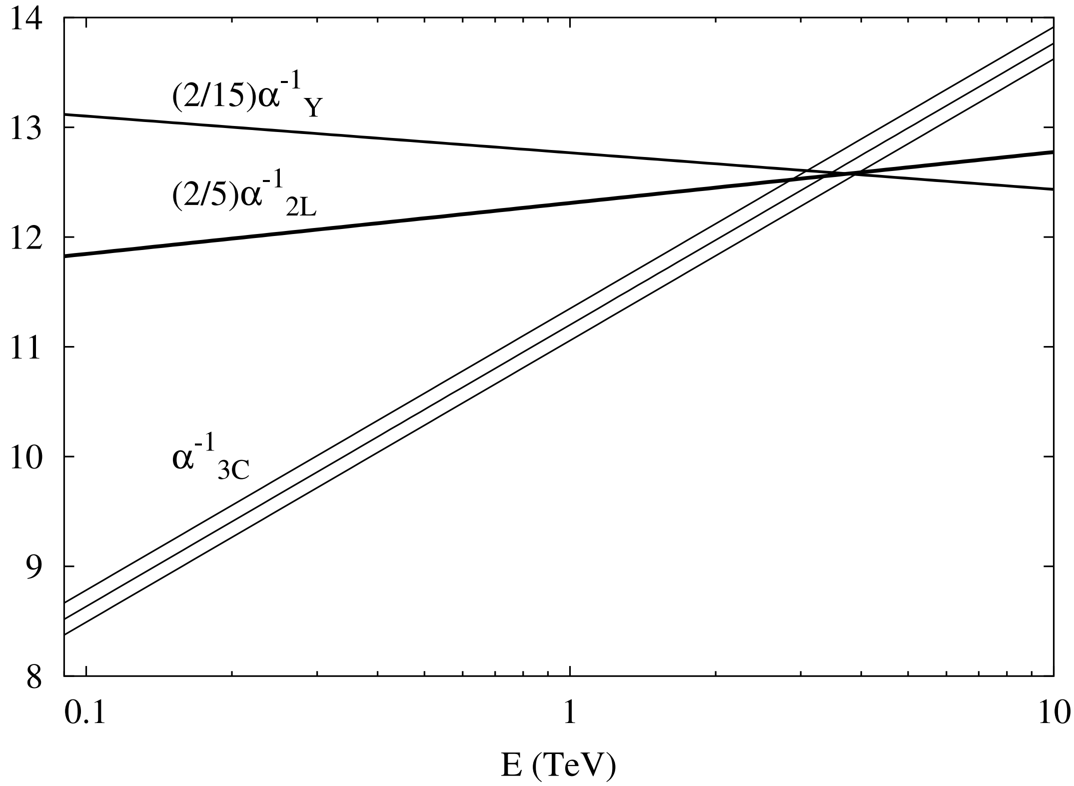
.

predictivity
hep-ph/0302074
PHF+Ryan Rohm + Tomo Takahashi



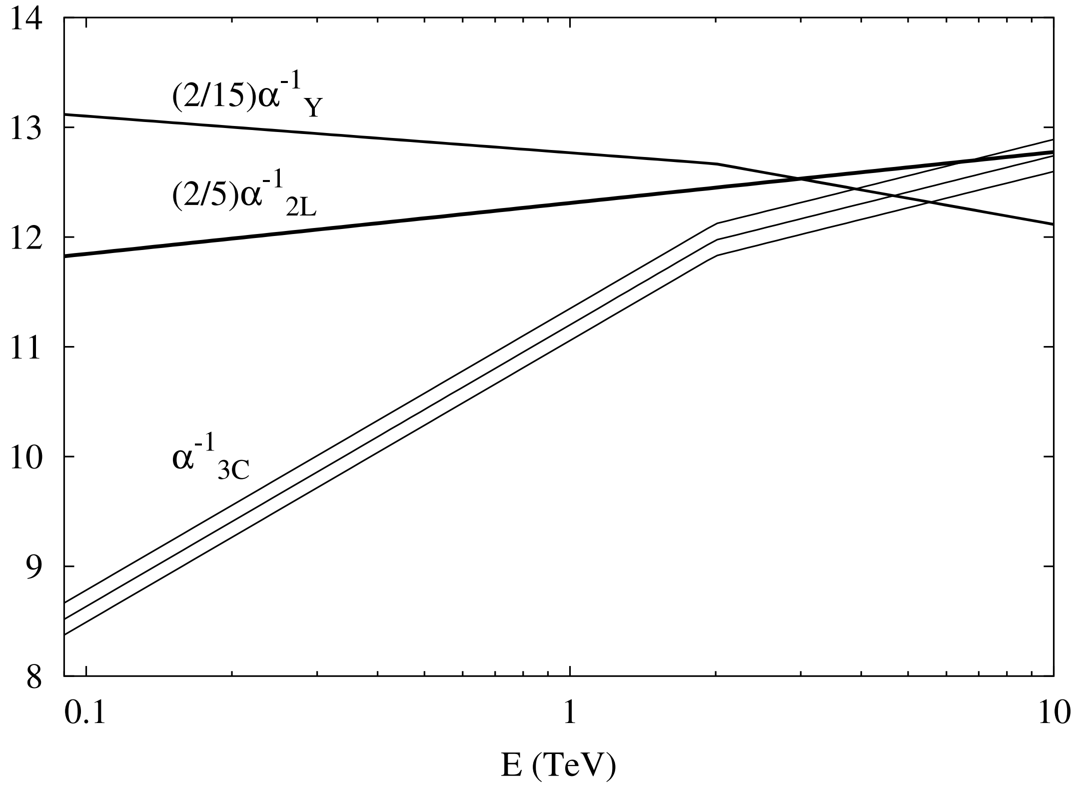
.

predictivity
hep-ph/0302074
PHF+Ryan Rohm + Tomo Takahashi



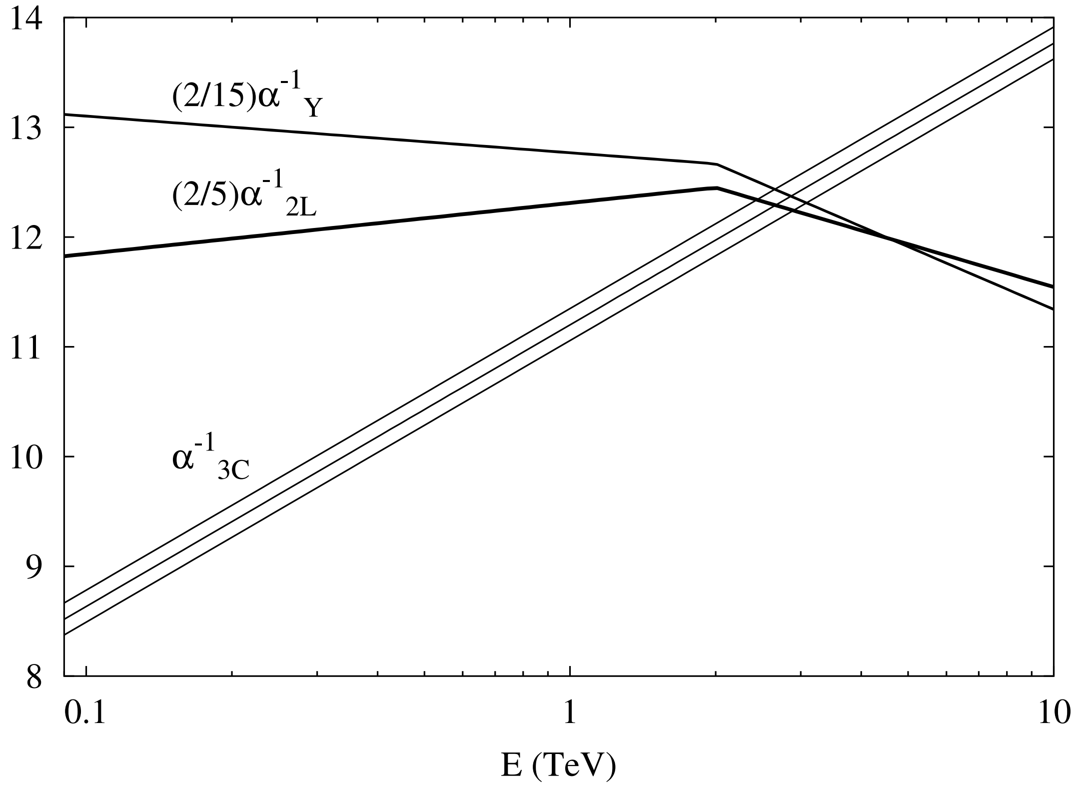
.

without thresholds: all states at M_U
hep-ph/0302074
PHF+Ryan Rohm + Tomo Takahashi



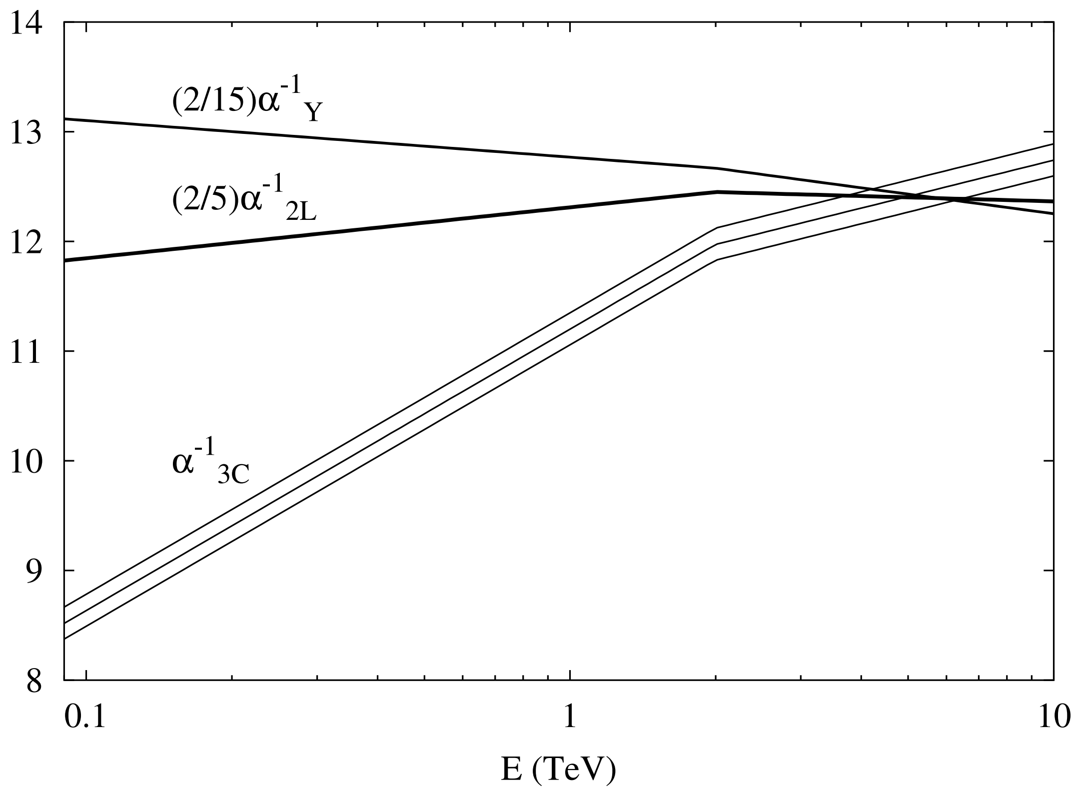
.

thresholds: CH fermions at 2 TeV
hep-ph/0302074
PHF+Ryan Rohm + Tomo Takahashi



.

thresholds: WH fermions at 2 TeV
hep-ph/0302074
PHF+Ryan Rohm + Tomo Takahashi



.

thresholds: CW fermions at 2 TeV
hep-ph/0302074
PHF+Ryan Rohm + Tomo Takahashi

4. QUADRATIC DIVERGENCES.

Classification of abelian quiver gauge theories

We consider the compactification of the type-IIB superstring on the orbifold $AdS_5 \times S^5/\Gamma$ where Γ is an abelian group $\Gamma = Z_p$ of order p with elements $\exp(2\pi i A/p)$, $0 \leq A \leq (p-1)$.

The resultant quiver gauge theory has $\mathcal{N}$ residual supersymmetries with $\mathcal{N} = 2, 1, 0$ depending on the details of the embedding of Γ in the $SU(4)$ group which is the isotropy of the S^5. This embedding is specified by the four integers $A_m, 1 \leq m \leq 4$ with

$$\Sigma_m A_m = 0(\mathrm{mod} p) \tag{2}$$

which characterize the transformation of the components of the defining representation of $SU(4)$. We are here interested in the non-supersymmetric case $\mathcal{N} = 0$ which occurs if and only if all four A_m are non-vanishing.

The gauge group is $U(N)^p$. The fermions are all in the bifundamental representations

$$\Sigma_{m=1}^{m=4}\Sigma_{j=1}^{j=p}(N_j, \bar{N}_{j+A_m}) \tag{3}$$

which are manifestly non-supersymmetric because no fermions are in adjoint representations of the gauge group. Scalars appear in representations

$$\Sigma_{i=1}^{i=3}\Sigma_{j=1}^{i=p}(N_j, \bar{N}_{j\pm a_i}) \tag{4}$$

in which the six integers $(a_i, -a_i)$ characterize the transformation of the antisymmetric second-rank tensor representation of $SU(4)$. The a_i are given by $a_1 = (A_2 + A_3)$, $a_2 = (A_3 + A_1)$, $a_3 = (A_1 + A_2)$

It is possible for one or more of the a_i to vanish in which case the corresponding scalar representation in the summation in Eq.(4) is to be interpreted as an adjoint representation of one particular $U(N)_j$. One may therefore have zero, two, four or all six of the scalar representations, in Eq.(4), in such adjoints.

For the lowest few orders of the group Γ, the members of the infinite class of $\mathcal{N} = 0$ abelian quiver gauge theories are tabulated below:

	p	A_m	a_i	scal bfds	scal adjs	chir frms	SM
1	2	(1111)	(000)	0	6	No	No
2	3	(1122)	(001)	2	4	No	No
3	4	(2222)	(000)	0	6	No	No
4	4	(1133)	(002)	2	4	No	No
5	4	(1223)	(011)	4	2	No	No
6	4	(1111)	(222)	6	0	Yes	No
7	5	(1144)	(002)	2	4	No	No
8	5	(2233)	(001)	2	4	No	No
9	5	(1234)	(012)	4	2	No	No
10	5	(1112)	(222)	6	0	Yes	No
11	5	(2224)	(111)	6	0	Yes	No
12	6	(3333)	(000)	0	6	No	No
13	6	(2244)	(002)	2	4	No	No
14	6	(1155)	(002)	2	4	No	No
15	6	(1245)	(013)	4	2	No	No
16	6	(2334)	(011)	4	2	No	No
17	6	(1113)	(222)	6	0	Yes	No
18	6	(2235)	(112)	6	0	Yes	No
19	6	(1122)	(233)	6	0	Yes	No

The Table continues to infinity but we stop at $p = 7$:

	p	A_m	a_i	scal bfds	scal adjs	chir frms	SM
20	7	(1166)	(002)	2	4	No	No
21	7	(3344)	(001)	2	4	No	No
22	7	(1256)	(013)	4	2	No	No
23	7	(1346)	(023)	4	2	No	No
24	7	(1355)	(113)	6	0	No	No
25	7	(1114)	(222)	6	0	Yes	No
26	7	(1222)	(333)	6	0	Yes	No
27	7	(2444)	(111)	6	0	Yes	No
28	7	(1123)	(223)	6	0	Yes	Yes
29	7	(1355)	(113)	6	0	Yes	Yes
30	7	(1445)	(113)	6	0	Yes	Yes

Note that there is one model with all scalars in adjoints for each even value of p (see Model Nos 1,3,12). For general even p the embedding is $A_m = (\frac{p}{2}, \frac{p}{2}, \frac{p}{2}, \frac{p}{2})$. This series is the complete list of $\mathcal{N} = 0$ abelian quivers with all scalars in adjoints.

To be of more phenomenolgical interest the model should contain chiral fermions. This requires that the embedding be complex: $A_m \not\equiv -A_m$ (mod p). It will now be shown that for the presence of chiral fermions all scalars must be in bifundamentals.

The proof of this assertion follows by assuming the contrary, that there is at least one adjoint arising from, say, $a_1 = 0$. Therefore $A_3 = -A_2$ (mod p). But then it follows from Eq.(2) that $A_1 = -A_4$ (mod p). The fundamental representation of $SU(4)$ is thus real and fermions are non-chiral[1].

[1] This is almost obvious but for a complete justification, see Frampton & Kephart (2004)

It follows that:

In an $\mathcal{N} = 0$ quiver gauge theory,
chiral fermions are present
if and only if all scalars
are in bifundamental representations.

Cancellation of quadratic divergences

The lagrangian for the nonsupersymmetric Z_p theory can be written in a convenient notation which accommodates simultaneously both adjoint and bifundamental scalars as

$$\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{4}F^{ab}_{\mu\nu;r,r}F^{ba}_{\mu\nu;r,r} + i\bar{\lambda}^{ab}_{r+A_4,r}\gamma^{\mu}D_{\mu}\lambda^{ba}_{r,r+A_4} + 2D_{\mu}\Phi^{ab\dagger}_{r+a_i,r}D_{\mu}\Phi^{ba}_{r,r+a_i} + i\bar{\Psi}^{ab}_{r+A_m,r}\gamma^{\mu}D_{\mu}\Psi^{ba}_{r,r+A_m} \\
& -2ig\left[\bar{\Psi}^{ab}_{r,r+A_i}P_L\lambda^{bc}_{r+A_i,r+A_i+A_4}\Phi^{\dagger ca}_{r+A_i+A_4,r} - \bar{\Psi}^{ab}_{r,r+A_i}P_L\Phi^{\dagger bc}_{r+A_i,r-A_4}\lambda^{ca}_{r-A_4,r}\right] \\
& -\sqrt{2}ig\epsilon_{ijk}\left[\bar{\Psi}^{ab}_{r,r+A_i}P_L\Psi^{bc}_{r+A_i,r+A_i+A_j}\Phi^{ca}_{r-A_k-A_4,r} - \bar{\Psi}^{ab}_{r,r+A_i}P_L\Phi^{bc}_{r+A_i,r+A_i+A_k+A_4}\Psi^{ca}_{r-A_j,r}\right] \\
& -g^2\left(\Phi^{ab}_{r,r+a_i}\Phi^{\dagger bc}_{r+a_i,r} - \Phi^{\dagger ab}_{r,r-a_i}\Phi^{bc}_{r-a_i,r}\right)\left(\Phi^{cd}_{r,r+a_j}\Phi^{\dagger da}_{r+a_j,r} - \Phi^{\dagger cd}_{r,r-a_j}\Phi^{da}_{r-a_j,r}\right) \\
& +4g^2\left(\Phi^{ab}_{r,r+a_i}\Phi^{bc}_{r+a_i,r+a_i+a_j}\Phi^{\dagger cd}_{r+a_i+a_j,r+a_j}\Phi^{\dagger da}_{r+a_j,r} - \Phi^{ab}_{r,r+a_i}\Phi^{bc}_{r+a_i,r+a_i+a_j}\Phi^{\dagger cd}_{r+a_i+a_j,r+a_i}\Phi^{\dagger da}_{r+a_i,r}\right)
\end{aligned}$$

where $\mu,\nu = 0,1,2,3$ are lorentz indices; $a,b,c,d = 1$ to N are $U(N)^p$ group labels; $r = 1$ to p labels the node of the quiver diagram ;$a_i \quad (i = \{1,2,3\})$ label the first three of the **6** of SU(4); $A_m \quad (m = \{1,2,3,4\}) = (A_i, A_4)$ label the **4** of SU(4). By definition A_4 denotes an arbitrarily-chosen fermion (λ) associated with the gauge boson, similarly to the notation in the $\mathcal{N} = 1$ supersymmetric case.

(This $\mathcal{N} = 0$ lagrangian typeface may be illegible. Explain notation if necessary.)

As we showed in the previous section, the infinite sequence of nonsupersymmetric Z_p models can have scalars in adjoints (corresponding to $a_i = 0$) and bifundamentals ($a_i \neq 0$). Denoting by x the number of the three a_i which are nonzero, the models with $x = 3$ have only bifundamental scalars, those with $x = 0$ have only adjoints while $x = 1, 2$ models contain both types of scalar representations. As we have seen, to contain the phenomenologically-desirable chiral fermions, it is necessary and sufficient that $x = 3$.

Let us first consider the quadratic divergence question in the mother $\mathcal{N} = 4$ theory. The $\mathcal{N} = 4$ lagrangian is like Eq.(5) but since there is only one node all those subscripts become unnecessary so the form is simply

$$\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{4}F^{ab}_{\mu\nu}F^{ba}_{\mu\nu} + i\bar{\lambda}^{ab}\gamma^{\mu}D_{\mu}\lambda^{ba} + 2D_{\mu}\Phi_i^{ab\dagger}D_{\mu}\Phi_i^{ba} + i\bar{\Psi}^{ab}_m\gamma^{\mu}D_{\mu}\Psi^{ba}_m \\
& -2ig\left[\bar{\Psi}^{ab}_i P_L \lambda^{bc}\Phi^{\dagger ca}_{i,r} - \bar{\Psi}^{ab}_i P_L \Phi^{bc}_i \lambda^{ca}\right] \\
& -\sqrt{2}ig\epsilon_{ijk}\left[\bar{\Psi}^{ab}_i P_L \Psi^{bc}_j \Phi^{\dagger ca}_k - \bar{\Psi}^{ab}_i P_L \Phi^{bc}_j \Psi^{ca}_k\right] \\
& -g^2\left(\Phi^{ab}_i\Phi^{\dagger bc}_i - \Phi^{\dagger ab}_i\Phi^{bc}_i\right)\left(\Phi^{cd}_j\Phi^{\dagger da}_j - \Phi^{\dagger cd}_j\Phi^{da}_j\right) \\
& +4g^2\left(\Phi^{ab}_i\Phi^{bc}_j\Phi^{\dagger cd}_i\Phi^{\dagger da}_j - \Phi^{ab}_i\Phi^{bc}_j\Phi^{\dagger cd}_j\Phi^{\dagger da}_i\right)
\end{aligned}$$

All $\mathcal{N} = 4$ scalars are in adjoints and the scalar propagator has one-loop quadratic divergences coming potentially from three scalar self-energy diagrams: (a) the gauge loop (one quartic vertex); (b) the fermion loop (two trilinear vertices); and (c) the scalar loop (one quartic vertex).

For $\mathcal{N} = 4$ the respective contributions of (a, b, c) are computable from Eq.(5) as proportional to $g^2 N(1, -4, 3)$ which cancel exactly.

The $\mathcal{N} = 0$ results for the scalar self-energies (a, b, c) are computable from the lagrangian of Eq.(5). Fortunately, the calculation was already done in the literature. The result is amazing! The quadratic divergences cancel if and only if x = 3, exactly the same "if and only if" as to have chiral fermions. It is pleasing that one can independently confirm the results directly from the interactions in Eq.(5) To give just one explicit example, in the contributions to diagram (c) from the last term in Eq.(5), the 1/N corrections arise from a contraction of Φ with $\Phi^\dagger$ when all the four color superscripts are distinct and there is consequently no sum over color in the loop. For this case, examination of the node subscripts then confirms proportionality to the kronecker delta, δ_{0,a_i}. If and only if all $a_i \neq 0$, all the other terms in Eq.(5) do not lead to 1/N corrections to the $\mathcal{N} = 4$.

Some comments on the literature are necessary. In a 1999 paper of Csaki, Skiba and Terning it was claimed that there are always $1/N$ corrections to spoil cancellation for finite N and that $N \geq 10^{28}$ is necessary! This was because of a technical error that the orbifolded gauge group is not $SU(N)^p$ but $U(N)^p$ and bifundamentals carry U(1) charges. A paper by Fuchs in 2000, which has been largely ignored, corrected this point.

The conclusion is that the chiral Z_7, Z_{12} models of the earlier discussion which contain the standard model are free of one-loop quadratic divergences in the scalar propagator. Nevertheless the overall conformal invariance would not be respected by U(1) factors which would have non-zero positive beta-functions. Clearly these factors must somehow be decoupled. This mysterious decoupling of U(1)'s from AdS/CFT which would not be conformally invariant has been commented upon as an unsolved cunundrum in the literature on AdS/CFT. A better understanding of these U(1)'s may be necessary to achieve the hope of a fully four-dimensionally conformally invariant extension of the standard model. There is the paradoxical requirement that the U(1) gauge factors must be present in the UV to cancel quadratic divergences but must decouple in the IR to preserve 4-dimensional conformal invariance at lower energy.

Eventually gravity, at the Planck scale, will inevitably break conformal invariance because Newton's constant is dimensionful. A realistic hope is that there is a substantial window of energy scales where conformal invariance is an excellent approximation between, say, 4 TeV for at least a few orders of magnitude in energy even towards a scale approaching the see-saw scale $\sim 10^{10}$ GeV. It is difficult to foresee how large the conformality window is. Finally it is interesting to note that the present models seem to have all the ingredients of the so-called little Higgs models, which were proposed later, with the quiver diagram here interpreted as the theory space there.

This conformality idea, that an augmented standard model possess an energy window of conformal invariance starting just above the weak interaction scale, requires the existence of new undiscovered particles accessible to the LHC: gauge bosons which fill out the unitary gauge group $U(N)^p$ which contains the established $SU(3)\times SU(2)\times U(1)$; chiral fermions in bifundamental representations of $U(N)^p$; and, as shown in the present article, complex scalars also in bifundamentals of $U(N)^p$. The new experimental results should be able to distinguish these definite predictions coming from the assumption of four-dimensional conformal invariance.

5. Anomaly cancellation by a compensatory term and U(1)'s in conformality

The lagrangian for the nonsupersymmetric Z_n theory can be written in a convenient notation which accommodates simultaneously both adjoint and bifundamental scalars as mentioned before.

As also mentioned above we shall restrict attention to models where all scalars are in bifundamentals which requires all a_i to be non zero. Recall that $a_1 = A_2 + A_3$, $a_2 = A_3 + A_1$; $a_3 = A_1 + A_2$.

The lagrangian is classically $U(N)^p$ gauge invariant. There are, however, triangle anomalies of the $U(1)_p U(1)_q^2$ and $U(1)_p SU(N)_q^2$ types. Making gauge transformations under the $U(1)_r$ (r = 1,2,...,n) with gauge parameters Λ_r leads to a variation

$$\delta\mathcal{L} = -\frac{g^2}{4\pi^2}\Sigma_{p=1}^{p=n} A_{pq} F_{\mu\nu}^{(p)} \tilde{F}^{(p)\mu\nu} \Lambda_q \qquad (5)$$

which defines an $n \times n$ matrix A_{pq} which is given by

$$A_{pq} = \mathrm{Tr}(Q_p Q_q^2) \qquad (6)$$

where the trace is over all chiral fermion links and Q_r is the charge of the bifundamental under $U(1)_r$. We shall adopt the sign convention that $\mathbf{N}$ has $Q = +1$ and $\mathbf{N}^*$ has $Q = -1$.

It is straightforward to write A_{pq} in terms of Kronecker deltas because the content of chiral fermions is

$$\Sigma_{m=1}^{m=4}\Sigma_{r=1}^{r=n}(\mathbf{N}_r, \mathbf{N^*}_{r+A_m}) \tag{7}$$

This implies that the antisymmetric matrix A_{pq} is explicitly

$$A_{pq} = -A_{qp} = \Sigma_{m=1}^{m=4}\left(\delta_{p,q-A_m} - \delta_{p,q+A_m}\right) \tag{8}$$

Now we are ready to construct $\mathcal{L}^{(1)}_{comp}$, the compensatory term. Under the $U(1)_r$ gauge transformations with gauge parameters Λ_r we require that

$$\begin{aligned}\delta\mathcal{L}^{(1)}_{comp} &= -\delta\mathcal{L} \\ &= +\frac{g^2}{4\pi^2}\Sigma_{p=1}^{p=n} A_{pq} F^{(p)}_{\mu\nu} \tilde{F}^{(p)\mu\nu} \Lambda_q \quad (9)\end{aligned}$$

To accomplish this property, we construct a compensatory term in the form

$$\mathcal{L}^{(1)}_{comp} = \frac{g^2}{4\pi}\Sigma_{p=1}^{p=n}\Sigma_k B_{pk} \mathrm{ImTrln}\left(\frac{\Phi_k}{v}\right) F^{(p)}_{\mu\nu}\tilde{F}^{(p)\mu\nu} \quad (10)$$

where Σ_k runs over scalar links. To see that $\mathcal{L}^{(1)}_{comr}$ of Eq.(10) has $SU(N)^n$. invariance rewrite Tr ln $\equiv$ exp det and note that the $SU(N)$ matrices have unit determinant.

We note *en passant* that one cannot take the $v \longrightarrow 0$ limit in Eq.(10); the chiral anomaly enforces a breaking of conformal invariance.

We define a matrix C_{kq} by

$$\delta\left(\Sigma_{p=1}^{p=n}\Sigma_k \mathrm{ImTrln}\left(\frac{\Phi_k}{v}\right)\right) = \Sigma_{q=1}^{q=n} C_{kq}\Lambda_q \quad (11)$$

whereupon Eq.(9) will be satisfied if the matrix B_{pk} satisfies $A = BC$. The inversion $B = AC^{-1}$ is non trivial because C is singular but C_{kq} can be written in terms of Kronecker deltas by noting that the content of complex scalar fields in the model implies that the matrix C_{kq} must be of the form

$$C_{kq} = 3\delta_{kq} - \Sigma_i \delta_{k+a_i,q} \quad (12)$$

Explicit construction of the matrix B in $\mathcal{L}^{(1)}_{comp}$

Construction of the anomaly compensatory term $\mathcal{L}^{(1)}_{comp}$ of Eq.(10) has been reduced to the explicit construction of the matrix B_{pk}. Although $B = AC^{-1}$ is inadequate because Rank(C) < n, a necessary and sufficient condition for the existence of B is Rank (A) $\leq$ Rank (C). Proving this in general would be one approach but the large number of special cases will make the proof lengthy. We have shown (in hep-th/0603065) explicit construction of B in two extremes which we call the totally degenerate and the totally nondegenerate cases respectively.

Evolution of U(1) gauge couplings.

In the absence of the compensatory term, the two independent $U(N)^n$ gauge couplings g_N for SU(N) and g_1 for U(1) are taken to be equal $g_N(\mu_0) = g_1(\mu_0)$ at a chosen scale, *e.g.* μ_0=4 TeV to enable cancellation of quadratic divergences. Note that the n SU(N) couplings $g_N^{(p)}$ are equal by the overall Z_n symmetry, as are the n U(1) couplings $g_1^{(p)}$, $1 \leq p \leq n$.

As one evolves to higher scales $\mu > \mu_0$, the renormalization group beta function β_N for SU(N) vanishes $\beta_N = 0$ at least at one-loop level so the $g_N(\mu)$ can behave independent of the scale as expected by conformality. On the other hand, the beta function β_1 for U(1) is positive definite in the unadorned theory, given at one loop by

$$b_1 = \frac{11N}{48\pi^2} \tag{13}$$

where N is the number of colors.

The corresponding coupling satisfies

$$\frac{1}{\alpha_1(\mu)} = \frac{1}{\alpha_1(M)} + 8\pi b_1 \ln\left(\frac{M}{\mu}\right) \qquad (14)$$

so the Landau pole, putting $\alpha(\mu) = 0.1$ and $N = 3$, occurs at

$$\frac{M}{\mu} = \exp\left[\frac{20\pi}{11}\right] \simeq 302 \qquad (15)$$

so for $\mu = 4$ TeV, $M \sim 1200$ TeV. The coupling becomes "strong" $\alpha(\mu) = 1$ at

$$\frac{M}{\mu} = \exp\left[\frac{18\pi}{11}\right] \simeq 171 \qquad (16)$$

or $M \sim 680$ TeV.

We may therefore ask whether the new term $\mathcal{L}_{comp}$ in the lagrangian, necessary for anomaly cancellation, can solve this problem for conformality?

Indeed there is the real counterpart of Eq,(10) which has the form

$$\mathcal{L}_{comp}^{(1),real} = \frac{g^2}{4\pi} \Sigma_{p=1}^{p=n} \Sigma_k B_{pk} \mathrm{ReTrln} \left(\frac{\Phi_k}{v} \right) F_{\mu\nu}^{(p)} F^{(p)\mu\nu} \tag{17}$$

and this contributes to the U(1) gauge propagator and to the U(1) $\beta-$function. Using our formula for B_{pk}, the one-loop quadratic divergence for a bifundamental scalar loop cancels because

$$\Sigma_k B_{pk} = 0 \tag{18}$$

which confirms the cancellation already expected.

Since the scale v breaks conformal invariance, the matter fields acquire mass, so the one-loop diagram [2] has a logarithmic divergence proportional to

$$\int \frac{d^4p}{v^2}\left[\frac{1}{(p^2-m_k^2)}-\frac{1}{(p^2-m_{k'}^2)}\right] \sim -\frac{\Delta m_{kk'}^2}{v^2}\ln\left(\frac{\Lambda}{v}\right) \tag{19}$$

the sign of which depends on $\delta m_{kk'}^2 = (m_k^2 - m_{k'}^2)$.

[2]The usual one-loop $\beta-$function is of order h^2 regarded as an expansion in Planck's constant: four propagators each $\sim h$ and two vertices each $\sim h^{-1}$ (c.f. Y. Nambu, Phys. Lett. **B26,** 626 (1968)). The diagram considered is also $\sim h^2$ since it has three propagators, one quantum vertex $\sim h$ and an additional h^{-2} associated with $\Delta m_{kk'}^2$.

To achieve conformality of U(1), a constraint must be imposed on the mass spectrum of matter bifundamentals, *viz*

$$\Delta m^2_{kk'} \propto v^2 \left(\frac{11N}{48\pi^2} \right) \tag{20}$$

with a proportionality constant of order one which depends on the choice of model, the n of Z_n and the values chosen for $A_m, m = 1, 2, 3$. This signals how conformal invariance must be broken at the TeV scale in order that it can be restored at high energy; it is interesting that such a constraint arises in connection with an anomaly cancellation mechanism which necessarily breaks conformal symmetry.

To give an explicit model, consider the case of Z_4 and $A_m = (1,1,1,1)$ treated earlier for which one finds:

$$\Delta m^2_{kk'} = \frac{3}{2} v^2 \left(\frac{11N}{48\pi^2} \right) \tag{21}$$

In a more general model, the analog of Eq.(21) involves replacement of $\frac{3}{2}$ by a generally different coefficient derivable for each case from the coefficient B_{pk} in Eq.(10).

With such a constraint, the one-loop β_1 vanishes in addition to β_N so that the couplings $\alpha_1(\mu)$ and $\alpha_N(\mu)$ can be scale invariant for $\mu \geq \mu_0$. For such conformal invariance at high energy to be maintained to higher orders of perturbation theory probably requires a global symmetry, for example an explicit form of misaligned supersymmetry.

6. DARK MATTER CANDIDATE

Definition of a Z_2 symmetry

In the nonsupersymmetric quiver gauge theories, the gauge group, for abelian orbifold $AdS_5 \times S^5/Z_n$ is $U(N)^n$. In phenomenological application $N = 3$ and n reduces eventually after symmetry breaking to $n = 3$ as in trinification. The chiral fermions are then in the representation of $SU(3)^3$:

$$(3, 3^*, 1) + (3^*, 1, 3) + (1, 3, 3^*) \tag{22}$$

This is as in the **27** of E_6 where the particles break down in to the following representations of the $SU(3) \times SU(2) \times U(1)$ standard model group:

$$Q, \quad u^c, \quad d^c, \quad L \quad e^c \quad N^c \tag{23}$$

transforming as

$$(3, 2), \quad (3^*, 1), \quad (3^*, 1), \quad (1, 2), \quad (1, 1), \quad (1, 1) \tag{24}$$

in a **16** of the $SO(10)$ subgroup.

In addition there are the states

$$h, \qquad h*, \qquad E, \qquad E* \tag{25}$$

transforming as

$$(3,1), \quad (3^*,1), \quad (2,1), \quad (2,1) \tag{26}$$

in a **10** of $SO(10)$ and finally

$$S \tag{27}$$

transforming as the singlet

$$(1,1) \tag{28}$$

It is natural to define a Z_2 symmetry R which commutes with the $SO(10)$ subgroup of $E_6 \longrightarrow O(10) \times U(1)$ such that $R = +1$ for the first **16** of states. Then it is mandated that $R = -1$ for the **10** and **1** of SO(10) because the following Yukawa couplings must be present to generate mass for the fermions:

$$16_f 16_f 10_s, \quad 16_f 10_f 16_s, \quad 10_f 10_f 1_s, \quad 10_f 1_f 1_s \tag{29}$$

which require $R = +1$ for $10_s, 1_s$ and $R = -1$ for 16_s.

The LSMP is the lightest linear combination of the three neutral components of E, E^* and S. It is expected to have mass ~ 1 TeV and is a WIMP candidate for dark matter.

Contribution of the LMSP to the Cosmological Energy Density

The LMSP act as cold dark matter WIMPs, and the calculation of the resultant energy density follows a well-known path. Here we follow the procedure in a recent technical book by Mukhanov.

The LMSP decouple at temperature T_*, considerably less than their mass M_{LMSP}; we define $x_* = M_{LMSP}/T_*$. Let the annihilation cross-section of the LMSP at decoupling be σ_*. Then the dark matter density Ω_m, relative to the critical density, is estimated as

$$\Omega_m h_{75}^2 = \frac{\tilde{g}_*^{1/2}}{g_*} x_*^{3/2} \left(\frac{3 \times 10^{-38} cm^2}{\sigma_*} \right) \qquad (30)$$

where h_{75} is the Hubble constant in units of $75km/s/Mpc$. $g_* = (g_b + \frac{7}{8} g_f)$ is the effective number of degrees of freedom (dof) at freeze-out for all particles which later convert their energy into photons; and $\tilde{g}_*$ is the number of dof which are relativistic at T_*.

Thus, to estimate the non-baryonic dark matter density arising from LMSPs, we need estimates of five quantities occurring in Eq.(30): $h_{75}, \tilde{g}_*, g_*, x_*$ and σ_*, and to this we now turn.

We start with h_{75} where the central value from WMAP3 is $H_0 = 72km/s/Mpc$ and so a good estimate of h_{75} is $h_{75} = 72/75 = 0.96$.

For the energy ranges we consider, $\tilde{g}_* = g_*$ and depends on the freeze-out temperature T_*. We consider masses in the range $30GeV \leq M_{LMSP} \leq 2TeV$. Since $x_* = M_{LMSP}/T_*$ is relatively insensitive to M_{LMSP}, as we shall see shortly, always within the values $20 \leq x_* \leq 30$, the freeze-out temperatures of relevance will be in the range $1GeV \leq T_* \leq 100GeV$.

For these T_* we compute:

For $100GeV \geq T_* \geq 10GeV$:

$$g_* = 86.25 \tag{31}$$

For $10GeV \geq T_* \geq 3GeV$:

$$g_* = 75.75 \tag{32}$$

For $3GeV \geq T_* \geq 1GeV$:

$$g_* = 61.75 \tag{33}$$

The value of x_* may be estimated using the formula

$$X = 0.038\sqrt{g_*}M_{Planck}M_{LMSP}\sigma_* \quad (34)$$

$$x_* = lnX - \frac{1}{2}lnlnX \quad (35)$$

We have already estimated g_*. We use $M_{Planck} = 10^{19}GeV$ and $30GeV \leq M_{LMSP} \leq 2TeV$.

The annihilation cross section σ_* for LMSPs at freeze-out may be estimated using analogs of the Feynman graphs used for LSPs. A naive estimate of σ_* follows from the formula

$$\sigma_* = G_F^2 T_*^2 \quad (36)$$

although detailed calculations give a cross-section smaller by about an order of magnitude.

From Eq.(30) and the estimates for $h_{75}, g_*, \tilde{g}_*, x_*$ already given, the cross-section must satisfy $\sigma_* \geq 3 \times 10^{-36} cm^2$. From Eq. (36) this implies that $T_* > 1.4 GeV$ and $M_{LMSP} > 28 GeV$ are *lower* bounds on the freeze-out temperature and LMSP mass respectively.

The low mass $M_{LMSP} = 28 GeV$ for a WIMP is likely ruled out by empirical bounds. More realistically we may take $M_{LMSP} = 100 GeV$ and $T_* = 4 GeV$. Then the naive estimate Eq.(36) give $\sigma_* = 2.5 \times 10^{-35} cm^2$. Substitution in Eq.(35) gives $x_* = 25$, consistent with these choices.

As for the contribution of an 100 GeV LMSP to the cold dark matter, substitution into Eq.(30) with these parameters suggests Ω_{LMSP} is only about 10% of the observed $\Omega_{CDM} = 0.24$. But the estimate Eq.(36) for the freeze-out annihilation cross-section, σ_*, is sufficiently unreliable (the physical σ_* can be an order of magnitude smaller) that the LMSP can still provide all the observed non-baryonic dark matter, even for $M_{LMSP} = 100 GeV$.

Discussion of LMSP Dark Matter

The LMSP is a viable candidate for a cold dark matter particle which can be produced at the LHC. The distinction from the neutralino will require establishment of the $U(3)^3$ gauge bosons, extending the 3-2-1 standard model and the discovery that the LMSP is in a bifundamental representation thereof.

To confirm that the LMSP is the dark matter particle would, however, require direct detection of dark matter.

Finally, it has been established that misaligned supersymmetry can provide (i) naturalness without one-loop quadratic divergence for the scalar mass and anomaly cancellation; (ii) precise unification of the coupling constants; and (iii) a viable dark matter candidate. This completes the demonstration that all of these three primary virtues of supersymmetry, (i)-(iii), can actually be achieved without supersymmetry and weakens the motivation for weak-scale supersymmetry. Quiver gauge theories with gauge group $U(3)^3$ or $U(3)^n$ with $n \geq 4$ seem at least equally as likely to be employed by Nature.

SUMMARY

- Nonsupersymmetric quiver gauge theories motivated by AdS/CFT correspondence are very interesting to model builders.
- Phenomenology of conformality has striking resonances with the standard model.
- 4 TeV Unification predicts three families and new particles around 4 TeV accessible to experiment (LHC).
- The scalar propagator in these theories has no quadratic divergence iff there are chiral fermions.
- Anomaly cancellation in effective lagrangian connected to consistency of U(1) factors.
- Dark matter candidate (LMSP) will be produced at LHC, then (in)directly detected.