Thank you for the invitation.

PRIMORDIAL BLACK HOLES
AS
ALL DARK MATTER

Conference Honoring
Murray Gell-Mann's 80th Birthday.

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Nanyang Executive Centre, Singapore.

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IPMU / UNC-Chapel Hill

OUTLINE

1. The Entropy of the Universe.
2. Intermediate Mass Black Holes (IMBHs).
3. Cosmological Entropy Considerations.
4. Observation of IMBHs
5. Formation of IMBHs

SUMMARY.

(If time) Brief comments on:
"Entropic Accelerating Universe"
(`arXiv:1002.4278 [hep-th]`)

References:

(1) P.H.F.

Identification of All Dark Matter
as Black Holes.

`arXiv:0905.3632 [hep-th]`

JCAP 0910:016 (2009).

(2) P.H.F., Masahiro Kawasaki,
Fumihiro Takahashi and Tsutomu Yanagida

IPMU-09-0157 (December 2009).

Primordial Black Holes as All Dark Matter

`arXiv:1001.2308 [hep-ph]`.

(3) Damien Easson, P.H.F.
and George F. Smoot,

IPMU-10-0036 (February 2010).

Entropic Accelerating Universe

`arXiv:1002.4278 [hep-th]`

1. The Entropy of the Universe.

As interest grows in pursuing alternatives to the Big Bang, including cyclic cosmologies, it becomes more pertinent to address the difficult question of what is the present entropy of the universe?

Entropy is particularly relevant to cyclicity because it does not naturally cycle but has the propensity only to increase monotonically. In one recent proposal, the entropy is jettisoned at turnaround. In any case, for cyclicity to be possible there must be a gigantic reduction in entropy (presumably without violation of the second law of thermodynamics) of the visible universe at some time during each cycle.

Standard treatises on cosmology address the question of the entropy of the universe and arrive at a generic formula for a thermalized gas of the form

$$S = \frac{2\pi^2}{45} g_* V_U T^3 \tag{1}$$

where g_* is the number of degrees of freedom, T is the Kelvin temperature and V_U is the volume of the visible universe. From Eq.(1) with $T_\gamma = 2.7^0$K and $T_\nu = T_\gamma (4/11)^{1/3} = 1.9^0$K we find the entropy in CMB photons and neutrinos are roughly equal today

$$S_\gamma(t_0) \sim S_\nu(t_0) \sim 10^{88}. \tag{2}$$

Our topic here is the gravitational entropy, $S_{grav}(t_0)$. Following the same path as in Eqs. (1,2) we obtain for a thermal gas of gravitons $T_{grav} = 0.91^0$K and then

$$S_{grav}^{(thermal)}(t_0) \sim 10^{86} \tag{3}$$

This graviton gas entropy is a couple of orders of magnitude below that for photons and neutrinos.

On the other hand, while radiation thermalizes at $T \sim 0.1 eV$ for which the measurement of the black body spectrum provides good evidence and there is every reason, though no direct evidence, to expect that the relic neutrinos were thermalized at $T \sim 1 MeV$, the thermal equilibibriation of the present gravitons is less definite. If gravitons did thermalize, it was at or above the Planck scale, $T \sim 10^{19} GeV$, when everything is uncertain because of quantum gravity effects. If the gravitons are in a non-thermalized gas their entropy will be lower than in Eq.(3), for the same number density.

But there are larger contributions to gravitational entropy from elsewhere!!!

2. Intermediate Mass Black Holes

If we consider normal baryonic matter, other than black holes, contributions to the entropy are far smaller. The background radiation and relic neutrinos each provide $\sim 10^{88}$. We have learned in the last decade about the dark side of the universe. WMAP suggests that the pie slices for the overall energy are 4% baryonic matter, 24% dark matter and 72% dark energy. Dark energy has no known microstructure, and especially if it is characterized only by a cosmological constant, may be assumed to have zero entropy. As already mentioned, the baryonic matter other than the SMBHs contributes far less than $(S_U)^{min}$.

This leaves the dark matter which is concentrated in halos of galaxies and clusters.

It is counter to the second law of thermodynamics when higher entropy states are available that essentially all the entropy of the universe is concentrated in SMBHs. The Schwarzschild radius for a $10^7 M_{\odot}$ SMBH is $\sim 3 \times 10^7$ km and so 10^{12} of them occupy only $\sim 10^{-36}$ of the volume of the visible universe.

Several years ago important work by Xu and Ostriker showed by numerical simulations that IMBHs with masses above $10^6 M_{\odot}$ would have the property of disrupting the dynamics of a galactic halo leading to runaway spiral into the center. This provides an upper limit $(M_{IMBH})^{max} \sim 10^6 M_{\odot}$.

Gravitational lensing observations are amongst the most useful for determining the mass distributions of dark matter. Weak lensing by, for example, the HST shows the strong distortion of radiation from more distant galaxies by the mass of the dark matter and leads to astonishing three-dimensional maps of the dark matter trapped within clusters. At the scales we consider $\sim 3 \times 10^7$ km, however, weak lensing has no realistic possibility of detecting IMBHs in the forseeable future.

Gravitational microlensing presents a much more optimistic possibility. This technique which exploits the amplification of a distant source was first emphasized in modern times (Einstein considered it in 1912 unpublished work) by Paczynski. Subsequent observations found many examples of MACHOs, yet insufficient to account for all of the halo by an order of magnitude. These MACHO searches looked for masses in the range $10^{-6}M_{\odot} \leq M \leq 10^{2}M_{\odot}$.

The time t_0 of a microlensing event is given by

$$t_0 \equiv \frac{r_E}{v} \tag{4}$$

where r_E is the Einstein radius and v is the lens velocity usually taken as $v = 200$ km/s. The radius r_E is proportional to the square root of the lens mass and numerically one finds

$$t_0 \simeq 0.2y \, \left(\frac{M}{M_\odot}\right)^{1/2} \tag{5}$$

so that, for the MACHO masses considered, $2h \leq t_0 \leq 2y$.

Although some of the already observed MACHOs may be BHs, they do not saturate the possible mass or entropy for dark matter so let us set as definition $(M_{IMBH})^{min} \sim 10^2 M_\odot$. This provides the range for IMBH mass

$$2 \leq \log_{10} \eta = \log_{10}(M_{IMBH}/M_\odot) \leq 6 \quad (6)$$

which, after Eq.(7), provides a second window of interest. It corresponds to $2y \leq t_0 \leq 200y$.

3. Cosmological Entropy Considerations.

The cosmological entropy range

$$102 \leq \log_{10} S_U \leq 112 \qquad (7)$$

is the first of two interesting windows which are the subject. Conventional wisdom is $S_U \sim (S_U)^{min} = 10^{102}$.

As mentioned already, the key guide will be the holographic principle which informs us that the cosmological entropy is in the window (7). It cannot be at the absolute maximum value because that is possible only if every halo has already completely collpsed into a single black hole.

Also, the absolute minimum although not excluded seems intuitively implausible because all the entropy is compressed into $10^{-36} V_U$.

The natural suggestion is that there exist IMBHs in the mass region (6). The number is limited by the total halo mass $10^{12} M_{\odot}$. The total entropy is higher for higher IMBH mass because $S \propto M^2$. Let n be the number of DMBHs per halo, η be the ratio $(M_{IMBH}/M_{\odot})$, S_U be the total entropy for 10^{12} halos and t_0 be the microlensing longevity. The table below shows five possibilities

Intermediate Mass Black Holes and Microlensing Longevity

$\log_{10} n_{max}$	$\log_{10} \eta$	$\log_{10} S_{halo}$	$\log_{10} S_U$	t_0 (years)
8	2	88	100	2
7	3	89	101	6
6	4	90	102	20
5	5	91	103	60
4	6	92	104	200

(Assumes $\rho_{IMBH} \sim 1\% \rho_{DM}$)

4. Observation of IMBHs

Since microlensing observations already impinge on the lower end of the range (6) and the Table, it is likely that observations which look at longer time periods, have higher statistics or sensitivity to the period of maximum amplification can detect heavier mass IMBHs in the halo. If this can be achieved, and it seems a worthwhile enterprise, then the known entropy of the universe could be increased by more than two orders of magnitude.

There exists interesting other analyses pertinent to existence of massive halo objects:

J. Yoo, J, Chanamé and A. Gould, Astrophys. J. **601,** 311 (2004). `astro-ph/0307437`.

C. Murali, P. Arras and I. Wasserman. `astro-ph/9902028`.

B. Moore, Atrophys. J. **415,** L93 (1993). `astro-ph/9306004`.

I shall return to Yoo et al.'s article.

The previous analyses have assigned upper limits on the fraction (f) of the halo mass that can be constituted by IMBHs.

We have no reason to suggest that all of the dark matter halo mass is from IMBHs so the fraction f could indeed be small. Yet IMBHs can still provide a very large fraction of the entropy of the universe. For example, taking $f = 0.01$ and $10^6 M_\odot$ as mass allows up to $\sim 10^4$ IMBHs per halo, a total of $\sim 10^{16}$ Mega-$M_\odot$ black holes in the universe and the fraction of the total entropy of the universe provided by $\sim 1\%$ of dark matter can be $\sim 99\%$!!

According to G. Bertone (private communication, 2009) the best upper limits (from Disk Stability and Wide Binaries) appear in Fig. 7 on page 317 of

J. Yoo, J, Chanamé and A. Gould, Astrophys. J. **601,** 311 (2004). `astro-ph/0307437`.

which permits 10 percent of dark matter for the range of IMBH from $20M_\odot$ to $10^6M_\odot$.

*** astro-ph/0307437

It is this entropy argument based on holography and the second law of thermodynamics which is the most compelling supportive argument for IMBHs. If each galaxy halo asymptotes to a black hole the final entropy of the universe will be $\sim 10^{112}$ as in Eq.(7) and the universe will contain just $\sim 10^{12}$ supergigantic black holes. Conventional wisdom is that the present entropy due entirely to SMBHs is only $\sim 10^{-10}$ of this asymptopic value. IMBHs can increase the fraction up to $\sim 10^{-8}$, closer to asymptopia and therefore more probable according to the second law of thermodynamics.

There are several previous arguments about the existence of IMBHs and they have put upper limits on their fraction of the halo mass. The entropy arguments are new and provide additional motivation to tighten these upper bounds or discover the halo black holes. One observational method is high longevity microlensing events. It is up to the ingenuity of observers to identify other, possibly more fruitful, methods some of which have already been explored in a preliminary way.

5. Formation of Black Holes.

The presence of dark matter (DM) has been firmly established by a host of observations, and its abundance was measured by the WMAP with an unprecedented precision:

$$\Omega_{\mathrm{DM}} h^2 = 0.1131 \pm 0.0034. \tag{8}$$

However it is not known yet what DM is made of, and the question remains a big mystery in modern cosmology as well as particle physics.

It is often claimed that there is no DM candidate in the framework of the standard model (SM), assuming that DM is made of elusive particles which have evaded all conventional DM searches. The DM particle must be electrically neutral, long-lived, and cold, but no such particle exists in the SM.

Thus we need to postulate a theory beyond SM and introduce a new degree of freedom, which is usually made stable by imposing an additional discrete symmetry. The introduction of such a discrete symmetry may be motivated by other phenomenological reason. For instance, in the supersymmetric standard model (SSM), it is customary to introduce an R-parity in order to forbid dangerous operators which would give rise to too fast proton decay. Once the R-parity is imposed, the lightest supersymmetric particle (LSP) becomes stable, and therefore a DM candidate. On the other hand, there is an argument that the R-parity violation may be a common phenomenon in the string landscape. If so, the dangerous operators must be absent due to some other reason(s) and the lifetime of LSP in the SSM may be too short to account for the DM.

If the DM is made of a weakly interacting massive particle (WIMP), we may be able to observe collider, direct and indirect DM signatures; the DM particles may be produced at LHC, and the next-generation direct search experiments will probe a significant portion of parameter space predicted by various theoretical DM models. In spite of thorough DM searches using widely different techniques, the results are negative so far.[#1] If no DM signature is found in the future experiments, it may suggest that the basic assumption that the DM is made of unknown particles is simply wrong.

[#1] One of the exceptions is the DAMA experiment. However, it is still controversial concerning the interpretation of the experimental data. Very recently, two DM-like events were found by the CDMS II experiment, but more data is clearly necessary to draw definite conclusions.

There actually *is* a DM candidate in the framework of SM, namely, a PBH. In the early Universe PBHs can form when the density perturbation becomes large, and it has been known that a PBH of mass greater than 10^{15}g survives the Hawking evaporation and therefore contributes to the DM density.

In consideration of the entropy of the universe it was pointed out that if all DM were in the form of $10^5 M_\odot$ black holes it would contribute a thousand times more entropy than the supermassive black holes at galactic centers and hence be a statistically favored configuration. Here we consider primordial black holes (PBHs) with masses from $10^5 M_\odot$ to $10^{-8} M_\odot$ and, subject to observational constraints, any of these masses can comprise all DM although the entropy argument favors the heaviest $10^5 M_\odot$ mass.

There are several ways to realize large density fluctuations leading to PBH formation. One is phase transition involving violent processes like bubble collision or the collapse of string loop. As we will see in the next section, however, both scenarios have difficulties. Another possibility is the production of PBHs from density fluctuations generated during inflation. Since the blue spectrum with a spectral index $n_s > 1$ is disfavored by the WMAP data, a single inflation may not be able to produce large density fluctuations at small scales unless some dynamics is introduced during inflation. On the other hand, the density fluctuations can be easily enhanced at small scales in a double inflation model, as first discussed in the context of the PBH formation.

Here, we discuss a double inflation model that consists of a smooth-hybrid inflation and a new inflation. The smooth-hybrid new double inflation was studied in the context of explaining the large running spectral index suggested by the WMAP 1st year data. In this set-up PBHs with a narrow mass distribution are formed as a result of an explosive particle production between the two inflations. We will show that the PBH mass can take a wide range of values from $10^{-8}M_\odot$ up to $10^5M_\odot$. Also, the resultant PBH mass has a correlation with running of spectral index, which was roughly estimated in a semi-analytical method. Here we numerically calculated the correlation, which can be tested by future observations.

The black hole mass and the formation epoch are related to each other due to the causality. In the early Universe, the mass contained in the Hubble horizon sets an upper bound on the PBH mass formed at that time. Assuming that the whole mass in the horizon is absorbed into one black hole, we obtain

$$\begin{aligned} M_{\rm BH} &= \frac{4\pi\sqrt{3}M_P^3}{\sqrt{\rho_f}} \simeq 0.05\, M_\odot \left(\frac{g_*}{100}\right)^{-\frac{1}{2}} \left(\frac{T_f}{\rm GeV}\right)^{-2}, \\ &\simeq 1.4 \times 10^{13}\, M_\odot \left(\frac{g_*}{100}\right)^{-\frac{1}{6}} \left(\frac{k_f}{{\rm Mpc}^{-1}}\right)^{-2}, \end{aligned} \tag{9}$$

where $M_{\rm BH}$ is the black hole mass, $M_P \simeq 2.4 \times 10^{18}\,{\rm GeV}$ is the reduced Planck mass, $M_\odot \simeq 2 \times 10^{33}\,{\rm g}$ is the solar mass, g_* counts the light degrees of freedom in thermal equilibrium, ρ_f, T_f and k_f are the energy density, the plasma temperature and the comoving wavenumber corresponding to the Hubble horizon at the formation, respectively. The radiation domination was assumed in the second equality.

As is well known, Hawking made a striking prediction about the evaporation of black holes; any black holes have a temperature inversely proportional to its mass and evaporates in a finite time.

$$\tau_{\rm BH} \simeq 10^{64} \left(\frac{M_{\rm BH}}{M_\odot}\right)^3 {\rm yr.} \tag{10}$$

Thus the black holes with mass less than 10^{15} g must have evaporated by now. PBHs which remain as (a part of) DM must therefore be created at a temperature below 10^9 GeV. In the following we assume that PBHs account for all DM in our Universe.

The cosmological effects of PBHs have been extensively studied so far. While PBHs with masses below 10^{15} g are significantly constrained, it is very difficult to detect PBHs heavier than 10^{15} g because of negligible amount of the Hawking radiation. The MACHO and EROS collaborations monitored millions of stars in the Magellanic Clouds to search for microlensing events caused by MAssive Compact Objects (MACHOs) passing near the line of sight. The MACHO collaboration excluded the objects in the mass range $0.3M_\odot$ to $30M_\odot$, and the latest result of the EROS-1 and EROS-2 excluded the mass range $0.6 \times 10^{-7}M_\odot < M < 15M_\odot$, as the bulk component of the galactic DM. On the other hand, if we assume that the PBH formation occurs before the big bang nucleosynthesis (BBN) epoch, the PBH mass should be lighter than $10^5 M_\odot$ (see Eq. (9)). Therefore we consider PBHs with masses (i) $M_{\rm BH} < 10^{-7}M_\odot$ and (ii) $30M_\odot < M_{\rm BH} < 10^5 M_\odot$.

Let us comment on other existing constraints. The PBHs with masses heavier than $43M_{\odot}$ were claimed to be excluded by the presence of wide binaries in 2004, but the question on the validity of the data used to set the limit was raised in 2009. Taking account of low averaged DM density experienced by the four binaries used in their analysis, the strong constraints set by the wide binaries were undermined. Recently, Ricotti, Ostriker and Mack investigated the effect of non-evaporating PBHs on the cosmic microwave background (CMB) spectrum and anisotropy and found that the PBHs with mass greater than $\sim 0.1M_{\odot}$ cannot account for the bulk component of DM. However, the authors made assumptions about accretion efficiency in obtaining strong limits on PBH abundances; if these assumptions are weakened, all DM could be PBHs for the masses we consider.

The above observational constraints provide us with information on the PBH formation. If PBHs are produced at different times, the mass function tends to be broad, thereby making it difficult to be consistent with observations. In order to realize the PBH mass function with a sharp peak, most of the PBHs should be produced at the same time. Thus the production mechanism must involve such a dynamics that only the density fluctuation of a certain wavelength rapidly grows.

What kind of dynamics can create PBHs? First of all, density perturbation must become large for PBHs to be formed. There are several ways to realize large density fluctuations leading to the PBH formation. One is the phase transition which leads to violent processes like bubble collision or the collapse of string loops. However, in the case of the bubble collision, the bubble formation rate must be tuned to produce the PBH, and the PBH produced from the strong loops tends to have a broad mass function. Another possibility is the production of PBHs from density fluctuations generated during inflation. In the standard picture of inflation, the inflation driven by a slow-rolling scalar field lasts for more than about 60 e-foldings to solve theoretical problems of the big bang cosmology. Then no dynamics for producing a sharp peak in the density perturbation is expected. However, there is no a priori reason to believe that our Universe experienced only one inflationary expansion. Indeed, the cosmological gravitino or modulus problem can be relaxed if the energy scale of the last inflation is rather low, and it is then quite likely that there was another inflation before the last one. If the multiple inflation is a common phenomenon, we expect that explosive particle production between the successive inflation periods may produce a sharp peak in the density perturbation at the desired scales, which leads to the PBH formation at a later time. In the next section, we show that this is actually feasible using a concrete double inflation model.

We are able to provide a double inflation model producing PBHs with a sharp mass function as an existing proof. The double inflation model we adopt consists of two stages of inflation; the first inflation is realized by smooth hybrid inflation and the second one by new inflation. As shown below, the cosmologically relevant density fluctuations are generated during smooth hybrid inflation. After the first inflation, the inflaton and waterfall fields of the smooth hybrid inflation start to oscillate and decay into their quanta via self-coupling and mutual coupling of the two fields. The interesting point is that the decays of the scalar fields are largely enhanced through parametric resonance and hence the fluctuations of the scalar fields exponentially grow. This process is called preheating. During the preheating phase, only the fluctuations at a specific wavenumber corresponding to the inflaton mass rapidly grow, and those fluctuations finally turn into density fluctuations leading to the production of PBHs with a sharp mass function. The role of the second inflation is to stretch the density fluctuations generated during the first inflation and the subsequent preheating phase to cosmologically large scales.

FOR TECHNICAL DETAILS BEYOND THIS POINT, PLEASE STUDY ASSIDUOUSLY:

P.H.F., Masahiro Kawasaki,
Fumihiro Takahashi and Tsutomu Yanagida

IPMU-09-0157 (December 2009).

Primordial Black Holes as All Dark Matter

`arXiv:1001.2308 [hep-ph]`.

In contrast to the conventional WIMP DM model, PBHs have only gravitational interactions. In order to detect PBHs, we need to carefully look at the effect induced by PBHs such as gravitational lensing, gravity waves, etc. Intermediate mass black holes in the range $30M_\odot < M_{BH} < 10^5 M_\odot$ can be sought, for example, by higher-longevity microlensing events and by higher-statistics analysis of wide binaries. In particular there appeared recently an interesting idea that if the DM is explained by the PBH of mass $10^5 M_\odot$, it may account for the size evolution of the elliptic galaxies by dynamical friction. Further observations and theoretical study may reveal the presence of the PBH DM.

What makes the PBH particularly attractive as a DM candidate is that it is naturally long-lived due to the gravitationally suppressed evaporation rate. No discrete symmetries need to be introduced in an ad hoc manner. Also the PBH DM may be motivated from the arguments based on entropy of the Universe.

We have argued that the PBH is a natural and unique candidate for the DM in the minimal theoretical framework, namely, the SM. Using the smooth-hybrid new double inflation model, we have shown that it is possible to produce PBHs of mass ranging from $10^{-8}M_{\odot}$ to $10^{5}M_{\odot}$. Importantly, the PBH mass relates the scalar spectral index and the running of the spectral index, which can be tested by the Planck satellite.

SUMMARY

The best summary is to repeat a table and discuss.

Intermediate Mass Black Holes and Microlensing Longevity

Maximum no. IMBH/halo	Mass of $IMBH$	Entropy of Universe	Microlensing longevity
10^{10}	$100M_\odot$	10^{103}	2y
10^{9}	$1,000M_\odot$	10^{104}	6y
10^{8}	$10^4M_\odot$	10^{105}	20y
10^{7}	$10^5M_\odot$	10^{106}	60y

(Assumes $\rho_{IMBH} \sim 100\% \rho_{DM}$)

Thank you for your attention