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HUNTING FOR NEW

PHYSICS WITH

UNITARITY BOOMERANGS

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OUTLINE

Introduction

Unitarity boomerang

Kobayashi-Maskawa parametrizarion

Diminution of Errors.

Optimal boomerang

CP Violation

Hunting for New Physics

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Introduction

As is well known, there are different ways of parameterizing the Kobayashi-Maskawa quark mixing matrix, V_{KM}. For three generations of quarks, V_{KM} is a 3×3 unitary mixing matrix with three rotation angles (θ_1, θ_2, θ_3) and one CP violating phase δ. The magnitudes of the elements V_{ij} of V_{KM} are physical quantities which do not depend on parametrization. However, the value of δ does. For example, in the Particle Data Group (PDG) parametrization adopted $\delta \sim 70^\circ$, whereas the phase in the original KM parametrization has a different value, $\delta \sim 90^\circ$. Care must be exercised in quoting a value of δ, as it depends on how the matrix is parameterized. For example, the statement made after Eq. (11.3) in the current edition of PDG is misleading, because it identifies, incorrectly, the phase δ of KM.

It can therefore be more useful to employ only physically-measurable quantities. To this end, it has long ago been suggested that a unitarity triangle (UT) be used as a useful presentation for the quark flavor mixing, especially of CP violation. Because of the unitary nature of the KM matrix, one has $\Sigma_i V_{ij} V_{ik}^* = \delta_{jk}$ and $\Sigma_i V_{ji} V_{ki}^* = \delta_{jk}$, where the first and second indices of V_{ij} take the values u, c, t, ... and d, s, b, ..., respectively. For three generations of quarks, when $j \neq k$, these equations form closed triangles in a plane, the UTs. Six UTs can be formed with all of them having the same area. $A(UT)$, which is equal to half of the value of the Jarlskog determinant J, so that $A(UT) = \frac{1}{2}J$. The inner angles of a given UT are therefore closely related to the CP violating measure J.

When the inner angles are measured independently, their sum, whether it turns out to be consistent with precisely 180°, provides a test for the unitarity of the KM matrix. The unitarity triangle is also a popular way, to present CP violation, with three generations of quarks. A UT, however, does not contain all the information encoded in the KM matrix, V_{KM}. Although a UT has three inner angles and three sides, it contains only three independent parameters. The three parameters can be chosen to be two of the three inner angles and the area, or the three sides, or some combination thereof. One needs an additional parameter fully to represent the physics: this is hardly surprising, as the original UT idea involved only two, of the three, rows or columns of the 3×3 matrix, V_{KM}, An improved presentation is thus rendered desirable, in order better to present the KM matrix, V_{KM}, diagrammatically. We propose such a new diagram, the unitarity boomerang. The unitarity boomerang contains information from a pair of UTs. The different ways of choosing the pair contain, of course, equivalent information. Nevertheless, the specific choice can be made, as we shall discuss in detail, such as to maximize the minimum vertex angle in the unitarity boomerang. This choice is, we believe, the most convenient.

Unitarity Boomerang

We indicate the KM matrix and its elements by $V_{KM} = (V_{KM})_{ij}$, with $i = u, c, t$ and $j = b, s, d$. The unitarity of this matrix implies $\Sigma_i V_{ij} V_{ik}^* = \delta_{jk}$ and $\Sigma_j V_{ij} V_{kj}^* = \delta_{ik}$. The $j \neq k$ and $i \neq k$ cases form, respectively, the six possible different UT presentations for V_{KM} in a convenient two-dimensional plane. There are, thus, a total of 18 inner angles in the six UTs. However, only 9 are different because, by Euclidean geometry, each angle, in any particular UT, must have its equal counterpart in another, different, UT. This coincides with the fact that there are 9 different phase expressions of the KM matrix for different parameterizations.

To understand this simple but crucial discussion consider the two UTs defined by

$$\begin{aligned}
UT(a) \qquad & (V_{KM})_{ud}(V_{KM})^*_{ub} \\
+ \quad & (V_{KM})_{cd}(V_{KM})^*_{cb} \\
+ \quad & (V_{KM})_{td}(V_{KM})^*_{tb} = 0 \\
UT(b) \qquad & (V_{KM})_{ud}(V_{KM})^*_{td} \\
+ \quad & (V_{KM})_{us}(V_{KM})^*_{ts} \\
+ \quad & (V_{KM})_{ub}(V_{KM})^*_{tb} = 0
\end{aligned} \tag{1}$$

The inner angles defined by UT (a), in Eq. (1), are

$$\begin{aligned}
\phi_1(\beta) &= \arg\left(-\frac{(V_{KM})_{cd}(V_{KM})^*_{cb}}{(V_{KM})_{td}(V_{KM})^*_{tb}}\right) \\
\phi_2(\alpha) &= \arg\left(-\frac{(V_{KM})_{td}(V_{KM})^*_{tb}}{(V_{KM})_{ud}(V_{KM})^*_{ub}}\right) \\
\phi_3(\gamma) &= \arg\left(-\frac{(V_{KM})_{ud}(V_{KM})^*_{ub}}{(V_{KM})_{cd}(V_{KM})^*_{cb}}\right) \qquad (2)
\end{aligned}$$

Correspondingly, the unitarity triangle, $UT(b)$ in Eq. (1), defines another three inner angles

$$\begin{aligned}
\phi'_1(\beta') &= \arg\left(-\frac{(V_{KM})_{us}(V_{KM})^*_{ts}}{(V_{KM})_{ub}(V_{KM})^*_{tb}}\right) \\
\phi'_2(\alpha') &= \arg\left(-\frac{(V_{KM})_{ub}(V_{KM})^*_{tb}}{(V_{KM})_{ud}(V_{KM})^*_{td}}\right) \\
\phi'_3(\gamma') &= \arg\left(-\frac{(V_{KM})_{ud}(V_{KM})^*_{td}}{(V_{KM})_{us}(V_{KM})^*_{ts}}\right)
\end{aligned}$$

It is clear that $\phi_2' = \phi_2$. Since all the six UTs have the same area $J/2$, not all the different nine angles are independent. For example

$$J = |(V_{KM})_{td}(V_{KM})^*_{tb}||(V_{KM})_{ud}(V_{KM})^*_{ub}| \sin\phi_2$$
$$= |(V_{KM})_{td}(V_{KM})^*_{tb}||(V_{KM})_{cd}(V_{KM})^*_{cb}| \sin\phi_1$$
$$= |(V_{KM})_{us}(V_{KM})^*_{ts}||(V_{KM})_{ub}(V_{KM})^*_{tb}| \sin\phi_1'$$
$$= |(V_{KM})_{ud}(V_{KM})^*_{td}||(V_{KM})_{us}(V_{KM})^*_{ts}| \sin\phi_3'.$$

It can be shown that only four independent parameters are needed to parameterize the six UTs, and that any two different UTs contain the needed 4 parameters.

The values for the angles in $UT(a)$, of Eq.(1), derived from various experiments given by PDG are: $\phi_1 = (21.46 \pm 0.98)^\circ$ (derived from data on $\sin(2\phi_1) = 0.681 \pm 0.025$), and the values for ϕ_2 and ϕ_3 are $(88^{+6}_{-5})^\circ$ and $(77^{+30}_{-32})^\circ$, respectively. These values are consistent with the unitarity of the KM matrix within error bars, and therefore also with a choice of presentation which we now formulate in terms of a novel combination of two different unitarity triangles (a) and (b). $UT(a)$, defined by Eq. (1), is almost a right triangle, by virtue of ϕ_2. Numerically, the angles ϕ_1' and ϕ_3' are close to ϕ_1 and ϕ_2, respectively. All the angles in the two UTs are sizable, making experimental determination of them merely challenging, while for the other four choices of UT there is always, at least, one small angle where measurement may be exceptionally difficult.

It is therefore easiest to work with the two UTs, $UT(a)$ and $UT(b)$, for practical purposes. We now show that, by combining information from these two UTs, into the boomerang diagram [1] displayed in Fig. 1, on the next slide, all information needed to specify the KM matrix, V_{KM}, can be extracted.

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[1] The name arises from resemblance to the hunting instrument.

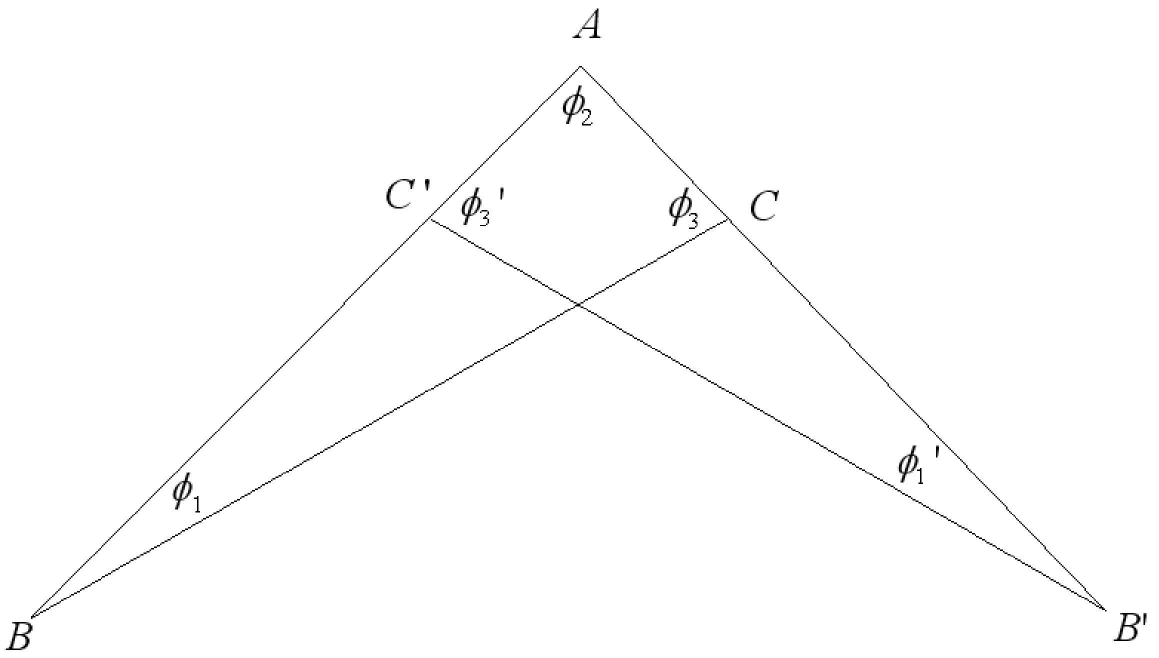

Figure 1: The unitarity boomerang. The sides are: $AC = |(V_{KM})_{ud}(V_{KM})^*_{ub}|$, $AC' = |(V_{KM})_{ub}(V_{KM})^*_{tb}|$, $AB = |(V_{KM})_{td}(V_{KM})^*_{tb}|$, $AB' = |(V_{KM})_{ud}(V_{KM})^*_{td}|$, $BC = |(V_{KM})_{cd}(V_{KM})^*_{cb}|$ and $B'C' = |(V_{KM})_{us}(V_{KM})^*_{ts}|$.

The unitarity boomerang is formed by locating the common angle $\phi_2' = \phi_2$ from the two UTs of $UT(a)$ and $UT(b)$ at the top point A and the shortest sides, $AC = |(V_{KM})_{ud}(V_{KM})^*_{ub}|$ and $AC' = |(V_{KM})_{ub}(V_{KM})^*_{tb}|$, on the opposite sides. The other sides are: $AB = |(V_{KM})_{td}(V_{KM})^*_{tb}|$, $AB' = |(V_{KM})_{ud}(V_{KM})^*_{td}|$, $BC = |(V_{KM})_{cd}(V_{KM})^*_{cb}|$ and $B'C' = |(V_{KM})_{us}(V_{KM})^*_{ts}|$. We emphasize that the Fig. is drawn with the central experimental values of $AC = 3.50 \times 10^{-3}$, $AC' = 3.59 \times 10^{-3}$, $AB = 8.73 \times 10^{-3}$, $AB' = 8.51 \times 10^{-3}$, $BC = 9.36 \times 10^{-3}$ and $B'C' = 9.19 \times 10^{-3}$. One can choose the area $(J/2)$ of the triangles, two inner angles from one of the UTs (for example ϕ_1 and ϕ_2), and a third angle from the other UT (for example ϕ'_3) as the four independent parameters.

Kobayashi-Maskawa parametrization

To show explicitly how the unitarity boomerang can provide all information needed to specify the quark flavor mixing, we work with a specific parametrization, V_{KM}, originally given by Kobayashi and Maskawa

$$V_{KM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}$$

One can also work with other parameterizations, such as that adopted by the PDG. But we find an interesting feature of the original KM parametrization which turns out to be very convenient for the discussions of the unitarity boomerang.

Using experimental values for for $(V_{KM})_{us} = 0.2257 \pm 0.0010$, $(V_{KM})_{ub} = 0.00359 \pm 0.00016$ and $(V_{KM})_{td} = 0.00874^{+0.00026}_{-0.00037}$, one finds that $s_2 s_3 << 1$. At a few percent level, one has $(V_{KM})_{tb} = (c_1 s_2 s_3 - c_2 c_3 e^{-i\delta}) \approx -c_2 c_3 e^{-i\delta}$.

Then

$$\begin{aligned} \phi_2 &= \arg\left(-\frac{s_1 s_2 * (c_1 s_2 s_3 - c_2 c_3 e^{-i\delta})}{c_1 * (-s_1 s_3)}\right) \\ &\approx \arg\left(\frac{s_1 s_2 * (-c_2 c_3 e^{-i\delta})}{c_1 * s_1 s_3}\right) = \pi - \delta. \end{aligned}$$

The CP violating phase δ, in this parametrization, is equal to $\pi - \phi_2$, to a good approximation.

The fact that $\phi_2 = (88^{+6}_{-5})^\circ$ implies $\delta \approx 90^\circ$. The approximate right angle at the top of the boomerang diagram may indicate that CP, from a deeper perspective, is maximally violated, Kobayashi and Maskawa, with remarkable prescience, made an excellent choice of parametrization. We suggest that the original parametrization of Kobayashi-Maskawa matrix be used as the standard parametrization.

A parametrization suggested by Fritzsch and Xing, which also has its phase close to ϕ_2, is another alternative interesting parametrization. From the unitarity boomerang, one can easily obtain approximation solutions for the four physical parameters. One first notices that the relation in Eq.(4) allows one to read off the δ from the top angle in the diagram. Taking the ratio, of the two sides AC/AC' or AB/AB', one obtains $|(V_{KM})_{ud}/(V_{KM})^*_{tb}| \approx c_1$ since $|(V_{KM})_{tb}|$ is very close to 1. With c_1 and therefore s_1 known, the length of the sides AB and AC' then provide the values for s_2 and s_3.

One can obtain more precise solutions by using the following information from four sides, $AC = a$, $BC = b$, $AB = c$ and $AB' = d$ of the unitarity boomerang:

$$\begin{aligned} a &= |(V_{KM})_{ud}(V_{KM})^*_{ub}| = c_1 s_1 s_3 \ , \quad b = |(V_{KM})_{c} \\ c &= |(V_{KM})_{td} V_{KM})^*_{tb}| = s_1 s_2 |c_1 s_2 s_3 - c_2 c_3 e^{-i\delta}| \end{aligned}$$

Using the above, one can express $s_{1,2,3}$ and δ as functions of a, b, c and d. The KM parameters can be determined. For example

$$a^2 - c_1^2 + c_1^4 \left(\frac{c^2}{d^2} - \frac{b^2}{c_1^4 - c_1^2 + d^2} \right) = 0 \ . \quad (5)$$

Solving for the roots of the above equations, the c_1^2 is determined up to four possible discrete solutions. Restricting to real positive solutions with magnitude less than 1, one can further limit the choices.

The other angles, and the phase, can be determined from the following relations

$$s_2 = \frac{d}{c_1 s_1} \; , \quad s_3 = \frac{a}{c_1 s_2} \; ,$$

$$\cos\delta = \frac{b^2/s_1^2 c_2^2 - (c_1^2 c_2^2 s_3^2 + s_2^2 c_3^2)}{2 c_1 c_2 s_2 c_3 s_3} = \frac{c_1^2 s_2^2 s_3^2 + c_2^2 c_3^2}{2 c_1 c_2 s_2}$$

After applying the constraint on $c^2_{2,3}$, that they satisfy $0 \leq c^2_{2,3} \leq 1$, the solution is even more restricted. Putting in numerical values, for the sides, and comparing with the approximate solution above, we find that a unique solution survives.

Numerically, with the current central values for a, b c and d, we obtain

$$c_1 = 0.97419 \; , \quad s_2 = 0.0387 \; , \quad s_3 = 0.0162 \; , \quad \delta = 88$$

and these numbers are self consistent.

Diminution of Errors.

One should be aware, that there remain errors, on the sides and angles of the boomerang. This leads to distortion of the UB away from the true one. When constructing the UB, one can first use measurable quantities without assuming unitarity to form one of the UT, say, the UT defined by triangle ABC in Fig. 1. This can be achieved by using the measured α and β and also the length of side AB, $c = |(V_{KM})_{td}(V_{KM})^*_{tb}|$. The major error comes from the uncertainty in $|(V_{KM})_{td}(V_{KM})^*_{tb}|$ measured from $B_b - \bar{B}_d$ mixing. Assuming $|(V_{KM})_{tb}|$ is almost one, then, $|(V_{KM})_{td}| = (8.09 \pm 0.6) \times 10^{-3}$. One then uses information on the values of $|(V_{KM})_{ud}|$ and $|(V_{KM})_{ub}|$ to construct the sides AB' and AC' to complete the boomerang. The error in $|(V_{KM})_{td}|$ will cause uncertainty in the side AB' of the UB with $d = (7.88 \pm 0.58) \times 10^{-3}$. At present within error bars, one cannot be sure which side, AB

or AB', is longer. Further reduce the errors in $|(V_{KM})_{td}(V_{KM})^*_{tb}|$ can be achieved by better understanding of the bag factor in $B_d - \bar{B}_d$ mixing.

Another way to improve the situation is to note that the value $|(V_{KM})_{tb}|/|(V_{KM})_{ud}|$ plays an important role which also determine the ratio of AC and AC'. Therefore precise measurement of $|(V_{KM})_{tb}|$ is crucial in constructing an accurate UB. Future studies of top quark decay and single top quark production at colliders, such as the LHC, will provide useful information.

To give a quantitative feeling, we have carried out an estimate assuming that the errors in a, b, c and d are given by the current PDG data with Gaussian errors to obtain the resultant errors in the KM angles. We obtain $\Delta c_1 = 0.046$, an error which is reasonably small. But errors on $s_{2,3}$ are large with $\Delta s_2 = 0.032$ and $\Delta s_3 = 0.077$. Such a larger error bolsters preference for the boomerang, to disentangle, most perspicuously, the quark flavor mixing. Note that errors, on $s_{2,3}$, are due to empirically-generated uncertainties on $(V_{KM})_{td}$, $(V_{KM})_{cb}$ and $(V_{KM})_{ub}$.

Indeed, when we look more closely at Eq. (5), it does turn out that the quantity c enters that equation, only in a combination (c^2/d^2), just so that $(V_{KM})_{td}$ cancels out. If one takes into account, the errors are reduced to $\Delta c_1 = 0.032$, $\Delta s_2 = 0.023$ and $\Delta s_3 = 0.055$.

If uncertainties on all four sides can be reduced, say by another factor of three, we project that errors can be reduced to $\Delta c_1 = 0.011$, $\Delta s_2 = 0.076$ and $\Delta s_3 = 0.018$, thus illustrating how the chosen boomerang may, in the foreseeable future, return to increase human knowledge. Our proposal, to move from a single triangle to a boomerang combination, therefore reflects, more than anything else, the increase in precision which is justifiably anticipated from the high-energy experiments.

Optimal Boomerang

Out of the six possible UTs, there are 9 different ways to have a common inner angle from two UTs. In addition one can align the two longer sides or one longer and one shorter sides to form a UB. Therefore total 18 possible UBs. We find that among these UBs, there is only one which does not involve very small angles and is good for practical uses. Although the UBs have different areas, there is an invariant quantity for all UBs which is equal to a quarter of the Jarlskog parameter J squared. We also discuss how new physics can be tested by using information from UBs.

The UBs are constructed by using two UTs. Let us summarize some of the relevant information for UTs here. There are six triangles. The inner angles of three UTs involving two columns are defined by, $\Sigma_i (V_{KM})_{ij}(V_{KM})^*_{ik} = 0\ , j \neq k$,

$$UT(1): \quad (V_{KM})_{ub}(V_{KM})^*_{us} + (V_{KM})_{cb}(V_{KM})^*_{cs}$$

$$\alpha_1 = arg\left(-\frac{(V_{KM})_{ub}(V_{KM})^*_{us}}{(V_{KM})_{cb}(V_{KM})^*_{cs}}\right)\ ,\ \beta_1 =$$

$$\gamma_1 = arg\left(-\frac{(V_{KM})_{cb}(V_{KM})^*_{cs}}{(V_{KM})_{tb}(V_{KM})^*_{ts}}\right)\ ,$$

$$UT(2): \quad (V_{KM})_{ud}(V_{KM})^*_{ub} + (V_{KM})_{cd}(V_{KM})^*_{cb}$$

$$\alpha_2 = arg\left(-\frac{(V_{KM})_{td}(V_{KM})^*_{tb}}{(V_{KM})_{ud}(V_{KM})^*_{ub}}\right)\ ,\ \beta_2 =$$

$$\gamma_2 = arg\left(-\frac{(V_{KM})_{ud}(V_{KM})^*_{ub}}{(V_{KM})_{cd}(V_{KM})^*_{cb}}\right)\ ,$$

$$UT(3): \quad (V_{KM})_{ud}(V_{KM})^*_{us} + (V_{KM})_{cd}(V_{KM})^*_{cs}$$

$$\alpha_3 = arg\left(-\frac{(V_{KM})_{td}(V_{KM})^*_{ts}}{(V_{KM})_{cd}(V_{KM})^*_{cs}}\right)\ ,\ \beta_3 =$$

$$\gamma_3 = arg\left(-\frac{(V_{KM})_{ud}(V_{KM})^*_{us}}{(V_{KM})_{td}(V_{KM})^*_{ts}}\right)\ .$$

For convenience, we have labeled the $UT(i)$ by the missing ith quark in the down-quark sector.

The inner angles of the three UTs involving two rows are given by, $\Sigma_i (V_{KM})_{ji} (V_{KM})^*_{ki} = 0\ , j \neq k$,

$$UT(1') : \quad (V_{KM})_{cd}(V_{KM})^*_{td} + (V_{KM})_{cs}(V_{KM})^*_{ts}$$

$$\alpha'_1 = arg\left(-\frac{(V_{KM})_{cs}(V_{KM})^*_{ts}}{(V_{KM})_{cd}(V_{KM})^*_{td}}\right)\ , \quad \beta'_1 =$$

$$\gamma'_1 = arg\left(-\frac{(V_{KM})_{cb}(V_{KM})^*_{tb}}{(V_{KM})_{cs}(V_{KM})^*_{ts}}\right)\ ,$$

$$UT(2') : \quad (V_{KM})_{ud}(V_{KM})^*_{td} + (V_{KM})_{us}(V_{KM})^*_{ts}$$

$$\alpha'_2 = arg\left(-\frac{(V_{KM})_{ub}(V_{KM})^*_{tb}}{(V_{KM})_{ud}(V_{KM})^*_{td}}\right)\ , \beta'_2 =$$

$$\gamma'_2 = arg\left(-\frac{(V_{KM})_{ud}(V_{KM})^*_{td}}{(V_{KM})_{us}(V_{KM})^*_{ts}}\right)\ ,$$

$$UT(3') : \quad (V_{KM})_{ud}(V_{KM})^*_{cd} + (V_{KM})_{us}(V_{KM})^*_{cs}$$

$$\alpha'_3 = arg\left(-\frac{(V_{KM})_{ub}(V_{KM})^*_{cb}}{(V_{KM})_{us}(V_{KM})^*_{cs}}\right)\ , \quad \beta'_3 =$$

$$\gamma'_3 = arg\left(-\frac{(V_{KM})_{ud}(V_{KM})^*_{cd}}{(V_{KM})_{ub}(V_{KM})^*_{cb}}\right)\ .$$

Here the $UT(i')$ labeled by the missing i'th quark in the up-quark sector.

It is clear from the above definitions that among the 18 inner angles of the six UTs, only 9 of them are different. Explicitly, we have

$$\begin{array}{lll} \alpha_1 = \alpha'_3 \;, & \beta_1 = \beta'_2 \;, & \gamma_1 = \gamma'_1 \;, \\ \alpha_2 = \alpha'_2 \;, & \beta_2 = \beta'_1 \;, & \gamma_2 = \gamma'_3 \;, \\ \alpha_3 = \alpha'_1 \;, & \beta_3 = \beta'_3 \;, & \gamma_3 = \gamma'_2 \;. \end{array} \tag{10}$$

We will choose the 9 inner angles without "prime" in our later discussions.

As pointed out earlier that for a given UT, it contains only 3 independent parameters which is not enough to completely determine parameters in the KM matrix. We have shown that using two triangles with one from $UT(i)$ and another from $UT(i')$ one can always form a boomerang like diagram, the unitarity boomerang and (UB) contains all information need to reconstruct the KM matrix. Let us show more details in the following.

There are total 18 different ways of constructing UBs. There are 9 different ways to pair up a common angle by taking one UT from $UT(i)$ and another from $UT(i')$. One can then overlap the longer side from on UT with the shorter side of the other UT, or overlap the longer sides of the two UTs. We will label the former 9 UBs as $B_{ii'a}$ and the later 9 UBs as $\tilde{B}_{ii'a}$. Here the index a indicates the common angle. The common angles and their current central experimental values of the UBs are given in the following

$$\begin{pmatrix} B(\tilde{B})_{11\gamma_1} & B(\tilde{B})_{12\beta_1} & B(\tilde{B})_{13\alpha_1} \\ B(\tilde{B})_{21\beta_2} & B(\tilde{B})_{22\alpha_2} & B(\tilde{B})_{23\gamma_2} \\ B(\tilde{B})_{31\alpha_3} & B(\tilde{B})_{32\gamma_3} & B(\tilde{B})_{33\beta_3} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \gamma_1 = 0.0207 & \beta_1 = 1.151 & \alpha_1 = 1.970 \\ \beta_2 = 0.435 & \alpha_2 = 1.535 & \gamma_2 = 1.171 \\ \alpha_3 = 2.686 & \gamma_3 = 0.455 & \beta_3 = 5.531 \times 10^{-4} \end{pmatrix} \tag{11}$$

In the above when calculating the inner angles, we have used the central values $|(V_{KM})_{ud}| = 0.97419$, $|(V_{KM})_{us}| = 0.2257$, $|(V_{KM})_{ub}| =$

0.00359 and $\alpha_2 = 88^\circ$. We will use these values in our later discussions for illustrations.

We now display the UBs. In Fig. 2 we show some details for the boomerang formed by using $UT(2)$ and $UT(2')$. Figs. 2. a) and 2. b) are for $B_{22\alpha_2}$, and $\tilde{B}_{22\alpha_2}$. The 9 $B_{ii'a}$ are shown in Fig. 3. One can also construct the 9 UBs of $\tilde{B}_{ii'a}$. But they contain similar information as those of $B_{ii'a}$, they can be obtained by flipping one of the UT in each of the UB, we therefore have not displayed them.

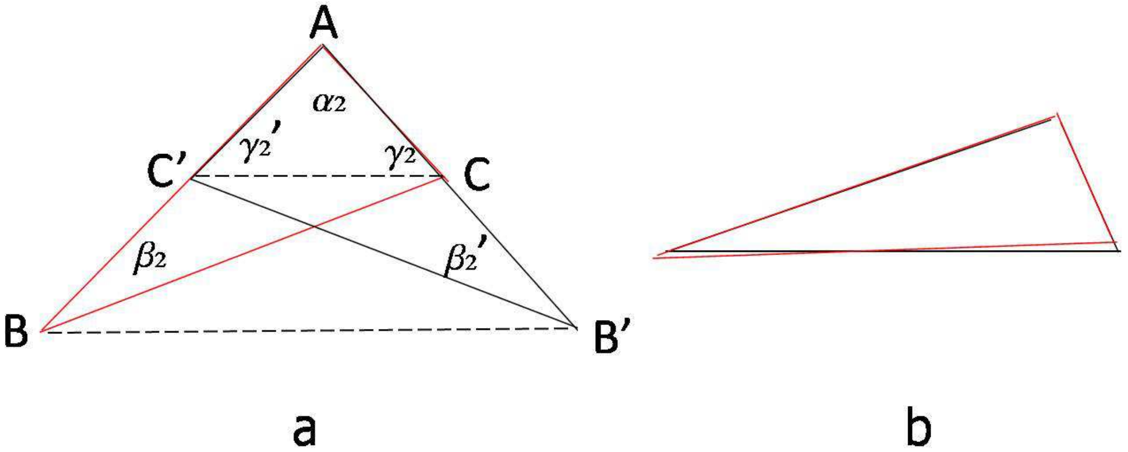

Figure 2: The unitarity boomerangs $B_{22\alpha_2}$ and $\tilde{B}_{22\alpha_2}$. Figure 2.a) overlaps one long and one short sides from the two UTs. The sides are: $AB = |(V_{KM})_{td}(V_{KM})^*_{tb}|$, $AC = |(V_{KM})_{ud}(V_{KM})^*_{ub}|$, $BC = |(V_{KM})_{cd}(V_{KM})^*_{cb}|$, $AB' = |(V_{KM})_{ud}(V_{KM})^*_{td}|$, $AC' = |(V_{KM})_{ub}(V_{KM})^*_{tb}|$, and $B'C' = |(V_{KM})_{us}(V_{KM})^*_{ts}|$. Figure 2.b) is for $\tilde{B}_{22\alpha}$. The red (left) lines and black (right) lines indicate the UT taking from $UT(i)$ and $UT(i')$, respectively.

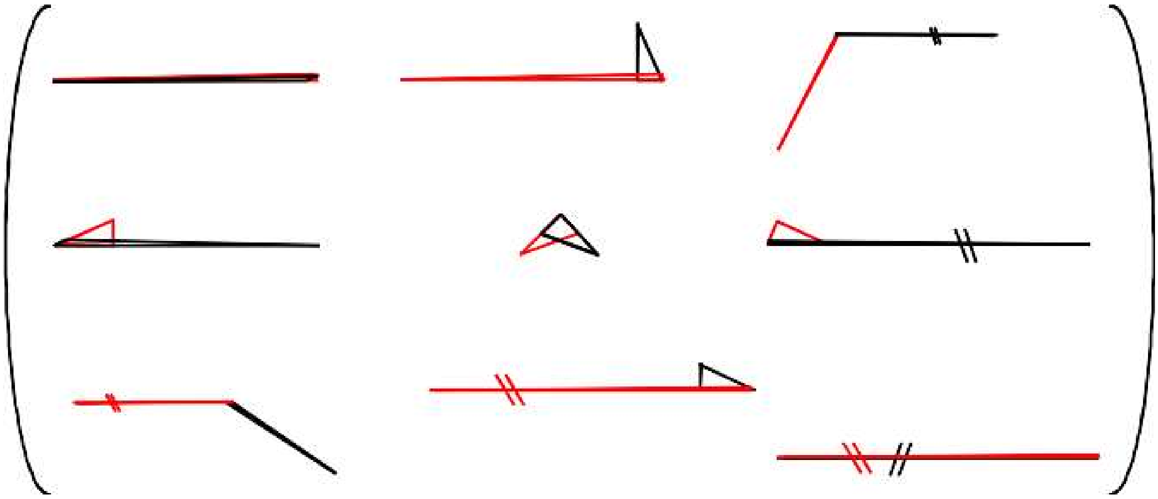

Figure 3: The 9 unitarity boomerangs $B_{ii'a}$. The red (left) lines and black (right) lines indicate the UTs taking from $UT(i)$ and $UT(i')$. In the figure, the symbols double backslash and double slash indicate the UT for $i = 3$ and $i' = 3$. For the 23, 32, 33 entries, the length in the figure for $UT(3)$ and $UT(3')$ should be scaled up by 5. For the 13 and 31 entries, the triangles involve $UT(3,3')$ and $UT(1,1')$ should be scaled up by 10 and 2, respectively.

As can be seen from Fig. 2, the UBs, except the $B_{22\alpha_2}$, all involve a very small angle in the diagram making it difficult to construct, with high accuracy. The $B_{22\alpha_2}$ can display the information more easily. One should therefore work with the $B_{22\alpha_2}$. From the $B_{22\alpha_2}$, one can easily obtain approximate solutions for the four physical parameters. Taking the ratio, of the two sides AC/AC' or AB/AB', one obtains, $|(V_{KM})_{ud}/(V_{KM})^*_{tb}| \approx c_1$ since $|(V_{KM})_{tb}|$ is very close to 1. With c_1 and therefore s_1 known, the length of the sides AB and AC' then provide the values for s_2 and s_3. Here the angles c_i and s_i i refer to the cosine and sine of the angles in the original KM matrix parametrization. One can also obtain more accurate expressions. In this parametrization, the CP violating phase δ is, to a very good approximation, equal to α_2.

CP violation

Information on CP violation are, also, fully encoded, in the UBs. The Jarlskog parameter J plays a fundamental role for CP violation with three generations. In the UT representation, it is related to the area of the UT. All UTs have the same area of $J/2$. One can also construct a geometric representation of CP violation invariant quantity for UB representation. Although the shapes of the UBs vary a lot, there exists a quantity related to the UB constructed areas represent CP violation in an invariant form.

Consider the areas $A_{ABB'}$ and $A_{ACC'}$ of the two triangles ACC' and ABB' in Fig. 1 for $B_{22\alpha_2}$, we find

$$A_{22} = A_{ABB'} = \frac{1}{2}\frac{|(V_{KM})_{td}|}{|(V_{KM})_{ub}|}J \ , \quad A'_{22} = A_{ACC'} =$$

The two triangles ABB' and ACC' are similar to each other.

Similarly for $\tilde{B}_{22\alpha_2}$, one has

$$\tilde{A}_{22} = \tilde{A}_{\tilde{A}\tilde{B}\tilde{B}'} = \frac{1}{2}\frac{|(V_{KM})_{td}|}{|(V_{KM})_{ub}|}J\ ,\quad \tilde{A}'_{22} = \tilde{A}_{\tilde{A}\tilde{C}\tilde{C}'} =$$

Again the two triangles are similar to each other.

This can be generalized to all UBs. The results can be collected in matrix forms. Writing A_{22}, A'_{22}, $\tilde{A}_{22}$ and $\tilde{A}'_{22}$ like areas for other UBs in similar ways, the resultant matrix forms A, A', $\tilde{A}$ and $\tilde{A}'$ are given by

$$A = \frac{1}{2}\begin{pmatrix} \frac{|(V_{KM})_{tb}|}{|(V_{KM})_{cs}|} & \frac{|(V_{KM})_{ts}|}{|(V_{KM})_{ub}|} & \frac{(V_{KM})_{cs}|}{|(V_{KM})_{ub}|} \\ \frac{|(V_{KM})_{cb}|}{|(V_{KM})_{td}|} & \frac{|(V_{KM})_{td}|}{|(V_{KM})_{ub}|} & \frac{(V_{KM})_{cd}|}{|(V_{KM})_{ub}|} \\ \frac{|(V_{KM})_{cs}|}{|(V_{KM})_{td}|} & \frac{|(V_{KM})_{us}|}{|(V_{KM})_{td}|} & \frac{(V_{KM})_{ud}|}{|(V_{KM})_{cs}|} \end{pmatrix} J\ ,\quad A' =$$

$$\tilde{A} = \frac{1}{2}\begin{pmatrix} \frac{|(V_{KM})_{ts}|}{|(V_{KM})_{cb}|} & \frac{|(V_{KM})_{tb}|}{|(V_{KM})_{us}|} & \frac{|(V_{KM})_{us}|}{|(V_{KM})_{cb}|} \\ \frac{|(V_{KM})_{tb}|}{|(V_{KM})_{cd}|} & \frac{|(V_{KM})_{tb}|}{|(V_{KM})_{ud}|} & \frac{|(V_{KM})_{ud}|}{|(V_{KM})_{cb}|} \\ \frac{|(V_{KM})_{cd}|}{|(V_{KM})_{ts}|} & \frac{|(V_{KM})_{ud}|}{|(V_{KM})_{ts}|} & \frac{|(V_{KM})_{us}|}{|(V_{KM})_{cd}|} \end{pmatrix} J\ ,\quad \tilde{A}' =$$

One immediately finds an invariant quantity: $I_{UB} = A_{ij}A^{'}_{ij} = \tilde{A}_{ij}\tilde{A'}_{ij} = J^2/4$. Here no summations on i and j. If this quantity is zero, there is no CP violation. So the universal nature of the Jarlskog parameter J is also present in the UB representation.

Hunting for New Physics.

We now discuss how possible new physics information may show up in the UB analysis. There are different ways the UBs can be used to hunt for new physics. We will discuss two ways to detect new physics beyond the three generation KM model.

One of them is to see if a UB can be formed as expected after the relevant sides, such as the sides shown in Fig.1 are measured. The construction of the UB uses the property that there is a common angle. With this constraint, if there is new physics to change the length of the sides in a fashion which is not a universal scaling, CP conserving or violating, then one cannot close the UB. Another way of presenting this situation is that, if one constructs the $UT(2)$ and $UT(2')$ first, then there may not be a common inner angle.

The above possibility may reveal information which cannot be obtained using only one UT. An example in which this might happen is the simple extension to four generation of quarks with the addition of t' and b'. In the case $V_{t'd} = 0$, the defining equation, Eq.(8), for $UT(2)$ is still the same and one can define effective inner angles and sides. If one just checks whether the sum of inner angles is 180° from direct measurements of individual angles, and the angles determined by the sides, they are consistent. No sign of new physics will show up by this analysis. However, the $UT(2')$ may be modified by a new term $V_{ub'}V^*_{tb'}$. Only information on $UT(2')$ is also compared with that from $UT(2)$, it is possible to decide if the type of new physics described above exists. The UB analysis contains this comparison together all in one.

Another is to use the property of the invariant quantity I_{UB} discussed earlier. It must equal to $J^2/4$. Taking the square root, one can check with one of the UT areas in the same UB. If more than one UB are constructed, one can also compare if their corresponding invariant quantities are equal.

Although the unitarity triangle carries information, about the Kobayashi-Maskawa quark mixing matrix, it explicitly contains just three parameters, which is one short to completely fix the KM matrix. The unitarity boomerangs formed using two UTs, with a common inner angle, can completely determine the KM matrix, and therefore better represents quark mixing information.

Out of the six possible UTs, there are 9 different ways to have a common inner angle from two UTs. In addition, one can align the two longer sides or one longer and one shorter side to form a UB. Therefore there are total 18 possible UBs. By studying the unitarity boomerangs, one can obtain all the information enshrined in KM matrix. We find that although the UBs have different areas, there is an invariant quantity for all UBs which is equal to a quarter of the Jarlskog parameter J squared. This is an universal representation of CP violation in the UB framework. We have also discussed how new physics can be hunted for by using information from UBs.

As far as graphic representation of the KM matrix, the proposal, to move from a single triangle to a boomerang combination, reflects, more than anything else, the impressive precision which has been attained by high-energy experiment. A unitarity boomerang contains all information of the KM matrix.

If new physics exists which modifies the unitarity nature, deviations from the expected UB can exist. The UB construction of the mixing matrix elements can also return, with information about new physics.

Thank you for your attention.