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NEUTRINO MASSES AND MIXING

OUTLINE

1. Rise and Fall of the Zee Model 1998-2001.

2. Two-Zero Textures.

3. Minimal See-Saw and Leptogenesis (FGY Model).

MINIMAL STANDARD MODEL (MSM)

- Three chiral neutrino states
- No renormalizable interactions can generate neutrino masses.
- In 1998 SuperKamiokande provided compelling evidence for non-zero neutrino mass.
- Therefore the MSM is insufficient.

RENORMALIZABLE

GAUGE-INVARIANT

EXTENSIONS OF THE MSM

1. Introduce complex triplet scalar coupled bilinearly to pairs of lepton doublets and to pairs of Higgs doublets (to avoid Majoron). This gives arbitrary Majorona matrix.

2. Introduce singlet right-handed neutrinos with large Majorans masses. See-saw mechanism gives arbitrary but naturally small Majorana matrix.

RENORMALIZABLE

GAUGE-INVARIANT

EXTENSIONS (continued)

3. Introduce a charged singlet coupled antisymmetrically to pairs of lepton doublets. Also a doubly- charged scalar coupled bilinearly both to pairs of lepton singlets and to pairs of the singly-charged scalars. This gives an arbitrary Majorana matrix at two-loops.

RENORMALIZABLE GAUGE-INVARIANT EXTENSIONS (concluded)

4.Introduce a charged singlet scalar coupled antisymmetrically both to pairs of lepton doublets and to a pair of Higgs doublets.

This is the Zee model which results in a particularly simple Majorana matrix at one loop order.

The model gives the neutrino Majorana neutrino mass matrix:

$$\mathcal{M} = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix}$$

We adopt this scenario. In particular we adopt the above ansatz for the mass matrix without committing to the Zee mechanism for its origin.

DIAGONAL ENTRIES

ARE ZERO

Neutrinoless double beta decay is zero at lowest order and cannot proceed at an observable rate.

The three mass parameters can be chosen real and non-negative.

The neutrino mixing becomes orthogonal; CP is conserved.

Since the trace of $\mathcal{M}$ vanishes the eigenvalues satisfy

$$m_1 + m_2 + m_3 = 0$$

Given the hierarchy of $\Delta_S/\Delta_a = r \ll 1$ when $r \rightarrow 0$ there are two possibilities:
Case A: $\quad m_1 + m_2 = 0 \quad$ and $\quad m_3 = 0.$
Case A requires at least one of the three parameters in $\mathcal{M}$ to vanish.
Case B: $\quad m_1 = m_2 \quad$ and $m_3 = -2m_1.$
For Case B the three entries in $\mathcal{M}$ must be equal.
In fact, r is small and nonzero but the relations between eigenvalues must be approximately true.

Case A

There are three subcases (1, 2, 3) of Case A, depending on which of the three parameters in $\mathcal{M}$ vanishes.

First consider $m_{e\mu} = 0$ (subcase 1).

This implies the conservation of $L_\tau - L_e - L_\mu$.

It follows that $\cos\theta_1$ is zero
and that $\theta_3 = \pi/4$.

This implies the atmospheric ν_μ oscillate exclusively into ν_e and *vice versa*.

(subcase1) is therefore excluded by SuperKamiokande data.

Case A (continued)

Next consider $m_{e\tau} = 0$ (subcase 2).

This implies the conservation of $L_\mu - L_e - L_\tau$.

It follows that $\sin\theta_1$ is zero
and that $\theta_3 = \pi/4$.

This implies the atmospheric ν_μ do not oscillate at all.

(subcase2) is therefore also excluded by SuperKamiokande data.

Case A (continued)

Next consider $m_{\mu\tau} = 0$ (subcase 3).

This implies the conservation of $L_e - L_\mu - L_\tau$.

It follows that $\sin\theta_2$ is zero
and that $\theta_3 = \pi/4$.

This implies that the solar neutrino oscillations are maximal:

$$P(\nu_e \to \nu_e)|_S = 1 - \sin^2(\Delta_S R_S/4E).$$

If we take a uniform energy-independent solar neutrino supression (drop the chlorine data) this is consistent with a large angle MSW interpretation.

Case A and subcase 3 (continued)

The atmospheric ν_μ neutrinos oscillate exclusively into ν_τ with unconstrained mixing angle θ_1:

$$P(\nu_\mu \to \nu_\tau)|_a = \sin^2 2\theta_1 \sin^2(\Delta_a R_a/4E).$$

while at the same time:

$$P(\nu_\mu \leftrightarrow \nu_e)|_a = 0$$

and

$$P(\nu_e \leftrightarrow \nu_\tau)|_a = 0$$

Note that the global conservation of $L_e - L_\mu - L_\tau$ protects the equality of $m_1^2 = m_2^2$ against radiative corrections.

Case A and subcase 3 (concluded)

subcase 3 predicts maximal mixing angle $\theta_3 = \pi/4$ for the solar neutrinos and hence is inconsistent with the small angle MSW explanation.

It is compatible with the large-angle MSW solution because the oscillation is maximal as shown by:

$$P(\nu_e \rightarrow \nu_e)|_S = 1 - \sin^2(\Delta_S R_S / 4E).$$

Making r differ slightly from zero the coefficient of the oscillatory term differs from one only by $r^2 \leq 10^{-4}$ and the solar neutrino oscillations remain virtually maximal.

Therefore subcase 3 requires maximal solar neutrino oscillation. An overall supression (energy-independent) of solar neutrinos is consistent if the chlorine data are dropped.

Case B

In Case B the mass realations are not protected by a global symmetry but we may nevertheless argue that Case B is excluded. It leads to the relation $\tan^2\theta_2 = 1/2$.

This imples that:

$$P(\nu_e \to \nu_e)|_a = 1 - (8/9)\sin^2(\Delta_a R_a/4E).$$

This leads to near-maximal oscillations of the atmospheric ν_e.

Case B (which has no subcases) is thus strongly disfavored by SuperKamiokande data.

PRE-SNO STATUS

OF ZEE MODEL

The ZEE extension of the MSM accommodates neutrino masses

No new fermion state is added.

Only one charged singlet scalar is added.

There are just four possibilities for hierarchical mass spectrum of neutrinos.

Before first SNO data (2001), it looked very promising!

PRE-SNO STATUS

(concluded)

For a viable solution, the consistency with the SuperKamiokande atmospheric data requires that the solar mixing is maximal - as favored by the pre-SNO data.

However, SNO data (2001), with SuperK data, now strongly disfavors the Zee model.

The SNO group announced their experimental data of solar neutrino flux from ^8B deay measured by the charged current reaction rate which is

$$\phi^{CC}(\nu_e) = 1.75 \pm 0,07^{+0.12}_{-0.11} \pm 0.05 \times 10^6 cm^{-2} s^{-1}$$

By combining this with the data from the superK experiment and using the values for the total flux expected in the Standard Solar Model (SSM) the survival probability of ν_e was reported as

$$P(\nu_e \to \nu_e) = \frac{\phi^{CC}}{\phi_{SSM}} = 0.347 \pm 0.029^{+0.056}_{-0.069}$$

where the first error is from SNO and the second from the SSM theoretical error.

Using a combination of $\phi^{CC}(\nu_e)$ and $\phi^{ES}(\nu_x)$ from SNO and Super-Kamiokande leads to the estimate

$$P(\nu_e \to \nu_e) = 0.322 \pm 0.076.$$

This suggests the survival probability $P(\nu_e \to \nu_e)$ is nearer to 1/3 than 1/2 and this is new information for us to analyze.

We must rediscuss the compatibility of the answer from Zee model with the LMA solution by comparing with the recent SNO data.

Under the Zee anzatz, the neutrino mass matrix in the flavor basis (e, μ, τ) is

$$\mathcal{M} = \begin{pmatrix} 0 & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & 0 & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & 0 \end{pmatrix} = U \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^\dagger$$

where m_1, m_2, m_3 are the eigenvalues of $\mathcal{M}$ and U is the unitary matrix to diagonalize it. $\mathcal{M}$ is real, traceless and symmetric. From the tracelessness condition,

$$m_1 + m_2 + m_3 = 0.$$

This condition is a strong constraint. The mass pattern of exact solutions which satisfy the atmospheric neutrino data is (as shown in PHF and Glashow, 1999)

$$m_1 = -m_2, \quad m_3 = 0,$$

With this situation, the allowed mixing matrix is the bimaximal one with $\theta_1 = \pi/4, \theta_2 = 0$ and $\theta_3 = \pi/4$, where the definition of the mixing angle is: (There is no CP violation in Zee model)

$$U = \begin{pmatrix} c_2 c_3 & c_2 s_3 & s_2 \\ -c_1 s_3 - s_1 s_2 c_3 & c_1 c_3 - s_1 s_2 s_3 & s_1 c_2 \\ s_1 s_3 - c_1 s_2 c_3 & -s_1 c_3 - c_1 s_2 s_3 & c_1 c_2 \end{pmatrix},$$

with s_i and c_i standing for sines and cosines of θ_i and the bimaximal mixing matrix is

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

To discuss the neutrino flux from the sun, we have to solve the neutrino propagation equation in the matter as follows:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \frac{1}{2E}M^2\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \tag{1}$$

$$= \frac{1}{2E}\left[U\begin{pmatrix}m_1^2 & & \\ & m_2^2 & \\ & & m_3^2\end{pmatrix}U^\dagger + \begin{pmatrix}A & & \\ & 0 & \\ & & 0\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}\right] \tag{2}$$

where $A = 2\sqrt{2}G_F N_e E$, N_e is the density of electron neutrino in the sun, E is the energy of the neutrino.

On the condition of eq.(5), $m_1^2 - m_2^2 = 0$ and $m_3^2 = 0$, the matrix M^2 can be written:

$$\begin{pmatrix}\frac{1}{2}(m_1^2+m_2^2)+A & 0 & 0\\ 0 & \frac{1}{4}(m_1^2+m_2^2) & -\frac{1}{4}(m_1^2+m_2^2)\\ 0 & -\frac{1}{4}(m_1^2+m_2^2) & \frac{1}{4}(m_1^2+m_2^2)\end{pmatrix}$$

The rotation among the weak eigenstates and the mass eigenstates in the center of the sun $(\nu_1^m, \nu_2^m, \nu_3^m)$ can be expressed as follows:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_m \begin{pmatrix} \nu_1^m \\ \nu_2^m \\ \nu_3^m \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \\ \nu_3^m \end{pmatrix},$$

At $t = 0$ an electron neutrino is produced in the sun and it is composed mainly of the state ν_2^m in the hierarchy we are considering in this work.

$$|\nu_e(0) >= |\nu_2^m > .$$

The time evolution of this state to time t is

$$|\nu_e(t) >= e^{i \int_0^t \frac{\lambda_2}{2E} dt} |\nu_2^m >$$

where λ_2 is the eigenvalue of M^2 for ν_2^m state.

Hence the survival probability is

$$P(\nu_e \to \nu_e) = | < \nu_e|\nu_e(t) > |^2 = \frac{1}{2}.$$

This is the result from the exact Zee anzatz with $r = 0$ and is significantly disfavored by SNO data.

In PHF, Oh and Yoshikawa (hep-ph/0110300, Phys. Rev D, 2002 in press) it is shown in detail that relaxing the hierarchy by making $r = \Delta_{\odot}/\Delta_{atmos} > 0$ cannot rescue the Zee model. For example, making $r = 0.1$ allows one to reduce $P(\nu_e \to \nu_e)$ from 0.5 to not less than 0.48.

To reduce the value sufficiently to agree with the SNO result would require $r > 1$ which seems a big stretch in terms of the present data.

The conclusion is that the Zee model is strongly disfavored, if not totally excluded, by the SNO/SuperK solar and atmospheric neutrino data.

ZEROES OF THE MASS MATRIX

What is the maximum number of zeroes possible? The Zee model had three and was ruled out. We begin by recalling the standard notation for three-flavor neutrino oscillations. With an appropriate choice of the phases we may express $\mathcal{M}$ as:

$$\mathcal{M} = U\,D\,U^{\dagger}$$

where U is the neutrino analog to the Kobayashi-Maskawa matrix expressing flavor eigenstates in terms of mass eigenstates:

$$\begin{pmatrix} c_2c_3 & c_2s_3 & s_2\,e^{-i\delta} \\ -c_1s_3 - s_1s_2c_3\,e^{i\delta} & +c_1c_3 - s_1s_2s_3\,e^{i\delta} & s_1c_2 \\ +s_1s_3 - c_1s_2c_3\,e^{i\delta} & -s_1c_3 - c_1s_2s_3\,e^{i\delta} & c_1c_2 \end{pmatrix}$$

with s_i and c_i standing for sines and cosines of θ_i. The remaining five parameters appear in

$$D = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \qquad (3)$$

where m_1 and m_2 are complex numbers while m_3 can be taken to be real.

In the analysis to follow we make the following hypotheses, all of which are favored (if not yet established) by current experimental data:

(1) For solar neutrinos, we assume oscillations to be large, but we also assume that they are measurably non-maximal. In particular, we take $0.6 \leq \sin^2 2\theta_3 \leq 0.96$, with 0.8 as a best-fit value. The relevant squared-mass difference $||m_1|^2 - |m_2|^2|$ is taken to be $\Delta_s \approx 5 \times 10^{-5}$ eV^2.

(2) For atmospheric neutrinos, we assume oscillations to be large (possibly maximal) and dominantly of the form $\nu_\mu \rightarrow \nu_\tau$. In particular, we take $\sin^2 2\theta_1 \simeq 1$. The relevant squared-mass difference $||m_{1,2}|^2 - |m_3|^2||$ is taken to be $\Delta_a \approx 3 \times 10^{-3}$ eV^2.

(3) For the subdominant angle θ_2 which controls atmospheric $\nu_\mu \leftrightarrow \nu_e$ oscillations we assume $\sin^2 2\theta_2 \leq 0.1$ in accordance with CHOOZ data.

It will be useful to define the ratio of squared-mass differences, whose estimated value is:

$$R_\nu \equiv \frac{\Delta_s}{\Delta_a} \approx 2 \times 10^{-2}.$$

Seven Two-Zero Textures

With the above hypotheses and notation, we turn to the question of which two independent entries of $\mathcal{M}$ can vanish in the basis wherein the charged lepton mass matrix is diagonal. Of the fifteen logical possibilities, we find just seven to be in accord with our empirical hypotheses. We discuss them individually, with the non-vanishing entries in each case denoted by X's. Our results are presented to leading order in the small parameter s_2. We begin with a texture in which $\mathcal{M}_{ee} = \mathcal{M}_{e\mu} = 0$:

$$\text{Case } A_1: \qquad \begin{pmatrix} 0 & 0 & X \\ 0 & X & X \\ X & X & X \end{pmatrix}$$

For this texture, $\mathcal{M}_{ee} = 0$ so that the amplitude for no-neutrino double beta decay vanishes to lowest order in neutrino masses. If Case A_1 is realized in nature, the neutrinoless process simply cannot be detected. There is even more to say. We find:

$$m_1 \simeq +(s_2 t_1 t_3)\, m_3\, e^{i\delta}$$

$$m_2 \simeq -(s_2 t_1 / t_3)\, m_3\, e^{i\delta}$$

$$R_\nu \simeq s_2^2\, t_1^2 |t_3^2 - 1/t_3^2|$$

where t_i stands for $\tan\theta_i$. Two of the squared neutrino masses are suppressed relative to the third by a factor of s_2^2. As a result we find that s_2 can lie close to its present experimental upper limit. This prediction will become more precise when θ_3 is better measured. However, the CP-violating parameter δ is entirely unconstrained.

Case A_2: $\begin{pmatrix} 0 & X & 0 \\ X & X & X \\ 0 & X & X \end{pmatrix}$ This texture, with $\mathcal{M}_{ee} = \mathcal{M}_{e\tau} = 0$, is described by the above with t_1 replaced by $-1/t_1$. Its phenomenological consequences are nearly the same as those of Case A_1.

Case B_1: $\begin{pmatrix} X & X & 0 \\ X & 0 & X \\ 0 & X & X \end{pmatrix}$ With $\mathcal{M}_{\mu\mu} = \mathcal{M}_{e\tau} = 0$, we find an acceptable solution if and only if $|s_2 \cos\delta \tan 2\theta_1| \ll 1$, in which case we obtain:

$$m_1 \simeq -(t_1^2 + s_2\,(e^{-i\delta}t_1 + e^{i\delta}/t_1)/t_3)\,m_3$$

$$m_{\simeq} - \,(t_1^2 - s_2\,(e^{-i\delta}t_1/ + e^{i\delta}/t_1)\,t_3\,)\,m_3$$

$$R_\nu \simeq |s_2\ \cos\delta\ \tan 2\theta_1(t_3 + 1/t_3)|$$

The three neutrinos are nearly degenerate in magnitude because $t_1^2 \simeq 1$.

Conversely, although atmospheric neutrino oscillations can be nearly maximal, the possibility that t_1 is exactly one is excluded. The appearance of the large factor $\tan 2\theta_1$ requires $s_2 \cos\delta$ to be tiny if R_ν is to be small. Thus s_2 is unlikely to be measureable *unless* $0 < |\cos\delta| \ll 1$. If this texture is correct, and if s_2 is found to depart significantly from zero, CP violation in the neutrino sector must be nearly maximal.

This texture is also promising in regard to the search for neutrinoless double beta decay. We obtain

$$\mathcal{M}_{ee} \simeq -t_1^2\sqrt{\Delta_a/|1-t_1^4|}$$

Because atmospheric neutrino oscillations are observed to be nearly maximal, this tells us that $\mathcal{M}_{ee}$ is likely to exceed 100 meV. The rate of neutrinoless double beta decay may approach its current experimental upper limit.

Case B_2: $$\begin{pmatrix} X & 0 & X \\ 0 & X & X \\ X & X & 0 \end{pmatrix}$$

This texture, with $\mathcal{M}_{\tau\tau} = \mathcal{M}_{e\mu} = 0$, is described as for B_1 with t_1 replaced by $-1/t_1$. Its phenomenological consequences are nearly the same as for B_1.

B_3: $$\begin{pmatrix} X & 0 & X \\ 0 & 0 & X \\ X & X & X \end{pmatrix}$$ B_4: $$\begin{pmatrix} X & X & 0 \\ X & X & X \\ 0 & X & 0 \end{pmatrix}$$

For Case B_3, with $\mathcal{M}_{\mu\mu} = \mathcal{M}_{e\mu} = 0$, we obtain the relations

$$m_1 \simeq -t_1^2(1 - s_2(e^{-i\delta}\, t_1 + e^{i\delta}/t_1)/t_3)\, m_3$$

$$m_2 \simeq -t_1^2(1 + s_2(e^{-i\delta}\, t_1 + e^{i\delta}\,/t_1)\, t_3)\, m_3$$

The phenomenology is substantially the same as Case B_1. The same is true for Case B_4 with $\mathcal{M}_{\tau\tau} = \mathcal{M}_{e\tau} = 0$. It results in Case B_3 with t_1 replaced by $-1/t_1$. Thus, the four B_i cases are experimentally almost indistinguishable.

Case C: $\begin{pmatrix} X & X & X \\ X & 0 & X \\ X & X & 0 \end{pmatrix}$

Our seventh and last allowed texture has $\mathcal{M}_{\mu\mu} = \mathcal{M}_{\tau\tau} = 0$. We obtain the relations

$$m_1 \simeq -(1 - e^{i\delta} \cot 2\theta_1/(t_3 s_2))\, m_3$$

$$m_2 \simeq -(1 + e^{i\delta} \cot 2\theta_1\, t_3/s_2)\, m_3$$

We can obtain a small value of R_ν if and only if $||m_1| - |m_2|| \ll m_3$, in which event we find

$$s_2 \cos\delta \approx \cot 2\theta_1 \cot 2\theta_3$$

This approximate equality shows that s_2 can be large enough to be measured if atmospheric neutrino oscillations are not too nearly maximal. The observed value of R_ν results from a small departure from the last equation by an amount of order s_2^2.
Furthermore, for Case C we obtain

$$|m_{1,2}|^2 \simeq (1 + \cos^2\delta\ \tan^2 2\theta_3) m_3^2$$

$$|\mathcal{M}_{ee}| \simeq m_3$$

From these results, we find

$$|\mathcal{M}_{ee}| \simeq |\sqrt{\Delta_a}\,\cos 2\theta_3/\cos\delta|$$

Thus the effective parameter governing neutrinoless double beta decay must be at least of order 30 meV and could be considerably larger.

No other two-zero texture of the neutrino mass matrix is compatible with our empirical hypotheses.

It is easily verified that no two of our allowed two-zero textures can be simultaneously satisfied while remaining consistent with our empirical hypotheses. It follows that there is no tolerable three-zero texture. For example, a neutrino mass matrix with vanishing diagonal entries (the Zee Ansatz) cannot yield a large enough value of R_ν unless solar neutrino oscillations are very nearly maximal, a situation that appears to be strongly disfavored by experiment.

Comment

The seven allowed two-zero neutrino textures fall into three classes: A (with two members), B (with four members), and C. The textures within each class are difficult or impossible to distinguish experimentally, but each of the three classes has radically different implications. For class A, the subdominant angle θ_2 is expected to be relatively large, but no-neutrino $\beta\beta$ decay is forbidden. For class B, the latter process should be measureable by the next generation of double beta decay experiments, whilst s_2 may or may not be large enough to be detected. However, if s_2 is comparable to its experimental upper limit, CP violation must be nearly maximal and should be readily detectable by proposed experiments. For class C, no-neutrino $\beta\beta$ decay is likely to be observable and θ_2 ought to be large enough to measure and to permit a search for CP violation.

KEY DATA NEEDED

(1) The third and final mixing angle $\theta_2 \equiv \theta_{13}$.

(2) The Majorana element M_{ee} from observation of neutrinoless double beta decay.

(3) CP violation in the neutrino sector which vanishes with $\theta_2 \rightarrow 0$.

INTRODUCTION TO LEPTOGENESIS

One of the most profound ideas in particle theory is that of Sakharov (1967). Following the discovery of CP violation in K decay (1964) - a surprise - he enunciated the conditions for baryogenesis:

1. B violation
2. C and CP violation.
3. Out-of thermal equilibrium era.

Early discussions were stated in terms of p decay but calculations gave much too small a baryon number of the universe. Now though we still have no evidence for B violation there is evidence for L violation in Majorana neutrino masses. Leptogenesis where $N \rightarrow e^- H^+$ followed by electroweak sphaleron conversion can give the correct B number.

Note that in $N \to e^- H^+$,
H^+ is massless since $E \gg M_W$.

L is subsequently converted to B through sphalerons which conserve (B - L).

We will study CP violation both at low energy (ξ_L = parameter) in ν oscillations and at high energy (ξ_H = parameter) in leptogenesis.

Can ξ_L and ξ_H be related?

Generally not, but our purpose here is to demonstrate the remarkable fact that in a class of models the answer is positive.

In such a case the sign of CP violation in neutrino oscillations can be predicted from the baryon number of the universe.

Present data on neutrinos:

ATMOSPHERIC NEUTRINOS

$\Delta_a \simeq 3 \times 10^{-3} eV^2$

$\tan^2\theta_a \simeq 1$

SOLAR NEUTRINOS

$\Delta_S \simeq 5 \times 10^{-5} eV^2$

$0.6 \leq \sin^2 2\theta_3 \leq 0.96$

$\sin^2 2\theta_3 = 0.8$ is best fit

THE THIRD MIXING ANGLE

$\sin^2 2\theta_2 \leq 0.1$ (CHOOZ)

θ_2 is sometimes called θ_{13}

These data must be accommodated successfully in our model.

THE MODEL

In the minimal SM: $m(\nu_i) = 0$.

Simplest extension of minimal SM which allows both $m(\nu) \neq 0$ and successful leptogenesis is:

TWO RIGHT-HANDED NEUTRINOS $N_{1,2}$

This, plus appropriate texture zeroes in the Dirac 3×2 rectangular matrix, is our model.

(Note that $N_{1,2,3}$ model suggested by SO(10) has an ESSENTIAL AMBIGUITY avoided here.)

New terms in the lagrangian are:

$$\mathcal{L} = \frac{1}{2}(N_1, N_2)\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} \tag{4}$$

$$+(N_1, N_2)\begin{pmatrix} a & a^{'} & 0 \\ 0 & b & b^{'} \end{pmatrix}\begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} + h.c. \tag{5}$$

D_{ij} is a rectangular 3×2 Dirac matrix.

We have assumed a texture

$$D_{ij} = \begin{pmatrix} x & x & 0 \\ 0 & x & x \end{pmatrix}$$

which leaves the exact number of parameters necessary and sufficient to account for the data.

Using the see-saw mechanism we compute:

$$
\begin{aligned}
L &= D^T M^{-1} D \\
&= \begin{pmatrix} a^2/M_1 & aa^{'}/M_1 & 0 \\ aa^{'}/M_1 & [(a^{'})^2/M_1 + b^2/M_2] & bb^{'}/M_2 \\ 0 & bb^{'}/M_2 & (b^{'})^2/M_2 \end{pmatrix}
\end{aligned}
$$

We can choose a basis in which $a, b, b^{'}$ are real and $a^{'} = |a^{'}|e^{i\delta}$.

To check consistency with low-energy data we put $a^{'} = \sqrt{2}a$ and $b^{'} = b$ (all real) whereupon:

whereupon:

putting $a^{'} = \sqrt{2}a$ and $b^{'} = b$ gives

$$L = \begin{pmatrix} a^2/M_1 & \sqrt{2}a^2/M_1 & 0 \\ \sqrt{2}a^2/M_1 & [2a^2/M_1 + b^2/M_2] & b^2/M_2 \\ 0 & b^2/M_2 & b^2/M_2 \end{pmatrix} \quad (6)$$

We diagonalize by rewriting:

$$\frac{1}{2}\nu^T L \nu = \frac{1}{2}\nu'^T U^T L U \nu'$$

where U is a real orthogonal matrix and $\nu^{'}$ are the three mass eigenstates.

We parametrize the unitary diagonalizing matrix as:

$$U = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix} \times$$
$$\times \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta & sin\theta \\ 0 & -sin\theta & cos\theta \end{pmatrix}$$

We deduce the mass eigenvalues and θ:

$M(\nu_3^{'}) = 2b^2/M_2 \gg M(\nu_2^{'}) = 2a^2/M_1 \gg M(\nu_1^{'}) \equiv 0$

The vanishing eigenvalue is exact (rank = 2)

This assumes $a^2/M_1 \ll b^2/M_2$. We also find:

$$\theta \simeq \frac{M(\nu_2^{'})}{\sqrt{2}M(\nu_3^{'})} \ll 1$$

For the unitary matrix relevant to neutrino oscillaions:

$U_{e3} = sin\theta/\sqrt{2} \simeq m(\nu_2^{'})/2m(\nu_3^{'})$.

Thus $A^{'} = \sqrt{2}a$ and $b^{'} = b$ adequately fits all the data. These values can be shimmied to improve the fit.

We deduce that:

$$\frac{2b^2}{M_2} \simeq \sqrt{\Delta_a} \simeq 0.05eV$$

and

$$\frac{2a^2}{M_1} \simeq \sqrt{\Delta_S} \simeq 0.007eV$$

These results imply that the N_1 state can satisfy the out-of -equlibrium condition but not N_2. Thus for leptogenesis to succeed it is necessary that $M(N_2) > M(N_1)$ and this resolves a sign ambiguity present in models with three right-handed neutrinos.

THE CONNECTING LINK

In our model (really a class of models) we can calculate the CP violation parameters ξ_L and ξ_H characterizing respectively the low- and high- energy.

THE RELATIVE SIGN OF THESE TWO PARAMETERS IS FIXED.

The magnitude itself is not predicted because it depends on the parameters.

The presence of texture zeroes in L and D implies only one phase, and that is why LE and HE are related. Let us therefore calculate ξ_H and ξ_L explicitly:

BARYON NUMBER THROUGH LEPTOGENESIS.

$$B \sim \xi_H = (ImDD^\dagger)^2_{12}$$

This crucial quantity and B proportional to ξ_H can be evaluated uniquely in the present model:

In the model

$$\xi_H = Im(a^{'}b)^2$$

$$= +Y^2a^2b^2sin^2\delta > 0$$

which has a definite sign.

Here $a^{'} = Yae^{i\delta}$ loosens up the previous assignment $a^{'} = \sqrt{2}a$.

Low-energy CP violation.

The relevant parameter is:

$$\xi_L = Im(h_{12}h_{23}h_{31})$$

where $h = (LL^\dagger)$ and ξ_L is like the Jarlskog determinant for quarks.

Simple algebra give:

$$\xi_L = -\frac{a^6 b^6}{M_1^3 M_2^3} \sin 2\delta Y^2(2 + Y^2)$$

which has a definite sign (negative).

The predicted sign is robust with respect to varying the phenomenological parameters.

So in a class of models having two right-handed neutrinos and a texture with the minimum number of parameters to accommodate the low-energy phenomenology we find that the

RELATIVE SIGN OF ξ_L and ξ_H IS UNIQUE

The essential ambiguity of normal versus inverted hierarchy for N_R's with three N_R's is evaded by including only two N_R's.

This provides a very interesting link between elementary particles (neutrinos) and the early universe.

SUMMARY

- The Zee model with three zeros on the diagonal of the 3×3 Majorana mass matrix is strongly disfavored by experimental data from SuperKamiokane, SNO and KamLAND.
- All three-zero textures, with the zeros in any entries are also empirically disfavored. Two zeros can occur in 7 out of 15 possible ways falling into 3 classes: A with 2 members and a normal hirearchy; B with 4 members and a degenerate spectrum; C with only one member and an inverted heirarchy.
- Use of texture zeros in a different environment with two right-handed neutrinos gives rise to a class of models where the CP phase occurring in leptogenesis is directly related (in sign) to the CP phase occurring in low-energy long-baseline oscillation experiments.