Thank you for the invitation.

TURNAROUND

IN

CYCLIC COSMOLOGY

Paul H Frampton

UNC-CHAPEL HILL

OUTLINE

1. Historical context.
2. Vanishing entropy of contracting universe
3. Infinite past
4. Constraints on deflation
5. Multifluid Models

SUMMARY.

References:

(1) L. Baum and P.H.F. *Turnaround in Cyclic Cosmology.* P.R.L. **98,** 071301 (2007) `hep-th/0610213`.

(2) L. Baum and P.H.F. *Entropy of Contracting Universe in Cyclic Cosmology* Mod. Phys. Lett. **A23,** 36 (2008). `hep-th/0703162`.

(3) P.H.F. *Cyclic Universe and Infinite Past.* Mod. Phys. Lett. **A22,** 2587 (2007). `arXiv:0706.1186`.

(4) L. Baum, P.H. Frampton and S. Matsuzaki. *Constraints on Deflation* .... `arViv:0801.4420[hep-th]`

(5) I. Arefeva, P.H.F. and S. Matsuzaki, *Multifluid Models for Cyclic Cosmology.* `arXiv:0802.1294 [hep-th]`

1. Historical context.

One of the oldest questions in theoretical cosmology is whether an infinitely oscillatory universe which avoids an initial singularity can be consistently constructed. As realized by Friedmann and especially by Tolman (also LeMaitre, Einstein, De Sitter ....) one principal obstacle is the second law of thermodynamics which dictates that the entropy increases from cycle to cycle. If the cycles thereby become longer, extrapolation into the past will lead back to an initial singularity again, thus removing the motivation to consider an oscillatory universe in the first place. This led to the abandonment of the oscillatory universe by the majority of workers.

Nevertheless, an oscillatory universe is an attractive alternative to the Big Bang. One new ingredient in the cosmic make-up is the dark energy discovered only in 1998 and so it natural to ask whether this can avoid the difficulties with entropy which have dogged previous attempts.

Some work has been started to exploit the dark energy in allowing cyclicity possibly without apparently the need for inflation in Steinhardt *et al* Another new ingredient is the use of branes and a fourth spatial dimension as in Randall *et al*, Binetruy *et al* which have examined the consequences for cosmology. The Big Rip and replacement of dark energy by modified gravity have been explored in PHF and Takahashi.

If the dark energy has a super-negative equation of state, $\omega_\Lambda = p_\Lambda/\rho_\Lambda < -1$, it leads to a Big Rip (R. Caldwell) at a finite time where there exist extraordinary conditions with regard to density and causality as one approaches the Big Rip. In the present article we explore whether these exceptional physical conditions can assist in providing an infinitely-cyclic cosmology.

We shall consider the situation where if, as we approach the Big Rip, the expansion stops due to the brane contribution just short of the Big Rip and there is a turnaround at $t = t_T$ when the scale factor is deflated to a very tiny fraction (f) of itself and only one causal patch is retained, while the other $1/f^3$ patches contract independently into separate universes. The turnaround takes place an extremely short time before the Big Rip would have occurred, at a time when the universe is fractionated into many independent causal patches, see *e.g.* PHF and Takahashi (2004).

We discuss the contraction phase which occurs with a very much smaller universe than in the expansion phase and with almost vanishing entropy because it is assumed empty of dust, matter and black holes all of which were jettisoned at turnaround. A bounce at $t = \tau$ takes place a short time before a would-be Big Bang. Then, immediately after the bounce, entropy is injected by inflation (Guth) where the scale factor is enhanced by large factor and hence so is entropy. Inflation can thus be a part of the present scenario which is one distinction from the work of Steinhardt *et al.*

For cyclicity of the entropy, $S(t) = S(t+\tau)$ to be consistent with thermodynamics it is necessary that the deflationary decrease by f^3 compensate the entire entropy increase acquired during contraction and expansion including the huge increase during inflation.

A possible shortcoming of the proposal could have been the persistence of spacetime singularities in cyclic cosmologies (Borde, Guth and Vilenkin, 2003) but to our understanding for the truly cyclic universe which we here outline this problem is avoided, provided a simple constraint on the time average of the Hubble parameter is respected.

This work is presented because our discussion seems to give a plausible realization of the infinitely oscillatory universe originally saught by cosmologists on the 1920s and 1930s ignorant of dark energy

(see, however, the discussion after Eq.(172.6) of R.C. Tolman in Relativity, Thermodynamics and Cosmology. Oxford University Press (1934))

and one whose minor shortcomings can hopefully be evolved by others into a convincing scenario.

2. Vanishing entropy of contracting universe.

The contracting universe of the cyclic model contains dark energy with zero entropy and possibly a small amount of radiation which could possess entropy. The deflation at turnaround reduces entropy from a gigantic value O($> 10^{88}$) to an extremely low value O(10^1). An unrealistic value for the dark energy equation of state $\omega = p/\rho = -4/3$ has been employed for algebraic simplicity as it makes $\rho_\Lambda \propto a$, and no attempt yet made at a realistic description of our universe. We shall now study the entropy of the contracting universe in this speculative scenario more quantitatively and now will use arbitrary $\omega = -1 - \phi$ with $\phi > 0$ so that $\rho_\Lambda \propto a^{3\phi}$.

The quantity ϕ is the most important parameter for observational discrimination between this cyclic model and a cosmological constant [#1] The next test of $\phi \neq 0$ will likely come from the Planck Surveyor satellite. One wonders, therefore, how different from zero ϕ is? There is no lower bound on ϕ to make the model work except that it must be non zero. We already know $\phi < 0.1$ from the WMAP3 data. If ϕ is truly infinitesimal, the test must await improved technology. To restore optimism we shall describe an anthropic fine tuning argument that shows that extremely small ϕ is unlikely.

[#1] and from the Steinhardt-Turok cyclic model.

The universe comes back empty of matter including black holes. The presence of matter during contraction causes apparently insuperable problems because accelerated structure formation will precipitate a premature bounce. Black holes, if present, will expand and proliferate with the same consequence. But the presence of radiation must also be carefully studied because although at turnaround the photon energy is infinitesimal ($E_\gamma < 10^{-200} eV$), the blue shifting during contraction leads before the bounce to production of e^+e^- pairs, undesirable because generically they will create problems with continued contraction. As we shall show there are fortunately no photons in the contracting phase of the cycle, only the truly innocuous dark energy.

The cyclic model contains one free parameter, the common density ρ_C at which the universe both turns around and bounces. Since the bounce is independent of ω we begin with it and take as bounce temperatures $T_B = 10^p$ GeV with, to be above the weak and below the Planck scales, $3 \leq p \leq 17$. This gives $\rho_C = \eta \rho_{H_2O}$ where $\eta = 10^{(19+4p)}$ and $\rho_{H_20} = 1 g/cm^3$ is the density of water, an easily imaginable unit somewhere between the unimaginably small present mean cosmic density and the unimaginably large critical density ρ_C at turnaround and bounce.

Going now to the turnaround at time $t = t_T$ the scale factor $a(t_T)$ is given by (since $a(t_0) = 1$ and putting $\rho_0 = 10^{-29}\rho_{H_2O}$) $a(t_T)^{3\phi} = 10^{29}\eta = 10^{48+4p}$

The present radiation temperature is $(T_\gamma)_0 = 2 \times 10^{-4}$ eV, and so the radiation temperature at turnaround is

$$(T_\gamma)_T = 2 \times 10^{-4} \left(10^{(48+4p)}\right)^{-1/3\phi} \text{eV} \quad (1)$$

which is infinitesimal: putting $\phi = 0.1$, Eq.(1) gives 10^{-200} eV for p=3 and 10^{-390} eV for p=17; with $\phi = 0.01$, the photon energy is 10^{-2000} eV for p=3 and 10^{-3900} eV for p=17. In all cases, the photon wavelength is an astronomical number of orders of magnitude longer than the present Hubble length.

To evaluate the contraction entropy we need to estimate many such photons are in one causal patch at turnaround. The deflationary factor multiplying entropy at turnaround must be much less than the inverse of the inflationary increase ($> 10^{84}$) of the early universe. We take the huge number of causal patches to be $10^{90}\alpha$ where $\alpha \gg 1$ is a parameter to allow an arbitrarily larger number, and $\alpha = 1$ will give an overestimate of contraction entropy.

At turnaround the scale factor is

$$a(t_T) = \left(10^{(48+4p)}\right)^{\frac{1}{3\phi}} \tag{2}$$

so taking the present volume as $10^{84} cm^3$ and the present radiation density as $\rho_r(t_0) = 10^{-33} g/cm^3 = 1 eV/cm^3$ gives for the radiation energy in one causal patch

$$(E_r)_{patch} = \frac{1}{(100\alpha)^3} \left(10^{(48+4p)}\right)^{-\frac{1}{3\phi}} \quad \text{eV} \tag{3}$$

Comparison with Eq.(1) then gives for the number of photons per causal patch

$$n_\gamma = \frac{1}{200\alpha^3} \ll 1 \tag{4}$$

which is small even for the unrealistic case $\alpha = 1$ and essentially zero for $\alpha \gg 1$. Thus, the entropy of the contracting universe (cu) vanishes $S_{cu} = 0$ for any value of equation of state of the dark energy $\omega = p/\rho = -1 - \phi$ since Eq.(4) has no ϕ dependence.

Anthropic fine tuning argument about ϕ

The time until turnaround is given by

$$(t_T - t_0) \simeq \frac{t_0}{\phi} \tag{5}$$

so if we take, for simplicity, the origin of life to have occurred at t_0 after the most recent bounce we see from Eq. (5) that given small $\phi \ll 1$ then ϕ measures the fraction of the expansion phase taken to originate life. An anthropic argument is: it is unreasonable for the fraction ϕ, assuming it is non zero, to be extremely close to zero.

The special case $\phi = 0$ is the standard cosmological model with a cosmological constant where there is no turnaround and the future lifetime is infinite so the origin of life necessarily takes place after a vanishing fraction of the expansion lifetime. Although such an infinite expansion seems to us unaesthetic, not all colleagues share our concern.

As soon as one commits to $\phi \neq 0$, however, the anthropic type argument emerges and it is unlikely that $\phi <<< 1$. For example, if $\phi = 10^{-3}$ the length of the expansion phase is 10^4 Gy whereas life orinated after only about 10 Gy which is only 0.1% of the expansion time. If life plays a central role in our universe, as in our understanding is the spirit of the anthropic principle, such a tiny value of ϕ is strongly disfavored; one expects at least $\phi > 0.01$ so the fraction before the origin of life is $> 1.0\%$ of the total expansion time.

This encouraging argument makes it more optimistic that the next generation of observations such as the Planck Surveyor will succeed in detecting a $\phi \neq 0$.

3. Constraints on deflation

1 Times of unbinding, causal disconnection and turnaround

In this section we analyze four relationships between cosmic times in the cyclic model expansion era: i) $t_{\rm unbound}$ (at which a bound system will become unbound due to the large dark energy force with $w < -1$); ii) $t_{\rm caus}$ (at which a previously bound system becomes casually disconnected, meaning that no light signal could exchange before the would-be Big Rip; this is how we estimate $N_{\rm cp}$); iii) t_T (time when the turnaround occurs); iv) $t_{\rm rip}$ (at which a "would-be" big rip takes place), in addition to the present time t_0.

In the BF model, there are three parameters; w (equation of state of dark energy); ρ_C (critical density which the total density in the system $\rho_{\rm tot}$ reached at $t = t_T$); f (the deflation fraction parameter related to the number of causal patches by $N_{\rm cp} = (1/f^3)$). We will analyze the model taking the value of w lying in a range,

$$-1.10000 \leq \omega \leq -1.00001\,, \tag{6}$$

and for ρ_C choosing the following range,

$$(10^3\,{\rm GeV})^4 \leq \rho_C \leq (10^{19}\,{\rm GeV})^4\,. \tag{7}$$

The choice of the range of w is motivated by the current lower bound from observations and the upper bound, by the cosmic variance uncertainty in this measurement.

Without showing derivations (see Appendices of BFM paper) we shall here refer to the resultant expressions:

$(t_{\rm rip} - t_0)$

$$t_{\rm rip} - t_0 \simeq \frac{11\,{\rm Gyr}}{|1+w|} \qquad (8)$$

$(t_{\rm rip} - t_{\rm unbound})$

$$t_{\rm rip} - t_{\rm unbound} = \alpha(w) P \qquad (9)$$

where

$$\alpha(w) = \frac{\sqrt{2|1+3w|}}{6\pi|1+w|}, \qquad (10)$$

and P denotes the period associated with the binding force which had been constraining objects into a certain bound system before $t = t_{\rm unbound}$.

$(t_{\rm rip} - t_{\rm caus})$

$$t_{\rm rip} - t_{\rm caus} = \left| \frac{1+3w}{3(1+w)} \right| \left(\frac{L}{c} \right) \quad (11)$$

where c is the speed of light and L stands for the length scale of the bound system.

$(t_{\rm rip} - t_T)$

$$t_{\rm rip} - t_T = \frac{11\,{\rm Gyr}}{|1+w|} 10^{-14.5} \eta^{-1/2} \quad (12)$$

where η is a scale factor of ρ_C defined by $\rho_C = \eta \rho_{H_2O}$ with ρ_{H_2O} being the density if water, $\rho_{H_2O} = 1{\rm g} \cdot {\rm cm}^{-3}$. Eq.(12) appeared as Eq.(4) in BF.

As a result, we find the lower bound for ρ_C,

$$\rho_C > (10^{18}\mathrm{GeV})^4, \tag{13}$$

which is obtained by imposing that the time for a presently point particle (PPP), with the size 10^{-33}m = L, satisfy $t_{\mathrm{rip}} > t_T > t^{\mathrm{PPP}}_{\mathrm{caus}} > t^{\mathrm{PPP}}_{\mathrm{unbound}}$. It should be emphasized that this result is almost independent of a choice of w in the range of interest. For a nucleon with $L \simeq 10^{-15}$m, the corresponding lower bound is

$$\rho_C > (10^9\mathrm{GeV})^4 \tag{14}$$

Various bound systems can be discussed including galaxies, the Earth-Sun system, the hydrogen atom and a nucleon. Each may be characterised by a present length scale L_0.

For the CBE condition we must insist that the smallest bound systems are disintegrated before turnaround which means that the size of a generic causal patch $L_{\rm cp}$ (to be defined below) is smaller than the size $L(t_T)$ at turnaround of the bound system whose present length scale is $L(t_0) = L_0$, namely

$$L_{\rm cp} \leq L(t_T) = L_0 \left(\frac{a(t_T)}{a(t_{\rm unbound})} \right). \qquad (15)$$

We recall that the CBE condition is mandatory because if the contracting universe contains matter it will not generally contract sufficiently but will undergo a premature bounce. Even if a causal patch contains only one very infra-red photon, this can blue-shift to an energy sufficient to create e^+e^- pairs before the bounce, again disallowing sufficient contraction for infinite cyclicity.

It was shown elsewhere (see references, BF2) that the mean number of low-energy photons per causal patch is much less than one and is essentially zero. There will always be a vanishing but strictly non-zero number of patches which fail to cycle but it can be shown in that the probability of a successful universe is equal to one; it was noted that the total number of universes has always been, and always will be constantly infinite and equal to $\aleph_0$ (Aleph-zero). $\aleph_0$ is a countable infinity, exemplified by the number of primes, of integers or of rational numbers.

To enable infinite cyclicity we must have the CBE condition, for the smallest bound systems. The smallest bound systems we know about are nucleons with $L_0 = 10^{-15}$m.

To be general, we consider PPPs (*Presently Point Particles*) meaning particles which are presently regarded as pointlike but may not be. We allow a bound state scale for PPPs to be anywhere between the present upper limit of about $(1\text{TeV})^{-1} = 10^{-18}$m and the Planck scale of 10^{-35}m. As we shall see shortly, the lower bound on $N_{\rm cp}$ is so sensitive to where L_0 is chosen within these twenty orders of magnitude that its presentation requires us to plot $\log_{10} \log_{10} N_{\rm cp}$ against the equation of state of the dark energy.

The present Hubble length $r_H(t_0)$ is given by

$$r_H(t_0) = \frac{1}{H_0} \tag{16}$$

which, at the turnaround, would naively become

$$r_H(t_T) = r_H(t_0)a(t_T) \tag{17}$$

since by definition $a(t_0) = 1$.

In the cyclic model, the size of a causal patch $L_{\rm cp}$ is instead defined by

$$L_{\rm cp} = \frac{r_H(t_T)}{N_{\rm cp}} \tag{18}$$

and therefore Eq.(15) have been calculated for different values of L_0. The results are illustrated (see references B-F-M) by plotting $\log_{10}\log_{10} N_{\rm cp}$ versus $w = -1 - \phi$.

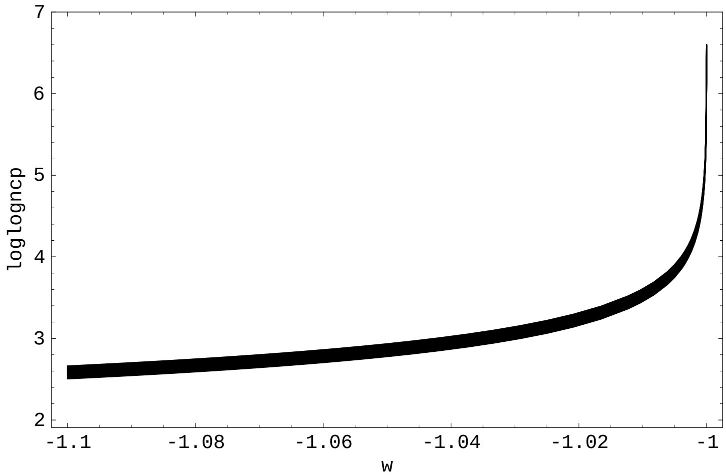

Figure 1: Plot of $log_{10}log_{10}N_{cp}$ vs. w. $(-1 > w > -1.1)$

From this we find that a measurement of w in the range anticipated for the Planck surveyor will provide a lower bound on $N_{\rm cp}$. For example $w = -1.05$ implies $N_{\rm cp} > 10^{630}$ for disintegration of nucleons and $N_{\rm cp} > 10^{1000}$ for disintegration of PPPs with bound scale at the Planck length.

Since we know the entropy of the present universe is at least $S(t_0) > 10^{102}$ one must impose

$$N_{\rm cp} > 10^{102} \tag{19}$$

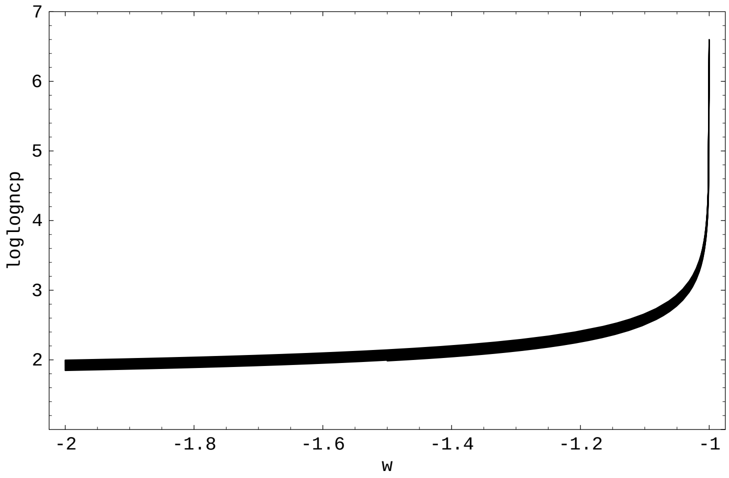

Figure 2: Plot of $log_{10}log_{10}N_{cp}$ vs. w. $(-1 > w > -2)$

and, by requiring only the dissociation of nucleons we see from Figure 2 that this implies

$$w > -2. \tag{20}$$

Of course, WMAP data already guarantee this condition but it is interesting that the cyclic model would be impossible if Eq.(20) had been violated.

3 Discussion of deflation constraints

What we have deduced is that the parameters ρ_c, w and $N_{\rm cp}$ in cyclic cosmology are already constrained by existing data. For example one requires $w > -2$ for the CBE aspect to work.

This constraint is already known to be respected in Nature but as better and more accurate cosmological data become available it will shed further light on the viability of the theory.

In particular, the accurate measurement of the equation of state $w = -1 - \phi$ is of special interest. Fortunately the Planck Surveyor is anticipated to acquire improved accuracy on w in the near future. As we have discussed, this will provide a lower bound on the number $N_{\rm cp}$ of causal patches necessary to dissociate the smallest bound systems at turnaround and hence to solve the entropy problem and, via CBE, enable the possibility of infinite cyclicity.

It is amusing that the physical conditions at the approach of deflation are so extraordinary that it is natural to ask whether the systems presently regarded as point particles may be composite because the phantom dark energy density grows to unimaginably large values and can disintegrate bound systems down to arbitrarily small scales. We have conservatively limited our attention to systems bigger than the Planck length. However, although this requirement seems dictated by considerations of quantum gravity, it is possible that the dark energy will dissociate even smaller systems if they exist.

The advantage of cyclic cosmology is that it removes the initial singularity associated with the Big Bang, about 13.7 billion years ago, and allows that time never began. The previous attempts to create a consistent infinite cyclicity were stymied between about 1934 and 2002 primarily because of the entropy problem and the second law of thermodynamics. The discovery of the accelerated expansion rate of the universe and the concomitant necessity of dark energy has permitted more optimism that the cyclic cosmology is, after all, on the right track.

4. Multifluid models.

Introduction. In the history of physics, it is impossible to exaggerate the fecundity of cross-fertilization between sub-disciplines. In theoretical physics, high-energy physics and cosmology have been repeatedly informed by condensed matter theory. One outstanding example is the idea of spontaneous symmetry breaking introduced by Nambu into particle theory inspired by study of the BCS theory of superconductivity. Few ideas have had more impact on our understanding of both high energy physics and cosmology. Here we take our inspiration for study of cyclic cosmology from the Landau two-fluid model of superfluidity, itself also applicable to superconductivity In this model, the energy density of superfluid liquid helium is expressed as a sum of two terms

$$\rho = \rho_n + \rho_s, \tag{21}$$

At the lambda temperature, only normal fluid is present. As temperature is decreased, more and more normal fluid is converted to superfluid until at absolute zero the liquid helium consists only of superfluid. Normal fluids behave like a Newtonian fluid with viscosity and entropy. The superfluid components have no viscosity, no entropy and do not carry any heat. The normal fluid and superfluid satisfy different equations of motion.

Similarly the different cosmological fluids will satisfy different equations of state. One dark energy superfluid (density ρ_1, equation of state w_1) is the one currently measurable. Of the others, ρ_2 (w_2) becomes important very close to the turnaround and ρ_3 (w_3) is important very close to the bounce.

Thus, in this analogy it is natural to associate superfluids with dark energy because it has no entropy. The normal fluids will correspond with the matter and radiation. Thus there are $n_n = 2$ normal fluids for cosmology, and we shall show that, for cyclic cosmology, we need $n_s = 3$ dark energy superfluids, making overall a five-fluid model.

Friedman equation. We write the Friedman equation as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left[\frac{(\rho_m)_0}{a(t)^3} + \frac{(\rho_r)_0}{a(t)^4} + \Sigma_{i=1}^{i=n_s}(\rho_i)_0 a(t)^{3\phi_i}\right], \tag{22}$$

where the equations of state for the normal fluids are $w_m = 0$ and $w_r = +1/3$ for matter and radiation respectively. The number of superfluids representing dark energy is n_s and the first component with "$i = 1$" will be the presently observed dark energy with $w_1 = -1 - \phi_1$ and $\phi_1 > 0$.

In Eq.(22), the n_s terms in the summation are analogs of the superfluid term in the Landau theory in that they carry zero entropy. To implement cyclic cosmology, it will be necessary that some (actually those with $i \geq 2$) superfluid energy densities (ρ_i) be negative. Let us first consider an $n_s = 2$ four-fluid model with $\phi_2 > \phi_1$, that is $w_2 = -1 - \phi_2 < w_1$. For turnaround from expansion to contraction at time $t = t_T$, and using $a(t_0) = 1$, we see that

$$(\rho_2)_0 = -(\rho_1)_0 (a(t_T))^{-3(\phi_2 - \phi_1)}, \qquad (23)$$

so that $(\rho_2)_0 < 0$ and, because $a(t_T) \gg 1$ and if $(\phi_2 - \phi_1)$ is sufficiently non-zero, it follows that $|(\rho_2)_0| \ll |(\rho_1)_0|$ and hence that the second ("$i = 2$" in Eq.(22)) dark energy superfluid is unobservably small at the present time $t = t_0$.

Can ρ_2 play the role of causing both the turnaround and the bounce in cyclic cosmology? The answer is negative as is now explained by a no-go theorem.

No-Go theorem. Let us prove a no-go theorem that $n_s = 2$ cannot produce an acceptable bounce at $t = t_B$ where contraction turns into expansion.

At the bounce when $t = t_B$, we would need

$$(\rho_2)_0 = -(\rho_r)_0(a(t_B))^{-(4+3\phi_2)} \qquad (24)$$

and, because $a(t_B) \ll 1$, this would require $\phi_2 < -4/3$, or $w_2 > +1/3$, clearly inconsistent with Eq.(23) which requires $\phi_2 > 0$ and $w_2 > +1/3$. This incompatibility of Eqs.(23) and (24) provide a No-Go theorem for any four-fluid ($n_n = 2$ and $n_s = 2$) model.

This is not surprising when we consider the brane-world Friedman equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left[\rho_\Lambda a(t)^{3\phi_\Lambda} + \frac{(\rho_m)_0}{a(t)^3} + \frac{(\rho_r)_0}{a(t)^4} - \frac{\rho_{total}^2}{\rho_c}\right] \tag{25}$$

where $\rho_{total} = (\rho_\Lambda + \rho_m + \rho_r)$. Thus, the final term on the right-hand-side of Eq.(25) has quite a different time dependence for $t \rightarrow t_T$ and $t \rightarrow t_B$. This underlies the No-Go theorem and mandates usage of the following five-fluid model.

Five-Fluid Model. We consider a five-fluid model with $n_n = 2$ and $n_s = 3$. The densities ρ_2 and ρ_3 are negative and the Friedman equation is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left[\frac{(\rho_m)_0}{a(t)^3} + \frac{(\rho_r)_0}{a(t)^4} + (\rho_1)_0 a(t)^{3\phi_1} - (\rho_2)_0 a(t)^{3\phi_2} - (\rho_3)_0 a(t)^{3\phi_3}\right], \quad (26)$$

and in this case we can arrange that at the turnaround

$$(\rho_2)_0 = -(\rho_1)_0 (a(t_T))^{-3(\phi_2 - \phi_1)}, \quad (27)$$

and at the bounce

$$(\rho_3)_0 = -(\rho_r)_0 (a(t_B))^{-(4+3\phi_3)}. \quad (28)$$

At the turnaround, as in the four-fluid model, we require (I) $(\phi_2 - \phi_1) > 0$ while at the bounce there is the new condition (II) $(\phi_3 + 4/3) < 0$.

Taking the two inequalities (I) and (II) together will ensure the turnaround and bounce occur and that $|(\rho_1)_0| > |(\rho_2)_0|$ and $|(\rho_r)_0| > |(\rho_3)_0|$, as necessary to preserve the successful description of the present universe. For special values of the equations of state w_2 and w_3 the five-fluid model becomes indistinguishable form the BF model, as follows.

Brane-world as special case of five-fluid model.

In the special case, consistent with the above inequalities, where $\phi_2 = 2\phi_1$ and $\phi_3 = -8/3$ the Friedman equation of Eq.(26) become indistinguishable for that of the brane -world model in Eq.(25). Although Eq.(27) is not *identical* to Eq.(25) there is no observable difference because at present the components (ρ_2) and (ρ_3) are negligible; at turnaround (ρ_2) duplicates the final term in Eq.(25) and at the bounce (ρ_3) plays precisely the same role. However, the five-fluid model is more general if we incorporate arbitrary values of ϕ_2 and ϕ_3 consistent with the above inequalities (I) and (II).

Distinguishing models. Let us consider a five-fluid model which is very disparate from the BF model. In such a case, accurate observations can distinguish the cyclic models. Of course, we do not yet know ϕ_1 precisely for the dark energy but let us suppose that $\phi_1 = 0.05$, consistent with present WMAP data. We need ϕ_2 to be bigger so let us assume $\phi_2 = 0.06$. In this case Eq.(27) requires that

$$(\rho_2)_0 = (\rho_1)_0 a(t_T)^{0.03}. \tag{29}$$

Just to complete an example, let us now assume $a(t_T) = 10^{33.33}$ whereupon Eq.(29) dictates that $(\rho_1)_0 = 10(\rho_2)_0$ and so the fit to dark energy should use a scale dependence

$$(\rho_{DE})_0 \left[a(t)^{0.15} - 0.1 a(t)^{0.18} \right]. \tag{30}$$

More generally the multifluid model suggests fitting to

$$(\rho_{DE})_0 \left[a(t)^{3\phi_1} - \eta a(t)^{3\phi_2} \right], \qquad (31)$$

where $\phi_2 > \phi_1$ and η is an additional parameter related, in general, to the turnaround scale. More accurate and complete data on dark energy will enable distinction between fitting with a two-term formula like Eq.(31) and fitting with only the first term.

Discussion of multifluids Inspired by previous successes, we have here attempted to emulate the two-fluid model of superfluidity in a five-fluid model of cyclic cosmology. The analogy is heightened by the zero entropy for the dark energy (superfluid) components, The multifluid models have certain advantages, including that they do not necessitate derivations from brane worlds in higher dimensionality. The analogies to the two-fluid model of superfluidity may be posited directly in four dimensions. In a certain limit, the five fluid model with two normal fluids, matter and radiation, and three superfluids for dark energy become indistinguishable from the brane-world BF model.

However, when the five-fluid model become very disparate from the brane-world model it will be possible to distinguish them by accurate observations of dark energy as we have discussed. It is therefore worth studying, as more and better observational data become available, whether fits to a two-term expression as in Eq.(31) are more successful for dark energy than those using only the first term thereof.

Summary of Talk

We have outlined here a cyclic cosmology resting on phantom dark energy where these objections are ameliorated: the classical density and temperature never become infinite and future expansion is truncated. Also, our proposal of deflation naturally leads to a multiverse picture, somewhat reminiscent of that predicted in eternal inflation, though here the proliferation of universes must be infinite and originates at the opposite end of a cyclic cosmology, at its maximum rather than at its minimum size.

We have shown that the entropy of the contracting universe is not only very small but actually vanishing so the entropy problem is solved more completely than originally envisioned. It offers an explanation of why the entropy pre inflation is zero.

We have seen how measurement of the dark energy equation of state can constrain the parameters (*e.g.* N_{cp}).

There is an alternative *multifluid* approach in which the dark energy is represented by superfluid components.

We present this cyclic universe proposal mainly in the hope that it will stimulate an improved and more consistent formulation by others.

Thank you for your attention