

Thank you for the invitation

QUIVER GAUGE THEORY AND

CONFORMALITY AT THE TeV SCALE

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OUTLINE

1. Quiver gauge theories.
2. Conformality phenomenology.
3. 4 TeV Unification.
4. Quadratic divergences.
5. Anomaly Cancellation - Conformal U(1)s
6. Dark Matter Candidate.

SUMMARY.

CONCISE HISTORY OF STRING THEORY

- Began 1968 with Veneziano model.
- 1968-1974 dual resonance models for strong interactions. Replaced by QCD around 1973. DRM book 1974. Hiatus 1974-1984
- 1984 Cancellation of hexagon anomaly.
- 1985 $E(8) \times E(8)$ heterotic string compactified on Calabi-Yau manifold gives temporary optimism of TOE.
- 1985-1997 Discovery of branes, dualities, M theory.
- 1997 Maldacena AdS/CFT correspondence relating 10 dimensional superstring to 4 dimensional gauge field theory.
- 1997-present Insights into gauge field theory including possible new states beyond standard model. String not only as quantum gravity but as powerful tool in nongravitational physics.

MORE ON STRING DUALITY:

Duality: Quite different looking descriptions of the same underlying theory.

The difference can be quite striking. For example, the AdS/CFT correspondence describes duality between a $\mathcal{N} = 4, d = 4$ SU(N) GFT and a $D = 10$ superstring. Nevertheless, a few non-trivial checks have confirmed this duality.

In its most popular version, one takes a Type IIB superstring (closed, chiral) in $d = 10$ and one compactifies on:

$$(AdS)_5 \quad \times \quad S^5$$

Perturbative finiteness of

$\mathcal{N} = 4$ SUSY Yang-Mills theory.

- Was proved by Mandelstam, Nucl. Phys. B213, 149 (1983); P.Howe and K.Stelle, ICL preprint (1983); Phys.Lett. **B137**, 135 (1984). L.Brink, conference talk at Johns Hopkins Workshop (1983).
- The Maldacena correspondence is primarily aimed at the $N \rightarrow \infty$ limit with the 't Hooft parameter of N times the squared gauge coupling held fixed.
- Conformal behavior valid here also for finite N .
- We exploit initially the duality thereafter focus exclusively on the field theory description.

1. QUIVER GAUGE THEORIES

BREAKING SUPERSYMMETRIES

To approach the real world, one needs less or no supersymmetry in the (conformal?) gauge theory.

By factoring out a discrete (abelian) group and composing an orbifold:

$$S^5/\Gamma$$

one may break $\mathcal{N} = 4$ supersymmetry to $\mathcal{N} = 2$, 1 , or 0 . Of special interest is the $\mathcal{N} = 0$ case.

We may take $\Gamma = Z_p$ which identifies p points in \mathcal{C}_3 .

The rule for breaking the $\mathcal{N} = 4$ supersymmetry is:

$$\Gamma \subset SU(2) \quad \Rightarrow \quad \mathcal{N} = 2$$

$$\Gamma \subset SU(3) \quad \Rightarrow \quad \mathcal{N} = 1$$

$$\Gamma \not\subset SU(3) \quad \Rightarrow \quad \mathcal{N} = 0$$

In fact to specify the embedding of $\Gamma = Z_p$ we need to identify three integers (a_1, a_2, a_3) :

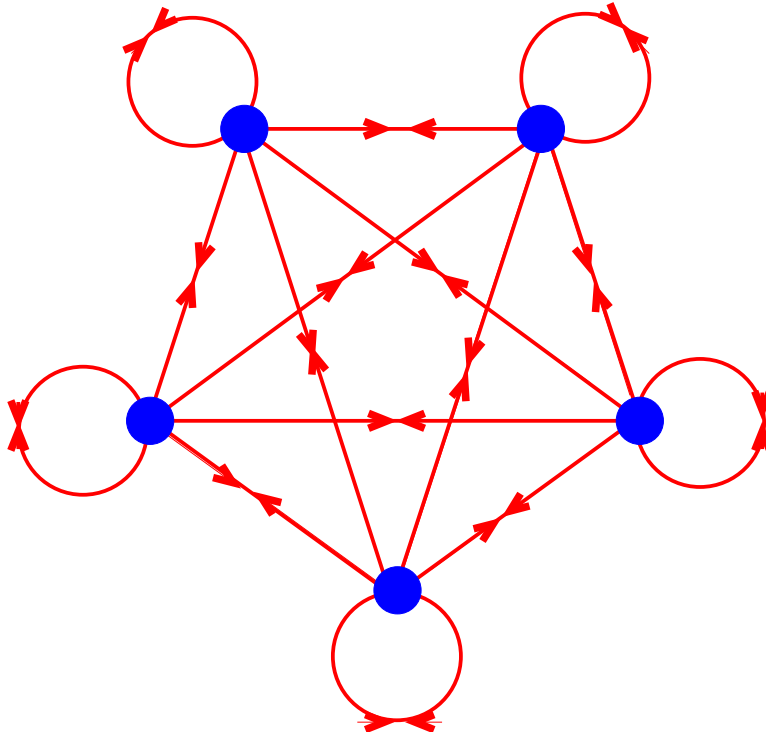
$$\mathcal{C}_3 : (X_1, X_2, X_3) \xrightarrow{Z_p} (\alpha^{a_1} X_1, \alpha^{a_2} X_2, \alpha^{a_3} X_3)$$

with

$$\alpha = \exp\left(\frac{2\pi i}{p}\right)$$

MATTER REPRESENTATIONS

- The Z_p discrete group identifies p points in \mathcal{C}_3 .
- The N converging D3-branes meet on all p copies, giving a gauge group: $U(N) \times U(N) \times \dots \times U(N)$.
- The matter (spin-1/2 and spin-0) which survives is invariant under a product of a gauge transformation and a Z_p transformation.



One can draw p points and arrows for a_1, a_2, a_3 .

e.g. $Z_5 (1, 3, 0)$

Quiver diagram (Douglas-Moore).

Scalar representation is:

$$\sum_{k=1}^3 \sum_{i=1}^p (N_1, \bar{N}_{i \pm a_k})$$

For fermions, one must construct the $\mathbf{4}$ of R-parity $SU(4)$:

From the $a_k = (a_1, a_2, a_3)$ one constructs the 4-spinor $A_\mu = (A_1, A_2, A_3, A_4)$:

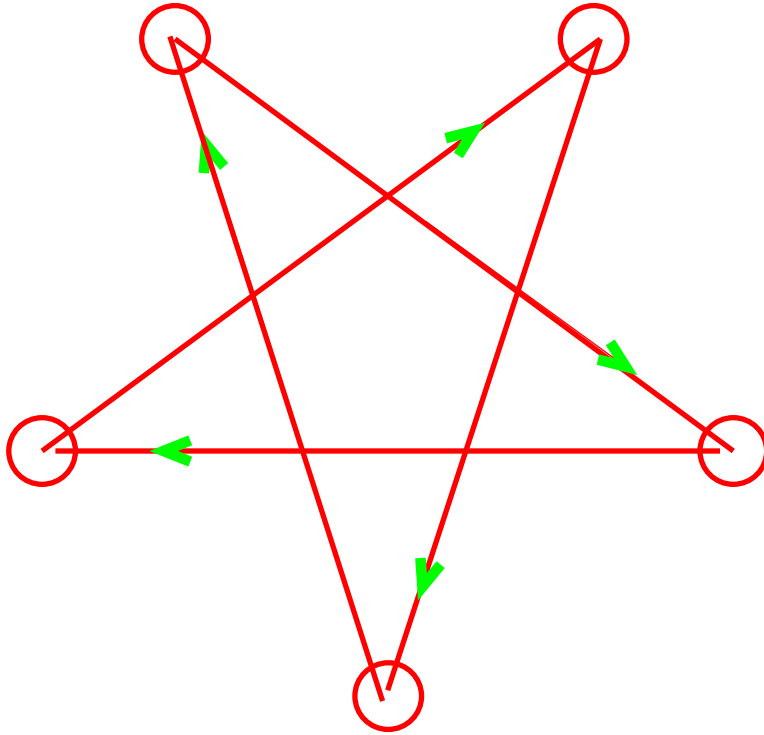
$$A_1 = \frac{1}{2}(a_1 + a_2 + a_3)$$

$$A_2 = \frac{1}{2}(a_1 - a_2 - a_3)$$

$$A_3 = \frac{1}{2}(-a_1 + a_2 - a_3)$$

$$A_4 = \frac{1}{2}(-a_1 - a_2 + a_3)$$

These transform as $\exp\left(\frac{2\pi i}{p}A_\mu\right)$ and the invariants may again be derived (by a different diagram):



e.g. $A_\mu = 2; \quad p = 5.$

These lines are oriented.

One finds for the fermion representation

$$\sum_{\mu=1}^4 \sum_{i=1}^p (N_i, \bar{N}_{i+A_\mu})$$

Two interesting references

Hierarchy and Naturalness

K.G. Wilson, Phys. Rev. **D3**, 1818 (1971).
(already discussed in the late 1960s)

K.G. Wilson hep-lat/0412043

What a difference 3 decades make.

SUPERCONFORMAL SYMMETRY

$\mathcal{N} = 4$ SU(N) Yang-Mills 1983
String theory 1997 for infinite N.

lessened to supersymmetry

Answered Wilson's objection 1974
> 10^4 papers.

or CONFORMALITY?

$\mathcal{N} = 4 \rightarrow \mathcal{N} = 0$ by orbifolding $\rightarrow U(N)^n$
< 10^2 papers. $3^3 \gg 422$

LHC (2009) SHOULD DISCRIMINATE

2. CONFORMALITY PHENOMENOLOGY

- Hierarchy between GUT scale and weak scale is 14 orders of magnitude. Why do these two very different scales exist?
- How is this hierarchy of scales stabilized under quantum corrections?
- Supersymmetry answers the second question but not the first.

Approach hierarchy problem by Conformality at TeV Scale.

- Will show idea is possible.
- Explicit examples containing standard model states.
- Conformality more rigid constraint than supersymmetry.
- Predicts additional states at TeV scale for conformality.
- Gauge coupling unification.
- Naturalness: cancellation of quadratic divergences.
- Anomaly cancellation: conformality of U(1) couplings.
- Dark Matter candidate.

Conformality as hierarchy solution

- Quark and lepton masses, QCD and weak scales small compared to TeV scale.
- May be put to zero suggesting:
- Add degrees of freedom to yield GFT with conformal invariance.
- 't Hooft naturalness since zero mass limit increases symmetry to conformal symmetry.

The theory is assumed to be given by the action:

$$S = S_0 + \int d^4x \alpha_i O_i \quad (1)$$

where S_0 is the action for the conformal theory and the O_i are operators with dimension below four which break conformal invariance softly.

The mass parameters α_i have mass dimension $4 - \Delta_i$ where Δ_i is the dimension of O_i at the conformal point.

Let M be the scale set by the parameters α_i and hence the scale at which conformal invariance is broken. Then for $E \gg M$ the couplings will not run while they start running for $E < M$. To solve the hierarchy problem we assume M is near the TeV scale.

GENERAL PREDICTIONS.

Consider embedding the standard model gauge group according to:

$$SU(3) \times SU(2) \times U(1) \subset \bigotimes_i U(Nd_i)$$

Each gauge group of the SM can lie entirely in a $SU(Nd_i)$ or in a diagonal subgroup of a number thereof.

Only bifundamentals (including adjoints) are possible. This implies no $(8, 2)$, etc. A conformality restriction which is new and satisfied in Nature!

No $U(1)$ factor can be conformal and so hypercharge is quantized through its incorporation in a non-abelian gauge group. This is the “conformality” equivalent to the GUT charge quantization condition in *e.g.* $SU(5)$!

Beyond these general consistencies, there are predictions of new particles necessary to render the theory conformal.

3. 4 TeV UNIFICATION

- Above 4 TeV scale couplings will not run.
- Couplings of 3-2-1 related, not equal, at conformality scale.
- Embeddings in different numbers of the equal-coupling $U(N)$ groups lead to the 4 TeV scale unification without logarithmic running over large desert.

When we arrive at $p = 7$ there are viable models. Actually three different quiver diagrams can give:

1) 3 chiral families.

2) Adequate scalars to spontaneously break $U(3)^7 \rightarrow SU(3) \times SU(2) \times U(1)$

and

3) $\sin^2\theta_W = 3/13 = 0.231$

The embeddings of $\Gamma = Z_7$ in $SU(4)$ are:

7A. $(\alpha, \alpha, \alpha, \alpha^4)$

7B. $(\alpha, \alpha, \alpha^2, \alpha^3)^*$ C-H-H-H-W-H-W

7C. $(\alpha, \alpha^2, \alpha^2, \alpha^2)$

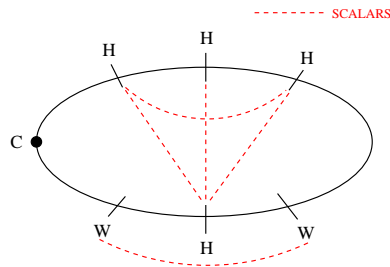
7D. $(\alpha, \alpha^3, \alpha^5, \alpha^5)^*$ C-H-W-H-H-H-W

7E. $(\alpha, \alpha^4, \alpha^4, \alpha^5)^*$ C-H-W-W-H-H-H

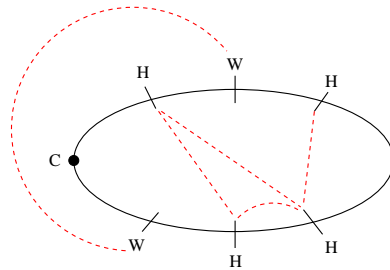
7F. $(\alpha^2, \alpha^4, \alpha^4, \alpha^4)$

* have properties 1), 2) and 3).

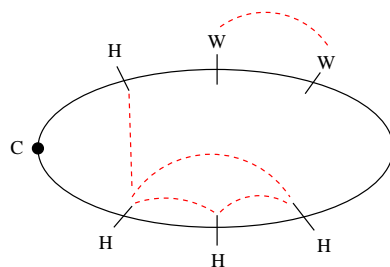
$$7B \quad 4 = (1, 1, 2, 3) \quad 6 = (2, 3, 3, -3, -3, -2)$$



$$7D \quad 4 = (1, 3, 5, 5) \quad 6 = (1, 1, 3, -3, -1, -1)$$



$$7E \quad 4 = (1, 4, 4, 5) \quad 6 = (1, 2, 2, -2, -2, -1)$$



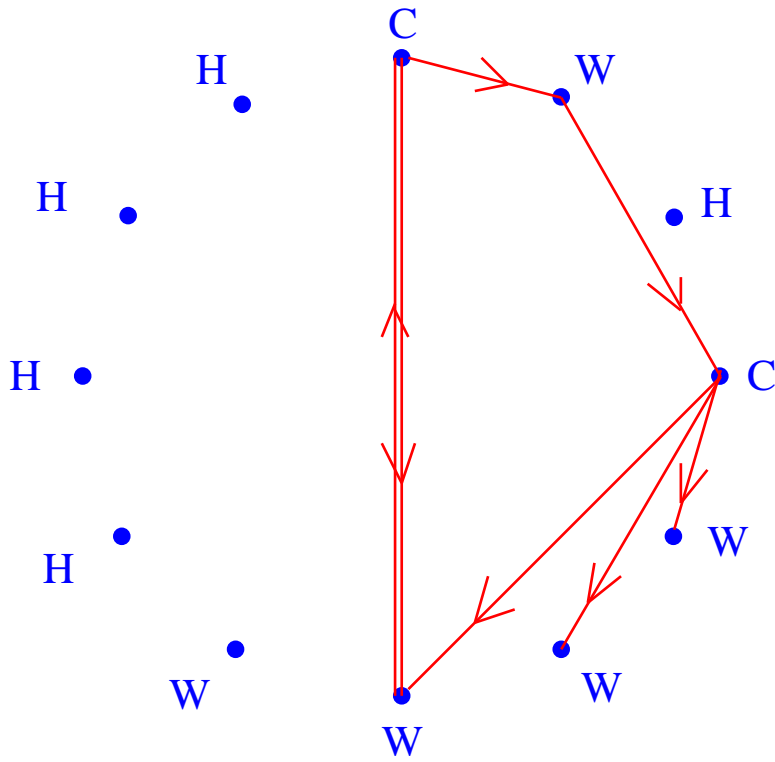
The simplest abelian orbifold conformal extension of the standard model has $U(3)^7 \rightarrow SU(3)^3$ trinification $\rightarrow (321)_{SM}$.

In this case we have α_2 and α_1 related correctly for low energy. But $\alpha_3(M) \simeq 0.07$ suggesting a conformal scale $M \geq 10$ TeV - too high for the L.H.C.

Taking as orbifold S^5/Z_{12} with embedding of Z_{12} in the $SU(4)$ R-parity specified by $\mathbf{4} \equiv (\alpha^{A_1}, \alpha^{A_2}, \alpha^{A_3}, \alpha^{A_4})$ and $A_\mu = (1, 2, 3, 6)$.

This accommodates the scalars necessary to spontaneously break to the SM.

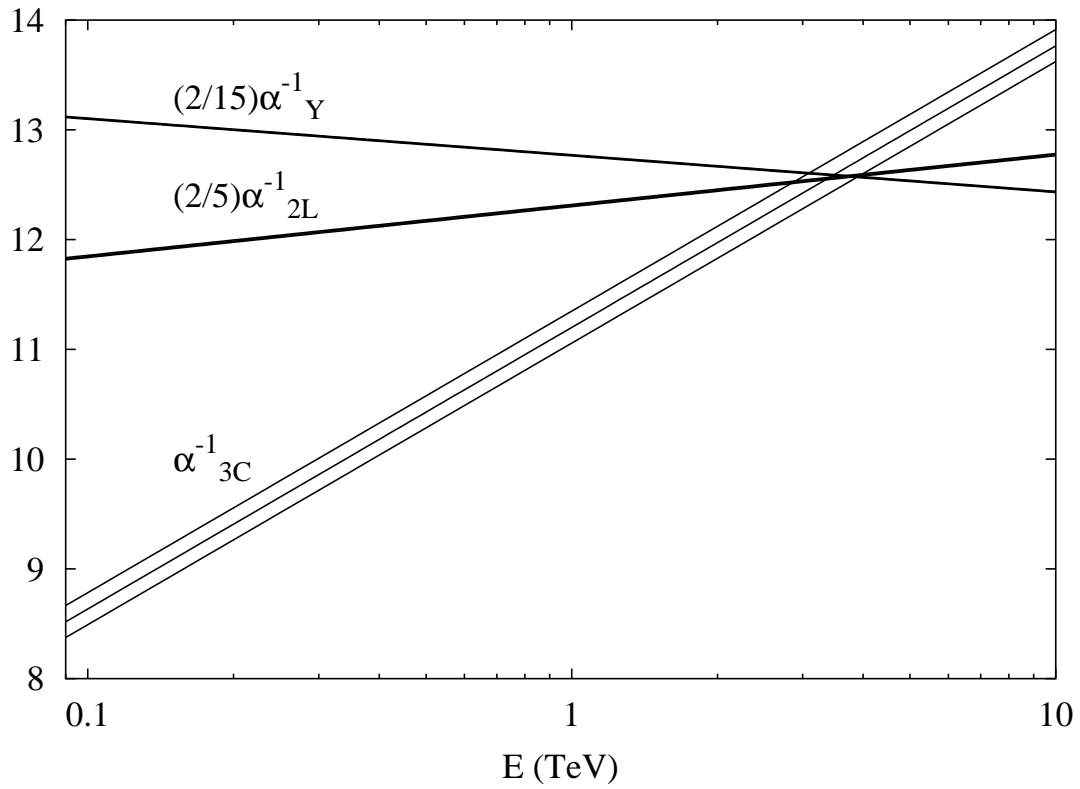
As a bonus, the dodecagonal quiver predicts three chiral families (see next transparency).



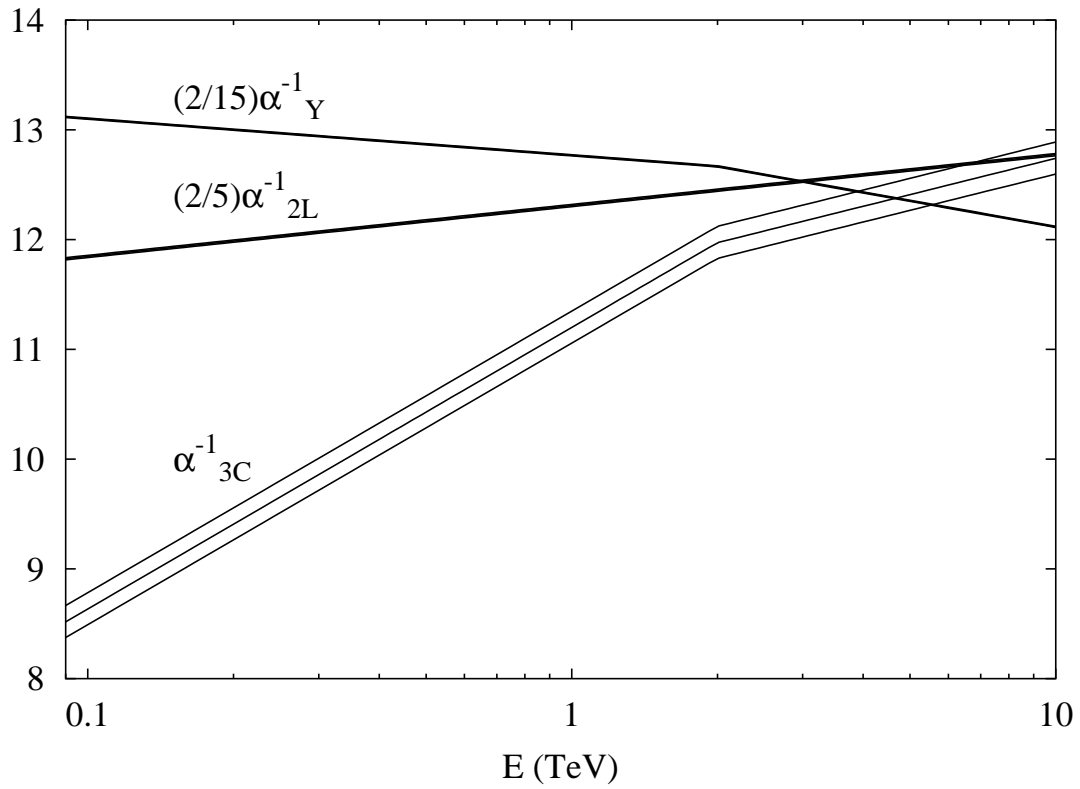
$$A_\mu = (1, 2, 3, 6)$$

$$SU(3)_C \times SU(3)_H \times SU(3)_H$$

$$5(3, \bar{3}, 1) + 2(\bar{3}, 3, 1)$$



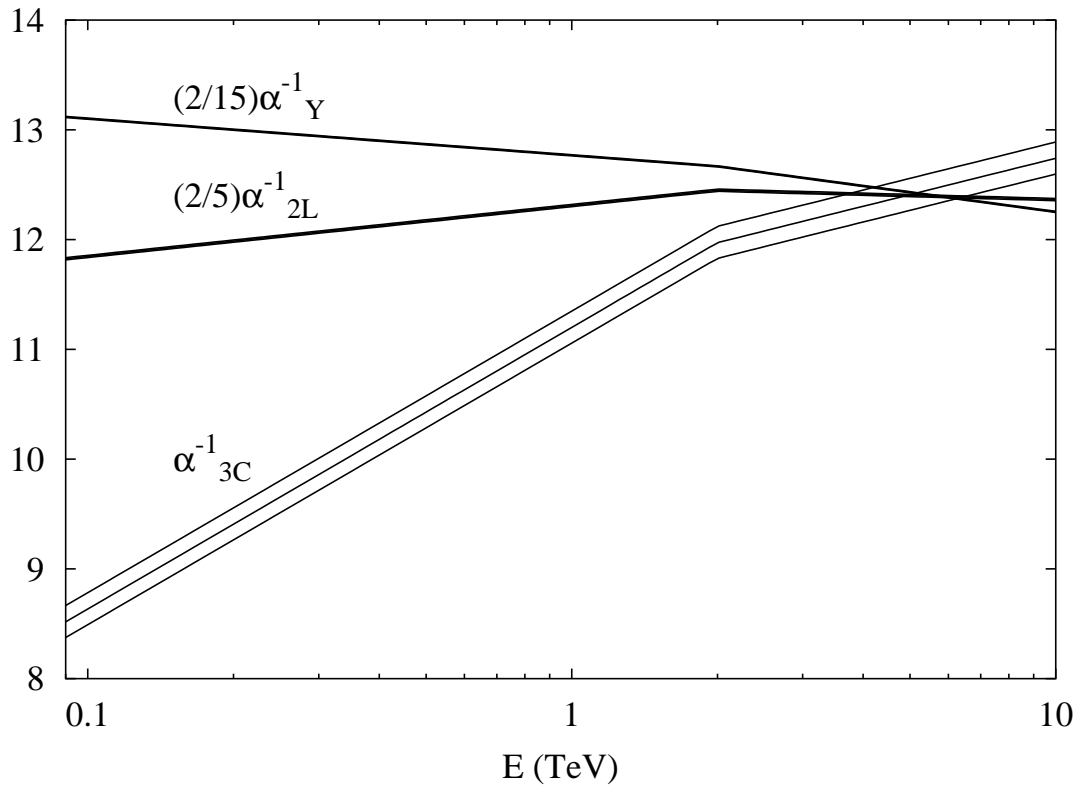
without thresholds: all states at M_U
 hep-ph/0302074
 PHF+Ryan Rohm + Tomo Takahashi



thresholds: CH fermions at 2 TeV

hep-ph/0302074

PHF+Ryan Rohm + Tomo Takahashi



thresholds: CW fermions at 2 TeV
 hep-ph/0302074
 PHF+Ryan Rohm + Tomo Takahashi

4. QUADRATIC DIVERGENCES.

Classification of abelian quiver gauge theories

We consider the compactification of the type-IIB superstring on the orbifold $AdS_5 \times S^5/\Gamma$ where Γ is an abelian group $\Gamma = Z_p$ of order p with elements $\exp(2\pi i A/p)$, $0 \leq A \leq (p-1)$.

The resultant quiver gauge theory has \mathcal{N} residual supersymmetries with $\mathcal{N} = 2, 1, 0$ depending on the details of the embedding of Γ in the $SU(4)$ group which is the isotropy of the S^5 . This embedding is specified by the four integers A_m , $1 \leq m \leq 4$ with

$$\sum_m A_m = 0(\text{mod } p) \quad (2)$$

which characterize the transformation of the components of the defining representation of $SU(4)$. We are here interested in the non-supersymmetric case $\mathcal{N} = 0$ which occurs if and only if all four A_m are non-vanishing.

	p	A_m	a_i	scal bfds	scal adjs	chir frms	SM
1	2	(1111)	(000)	0	6	No	No
2	3	(1122)	(001)	2	4	No	No
3	4	(2222)	(000)	0	6	No	No
4	4	(1133)	(002)	2	4	No	No
5	4	(1223)	(011)	4	2	No	No
6	4	(1111)	(222)	6	0	Yes	No
7	5	(1144)	(002)	2	4	No	No
8	5	(2233)	(001)	2	4	No	No
9	5	(1234)	(012)	4	2	No	No
10	5	(1112)	(222)	6	0	Yes	No
11	5	(2224)	(111)	6	0	Yes	No
12	6	(3333)	(000)	0	6	No	No
13	6	(2244)	(002)	2	4	No	No
14	6	(1155)	(002)	2	4	No	No
15	6	(1245)	(013)	4	2	No	No
16	6	(2334)	(011)	4	2	No	No
17	6	(1113)	(222)	6	0	Yes	No
18	6	(2235)	(112)	6	0	Yes	No
19	6	(1122)	(233)	6	0	Yes	No

The Table continues to infinity but we stop at $p = 7$:

	p	A_m	a_i	scal bfds	scal adjs	chir frms	SM
20	7	(1166)	(002)	2	4	No	No
21	7	(3344)	(001)	2	4	No	No
22	7	(1256)	(013)	4	2	No	No
23	7	(1346)	(023)	4	2	No	No
24	7	(1355)	(113)	6	0	No	No
25	7	(1114)	(222)	6	0	Yes	No
26	7	(1222)	(333)	6	0	Yes	No
27	7	(2444)	(111)	6	0	Yes	No
28	7	(1123)	(223)	6	0	Yes	Yes
29	7	(1355)	(113)	6	0	Yes	Yes
30	7	(1445)	(113)	6	0	Yes	Yes

It follows that:

In an $\mathcal{N} = 0$ quiver gauge theory,
chiral fermions are present
if and only if all scalars
are in bifundamental representations.

Cancellation of quadratic divergences

The lagrangian for the nonsupersymmetric Z_p theory can be written in a convenient notation which accommodates simultaneously both adjoint and bifundamental scalars as

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4}F_{\mu\nu;r,r}^{ab}F_{\mu\nu;r,r}^{ba} + i\bar{\lambda}_{r+A_4,r}^{ab}\gamma^\mu D_\mu\lambda_{r,r+A_4}^{ba} + 2D_\mu\Phi_{r+a_i,r}^{ab\dagger}D_\mu\Phi_{r,r+a_i}^{ba} + i\bar{\Psi}_{r+A_m,r}^{ab}\gamma^\mu D_\mu\Psi_{r,r+A_m}^{ba} \\
& -2ig\left[\bar{\Psi}_{r,r+A_i}^{ab}P_L\lambda_{r+A_i,r+A_i+A_4}^{bc}\Phi_{r+A_i+A_4,r}^{\dagger ca} - \bar{\Psi}_{r,r+A_i}^{ab}P_L\Phi_{r+A_i,r-A_4}^{\dagger bc}\lambda_{r-A_4,r}^{ca}\right] \\
& -\sqrt{2}ig\epsilon_{ijk}\left[\bar{\Psi}_{r,r+A_i}^{ab}P_L\Psi_{r+A_i,r+A_i+A_j}^{bc}\Phi_{r-A_k-A_4,r}^{ca} - \bar{\Psi}_{r,r+A_i}^{ab}P_L\Phi_{r+A_i,r+A_i+A_k+A_4}^{bc}\Psi_{r-A_j,r}^{ca}\right] \\
& -g^2\left(\Phi_{r,r+a_i}^{ab}\Phi_{r+a_i,r}^{\dagger bc} - \Phi_{r,r-a_i}^{\dagger ab}\Phi_{r-a_i,r}^{bc}\right)\left(\Phi_{r,r+a_j}^{cd}\Phi_{r+a_j,r}^{\dagger da} - \Phi_{r,r-a_j}^{\dagger cd}\Phi_{r-a_j,r}^{da}\right) \\
& +4g^2\left(\Phi_{r,r+a_i}^{ab}\Phi_{r+a_i,r+a_i+a_j}^{bc}\Phi_{r+a_i+a_j,r+a_j}^{\dagger cd} - \Phi_{r,r+a_i}^{ab}\Phi_{r+a_i,r+a_i+a_j}^{bc}\Phi_{r+a_i+a_j,r+a_i}^{\dagger cd}\right)
\end{aligned}$$

where $\mu, \nu = 0, 1, 2, 3$ are lorentz indices; $a, b, c, d = 1$ to N are $U(N)^p$ group labels; $r = 1$ to p labels the node of the quiver diagram ; a_i ($i = \{1, 2, 3\}$) label the first three of the **6** of $SU(4)$; A_m ($m = \{1, 2, 3, 4\}$) = (A_i, A_4) label the **4** of $SU(4)$. By definition A_4 denotes an arbitrarily-chosen fermion (λ) associated with the gauge boson, similarly to the notation in the $\mathcal{N} = 1$ supersymmetric case.

*(This $\mathcal{N} = 0$ lagrangian typeface may be illegible.
Explain notation if necessary.)*

Let us first consider the quadratic divergence question in the mother $\mathcal{N} = 4$ theory. The $\mathcal{N} = 4$ lagrangian is like Eq.(3) but since there is only one node all those subscripts become unnecessary so the form is simply

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}^{ab}F_{\mu\nu}^{ba} + i\bar{\lambda}^{ab}\gamma^\mu D_\mu\lambda^{ba} + 2D_\mu\Phi_i^{ab\dagger}D_\mu\Phi_i^{ba} + i\bar{\Psi}_m^{ab}\gamma^\mu D_\mu\Psi_m^{ba} \\
& -2ig\left[\bar{\Psi}_i^{ab}P_L\lambda^{bc}\Phi_{i,r}^{\dagger ca} - \bar{\Psi}_i^{ab}P_L\Phi_i^{bc}\lambda^{ca}\right] \\
& -\sqrt{2}ig\epsilon_{ijk}\left[\bar{\Psi}_i^{ab}P_L\Psi_j^{bc}\Phi_k^{\dagger ca} - \bar{\Psi}_i^{ab}P_L\Phi_j^{bc}\Psi_k^{ca}\right] \\
& -g^2\left(\Phi_i^{ab}\Phi_i^{\dagger bc} - \Phi_i^{\dagger ab}\Phi_i^{bc}\right)\left(\Phi_j^{cd}\Phi_j^{\dagger da} - \Phi_j^{\dagger cd}\Phi_j^{da}\right) \\
& +4g^2\left(\Phi_i^{ab}\Phi_j^{bc}\Phi_i^{\dagger cd}\Phi_j^{\dagger da} - \Phi_i^{ab}\Phi_j^{bc}\Phi_j^{\dagger cd}\Phi_i^{\dagger da}\right)
\end{aligned}$$

All $\mathcal{N} = 4$ scalars are in adjoints and the scalar propagator has one-loop quadratic divergences coming potentially from three scalar self-energy diagrams: (a) the gauge loop (one quartic vertex); (b) the fermion loop (two trilinear vertices); and (c) the scalar loop (one quartic vertex).

For $\mathcal{N} = 4$ the respective contributions of (a, b, c) are computable from Eq.(3) as proportional to $g^2 N(1, -4, 3)$ which cancel exactly.

The $\mathcal{N} = 0$ results for the scalar self-energies (a, b, c) are computable from the lagrangian of Eq.(3). Fortunately, the calculation was already done in the literature. The result is amazing! The quadratic divergences cancel if and only if $x = 3$, exactly the same “if and only if” as to have chiral fermions. It is pleasing that one can independently confirm the results directly from the interactions in Eq.(3) To give just one explicit example, in the contributions to diagram (c) from the last term in Eq.(3), the $1/N$ corrections arise from a contraction of Φ with Φ^\dagger when all the four color superscripts are distinct and there is consequently no sum over color in the loop. For this case, examination of the node subscripts then confirms proportionality to the kronecker delta, δ_{0,a_i} . If and only if all $a_i \neq 0$, all the other terms in Eq.(3) do not lead to $1/N$ corrections to the $\mathcal{N} = 4$.

5. Anomaly cancellation by a compensatory term and $U(1)$'s in conformality

The lagrangian for the nonsupersymmetric Z_n theory can be written in a convenient notation which accommodates simultaneously both adjoint and bifundamental scalars as mentioned before.

As also mentioned above we shall restrict attention to models where all scalars are in bifundamentals which requires all a_i to be non zero. Recall that $a_1 = A_2 + A_3$, $a_2 = A_3 + A_1$; $a_3 = A_1 + A_2$.

The lagrangian is classically $U(N)^p$ gauge invariant. There are, however, triangle anomalies of the $U(1)_p U(1)_q^2$ and $U(1)_p SU(N)_q^2$ types. Making gauge transformations under the $U(1)_r$ ($r = 1, 2, \dots, n$) with gauge parameters Λ_r leads to a variation

$$\delta \mathcal{L} = -\frac{g^2}{4\pi^2} \sum_{p=1}^{p=n} A_{pq} F_{\mu\nu}^{(p)} \tilde{F}^{(p)\mu\nu} \Lambda_q \quad (3)$$

which defines an $n \times n$ matrix A_{pq} which is given by

$$A_{pq} = \text{Tr}(Q_p Q_q^2) \quad (4)$$

where the trace is over all chiral fermion links and Q_r is the charge of the bifundamental under $U(1)_r$. We shall adopt the sign convention that \mathbf{N} has $Q = +1$ and \mathbf{N}^* has $Q = -1$.

It is straightforward to write A_{pq} in terms of Kronecker deltas because the content of chiral fermions is

$$\sum_{m=1}^{m=4} \sum_{r=1}^{r=n} (\mathbf{N}_r, \mathbf{N}_{r+A_m}^*) \quad (5)$$

This implies that the antisymmetric matrix A_{pq} is explicitly

$$A_{pq} = -A_{qp} = \sum_{m=1}^{m=4} (\delta_{p,q-A_m} - \delta_{p,q+A_m}) \quad (6)$$

Now we are ready to construct $\mathcal{L}_{comp}^{(1)}$, the compensatory term. Under the $U(1)_r$ gauge transformations with gauge parameters Λ_r we require that

$$\begin{aligned} \delta \mathcal{L}_{comp}^{(1)} &= -\delta \mathcal{L} \\ &= +\frac{g^2}{4\pi^2} \sum_{p=1}^{p=n} A_{pq} F_{\mu\nu}^{(p)} \tilde{F}^{(p)\mu\nu} \Lambda_q \end{aligned} \quad (7)$$

To accomplish this property, we construct a compensatory term in the form

$$\mathcal{L}_{comp}^{(1)} = \frac{g^2}{4\pi} \sum_{p=1}^{p=n} \sum_k B_{pk} \text{Im Tr ln} \left(\frac{\Phi_k}{v} \right) F_{\mu\nu}^{(p)} \tilde{F}^{(p)\mu\nu} \quad (8)$$

where \sum_k runs over scalar links. To see that $\mathcal{L}_{comp}^{(1)}$ of Eq.(8) has $SU(N)^n$ invariance rewrite $\text{Tr ln} \equiv \exp \det$ and note that the $SU(N)$ matrices have unit determinant.

We note *en passant* that one cannot take the $v \rightarrow 0$ limit in Eq.(8); the chiral anomaly enforces a breaking of conformal invariance.

We define a matrix C_{kq} by

$$\delta \left(\sum_{p=1}^{p=n} \sum_k \text{Im Tr} \ln \left(\frac{\Phi_k}{v} \right) \right) = \sum_{q=1}^{q=n} C_{kq} \Lambda_q \quad (9)$$

whereupon Eq.(7) will be satisfied if the matrix B_{pk} satisfies $A = BC$. The inversion $B = AC^{-1}$ is non trivial because C is singular but C_{kq} can be written in terms of Kronecker deltas by noting that the content of complex scalar fields in the model implies that the matrix C_{kq} must be of the form

$$C_{kq} = 3\delta_{kq} - \sum_i \delta_{k+a_i, q} \quad (10)$$

Evolution of $U(1)$ gauge couplings.

In the absence of the compensatory term, the two independent $U(N)^n$ gauge couplings g_N for $SU(N)$ and g_1 for $U(1)$ are taken to be equal $g_N(\mu_0) = g_1(\mu_0)$ at a chosen scale, *e.g.* $\mu_0=4$ TeV to enable cancellation of quadratic divergences. Note that the n $SU(N)$ couplings $g_N^{(p)}$ are equal by the overall Z_n symmetry, as are the n $U(1)$ couplings $g_1^{(p)}$, $1 \leq p \leq n$.

As one evolves to higher scales $\mu > \mu_0$, the renormalization group beta function β_N for $SU(N)$ vanishes $\beta_N = 0$ at least at one-loop level so the $g_N(\mu)$ can behave independent of the scale as expected by conformality. On the other hand, the beta function β_1 for $U(1)$ is positive definite in the unadorned theory, given at one loop by

$$b_1 = \frac{11N}{48\pi^2} \quad (11)$$

where N is the number of colors.

The corresponding coupling satisfies

$$\frac{1}{\alpha_1(\mu)} = \frac{1}{\alpha_1(M)} + 8\pi b_1 \ln\left(\frac{M}{\mu}\right) \quad (12)$$

so the Landau pole, putting $\alpha(\mu) = 0.1$ and $N = 3$, occurs at

$$\frac{M}{\mu} = \exp\left[\frac{20\pi}{11}\right] \simeq 302 \quad (13)$$

so for $\mu = 4$ TeV, $M \sim 1200$ TeV. The coupling becomes “strong” $\alpha(\mu) = 1$ at

$$\frac{M}{\mu} = \exp\left[\frac{18\pi}{11}\right] \simeq 171 \quad (14)$$

or $M \sim 680$ TeV.

We may therefore ask whether the new term \mathcal{L}_{comp} in the lagrangian, necessary for anomaly cancellation, can solve this problem for conformality?

Since the scale v breaks conformal invariance, the matter fields acquire mass, so the one-loop diagram ¹ has a logarithmic divergence proportional to

$$\int \frac{d^4 p}{v^2} \left[\frac{1}{(p^2 - m_k^2)} - \frac{1}{(p^2 - m_{k'}^2)} \right] \sim -\frac{\Delta m_{kk'}^2}{v^2} \ln \left(\frac{\Lambda}{v} \right) \quad (15)$$

the sign of which depends on $\delta m_{kk'}^2 = (m_k^2 - m_{k'}^2)$.

¹The usual one-loop β -function is of order h^2 regarded as an expansion in Planck's constant: four propagators each $\sim h$ and two vertices each $\sim h^{-1}$ (c.f. Y. Nambu, Phys. Lett. **B26**, 626 (1968)). The diagram considered is also $\sim h^2$ since it has three propagators, one quantum vertex $\sim h$ and an additional h^{-2} associated with $\Delta m_{kk'}^2$.

To achieve conformality of $U(1)$, a constraint must be imposed on the mass spectrum of matter bifundamentals, *viz*

$$\Delta m_{kk'}^2 \propto v^2 \left(\frac{11N}{48\pi^2} \right) \quad (16)$$

with a proportionality constant of order one which depends on the choice of model, the n of Z_n and the values chosen for $A_m, m = 1, 2, 3$. This signals how conformal invariance must be broken at the TeV scale in order that it can be restored at high energy; it is interesting that such a constraint arises in connection with an anomaly cancellation mechanism which necessarily breaks conformal symmetry.

6. DARK MATTER CANDIDATE

Definition of a Z_2 symmetry

In the nonsupersymmetric quiver gauge theories, the gauge group, for abelian orbifold $AdS_5 \times S^5/Z_n$ is $U(N)^n$. In phenomenological application $N = 3$ and n reduces eventually after symmetry breaking to $n = 3$ as in trification. The chiral fermions are then in the representation of $SU(3)^3$:

$$(3, 3^*, 1) + (3^*, 1, 3) + (1, 3, 3^*) \quad (17)$$

This is as in the **27** of E_6 where the particles break down in to the following representations of the $SU(3) \times SU(2) \times U(1)$ standard model group:

$$Q, \quad u^c, \quad d^c, \quad L \quad e^c \quad N^c \quad (18)$$

transforming as

$$(3, 2), \quad (3^*, 1), \quad (3^*, 1), \quad (1, 2), \quad (1, 1), \quad (1, 1) \quad (19)$$

in a **16** of the $SO(10)$ subgroup.

In addition there are the states

$$h, \quad h^*, \quad E, \quad E^* \quad (20)$$

transforming as

$$(3, 1), \quad (3^*, 1), \quad (2, 1), \quad (2, 1) \quad (21)$$

in a $\mathbf{10}$ of $SO(10)$ and finally

$$S \quad (22)$$

transforming as the singlet

$$(1, 1) \quad (23)$$

It is natural to define a Z_2 symmetry R which commutes with the $SO(10)$ subgroup of $E_6 \rightarrow O(10) \times U(1)$ such that $R = +1$ for the first $\mathbf{16}$ of states. Then it is mandated that $R = -1$ for the $\mathbf{10}$ and $\mathbf{1}$ of $SO(10)$ because the following Yukawa couplings must be present to generate mass for the fermions:

$$16_f 16_f 10_s, \quad 16_f 10_f 16_s, \quad 10_f 10_f 1_s, \quad 10_f 1_f 1_s \quad (24)$$

which require $R = +1$ for $10_s, 1_s$ and $R = -1$ for 16_s .

Contribution of the Lightest Conformality Particle (LCP) to the cosmological energy density

The LCP act as cold dark matter WIMPs, and the calculation of the resultant energy density follows a well-known path. Here we follow the procedure in a recent technical book by Mukhanov.

The LCP decouple at temperature T_* , considerably less than their mass M_{LCP} ; we define $x_* = M_{LCP}/T_*$. Let the annihilation cross-section of the LCP at decoupling be σ_* . Then the dark matter density Ω_m , relative to the critical density, is estimated as

$$\Omega_m h_{75}^2 = \frac{\tilde{g}_*^{1/2}}{g_*} x_*^{3/2} \left(\frac{3 \times 10^{-38} \text{cm}^2}{\sigma_*} \right) \quad (25)$$

where h_{75} is the Hubble constant in units of 75km/s/Mpc . $g_* = (g_b + \frac{7}{8}g_f)$ is the effective number of degrees of freedom (dof) at freeze-out for all particles which later convert their energy into photons; and \tilde{g}_* is the number of dof which are relativistic at T_* .

Discussion of LCP Dark Matter

The LCP is a viable candidate for a cold dark matter particle which can be produced at the LHC. The distinction from other candidates will require establishment of the $U(3)^3$ gauge bosons, extending the 3-2-1 standard model and the discovery that the LCP is in a bifundamental representation thereof.

To confirm that the LCP is the dark matter particle would, however, require direct detection of dark matter.

Finally, it has been established that conformality can provide (i) naturalness without one-loop quadratic divergence for the scalar mass and anomaly cancellation; (ii) precise unification of the coupling constants; and (iii) a viable dark matter candidate. It remains for experiment to check that quiver gauge theories with gauge group $U(3)^3$ or $U(3)^n$ with $n \geq 4$ are actually employed by Nature.

SUMMARY

- Phenomenology of conformality has striking resonances with the standard model.
- 4 TeV Unification predicts three families and new particles around 4 TeV accessible to experiment (LHC).
- The scalar propagator in these theories has no quadratic divergence iff there are chiral fermions.
- Anomaly cancellation in effective lagrangian connected to consistency of $U(1)$ factors.
- Dark matter candidate (LCP) will be produced at LHC, then (in)directly detected.

Thank you for your attention