Final Draft, December 6, 2004

Dynamical Evolution of Planetesimals: The Key to Terrestrial Planet Formation

Padraic S. Finnerty

1. Introduction

It is widely believed that the formation of Earth-type planets proceeded via agglomeration of large numbers of planetesimals: asteroid/comet-like rocky or icy bodies. The theory of this process was pioneered by a famous scientist, Safranov, who was first to point out the importance of the evolution of the dynamical properties of the planetesimal disk for the evolution of its mass distribution. Significant progress has been made recently in understanding the dynamical evolution of homogeneous planetesimal disks - those in which gradients of surface density or dynamical properties (such as the dispersions of eccentricity and inclination of various planetesimal populations) are absent. The assumption of homogeneity should suffice during the initial stages of planetesimal growth, when there are no massive bodies in the disk. However, as coagulation proceeds and planetary embryos - precursors of terrestrial planets - emerge, this assumption runs into problems. Rafikov (2003a) demonstrated this using N-body simulations and then confirmed it using orbit integrations that under a variety of conditions massive protoplanetary embryos would tend to repel planetesimal orbits, clearing out an annular gap in the disk around them. This process introduces a new spatial dimension into the problem and makes it much more difficult to treat.

N-body simulations are not good tools with which to study the details of planetesimal dynamics. The primary reason is that they become too time-consuming when one needs to follow the spatial and dynamical properties of a many-bodied system for many orbital periods. Moreover, the number of planetesimals that they can handle is not very large ($< 10^4$), which precludes consideration of realistic planetsimal disks, containing a huge number of bodies with masses spanning an enormous range - from 1m rocks to 100km planetesimals. Finally, one would need to perform a large number of such simulations to explore the whole space of physical parameters relevant for protoplanetary disk evolution. Similar problems, although less severe, plague performance of methods based on the direct integration of planetesimal orbits using some simplifying assumptions.

Planetesimals interact with eachother and also interact with embryos. The goal of this paper is to communicate to the reader the dynamics of both situations. In §2, I will talk about the basics of the N-body problem. In §3, I will discuss planetesimal - planetesimal interactions detailing the specific velocity regimes. In §4, I will describe in a similar fashion to §3, the dynamics of embryo-planetesimal interactions. In §5, I will make my concluding remarks.

2. The *N*-Body Problem

2.1. Code Used by Tanga et al. (2004)

The N-body heirarchial tree code `pkdgrav`, which is able to treat physical collisions between particles in a consistent way, simulating their coalescence into larger bodies was used by Tanga et al. (2004). Given the large numbers of interactions involved, all the runs presented in his paper required the use of high-performance parallel computers. In the following paragraph, I will address the numerical method employed by Tanga et al. (2004), together with boundary conditions.

The `pkdgrav` parallel N-body code allows in principle for treatment of a large number of (N) particles, and numerical experiments have been successfully performed with N ranging from 10^5 to 10^6. The main advantage provided by this code is the possibility to assign a physical size to particles (considered to be spherical) and to efficiently detect mutual collisions. The collisional outcome can be treated in different ways; by either imposing perfectly elastic bouncing between the colliders, or by making the colliders merge into a single body of mass equal to the sum of their masses, with the same internal density. Considering the colliding bodies as an isolated system, the first case implies energy conservation in the reference frame referred to the center of mass. In the second case, dissipation occurs. All simulations done by Tanga et al. (2004) have been performed in the assumption that a dense ring of planetesimals at a distance a from the central body can be divided into local boxes (or "patches") of size L, with $L \ll a$. In a coordinate frame centered on the box, with the $x - axis$ pointing away from the Sun, the $z - axis$ perpendicular to the orbital plane, and the $y - axis$ being tangential to the direction of motion, the equation of motion can be linearized:

$$\ddot{x} = F_x + 3\Omega^2 x + 2\Omega\dot{y}, \tag{1}$$

$$\ddot{y} = F_y - 2\Omega\dot{x}, \tag{2}$$

$$\ddot{x} = F_z - \Omega^2 z, \tag{3}$$

in which Ω is the angular frequency at distance a from the Sun, and F represents the gravitational force (per unit mass) due to the other particles.

The box in which the simulation done by Tanga et al. (2004) was performed is considered to be periodic in the (x, y) plane. In order to reduce the boundary effects, the gravitational contribution coming from three orders of ghost patches, each containing a copy of the particles in the main domain, was considered. The ghost patches were nested around the central one; those having a distance from the Sun different from a have a velocity tangential to the orbit consistent with the Keplerian shear.

It must be noted that the collective particle behavior can be associated to structures (such as clusters, waves, ect.) whose typical scale length must be much smaller than L to avoid boundary effects. However, given the limited resolution availiable in terms of N, it may happen that adequate sampling of all interesting scales present in the disk is not possible. In other words, while the largest scales are always limited by the size of L, the smallest possible scales that can be represented in the simulation have a size corresponding to several times the average particle size.

The spatial scales that are of interest can be estimated from the linear theory of gravitational instability in a rotating disk of planetesimals. The dispersion relation that is obtained, in the case of a thin disk locally rotating at a frequency Ω, for a slightly overdense domain of size λ, can be written as:

$$F(\lambda) = 2\pi^2 c^2 - 4\pi^2 F \Sigma \lambda + \lambda^2 \Omega^2 < 0, \tag{4}$$

in which c represents the velocity dispersion and Σ the local surface density of solids. An unstable solution exists when $F(\lambda) < 0$, i.e. for $c < c_{crit} = \frac{\pi G \Sigma}{\Omega}$. The widest range of unstable wavelengths is obtained when $c = 0$; in this case, for $\lambda < 2\lambda_{crit}$ instability is possible. λ_{crit} is the value of the most unstable wavelength (minimum of the dispersion relation) and is given by:

$$\lambda_{crit} = \frac{2\pi^2 G \Sigma}{\Omega^2}. \tag{5}$$

The overdense regions grow by heirarchial merging into larger and larger structures. However, before their sizes become comparable to L, the increase in velocity dispersion stabilizes the dust layer, so that these structures dissipate. Some examples of output for the `pkdgrav` package can be seen in Figure 1 and Figure 2.

3. Planetesimal - Planetesimal Dynamics

This process can proceed in two distinct regimes, depending on the amplitude of the planetesimals' random motion. The gravitational attraction between two planetesimals with masses m_1 and m_2 becomes stronger than the tidal field of a central star of mass M_c when their mutual separation is less than their Hill(or tidal, or Roche) radius r_H, defined as:

$$r_H = a_0(\frac{m_1 + m_2}{M_c})^{1/3}, \tag{6}$$

where a_0 is the distance from the central star. When the random velocities of the epicyclic motion of interacting planetesimals are smaller than $\Omega r_H[\Omega = (\frac{GM_c}{a_0^3})^{1/2}$ is the disk orbital frequency at $a_0]$, their relative approach velocities are small, and close interactions can lead to a considerable change of the orbital elements of the planetesimals. This velocity regime is called *shear dominated* (or "cold"). It should be contrasted with the other extreme - the so called *dispersion-dominated* ("hot") regime, which occurs when planetesimal velocity dispersions are larger than $\sim \Omega r_H$. In this latter case, scattering is typically weak, which often allows analytical treatment of this velocity regime.

The development of planteltesimal disk inhomogeneities driven by a protoplanetary embryo was explored by Rafikov (2001) by assuming that shear-dominated scattering of planetesimals prevails. It was also assumed in that study that the dynamical properties of planetesimals do not evolve as a result of scattering and that the disk always stays dynamically cold. Planetesimal-planetesimal interactions play the role of effective viscosity in the disk and tend to homogenize it and close up any gap. Nevertheless, this study demonstrated that gap formation is the natural outcome of the embryo-planetesimal interaction when the embryo is massive enough. These interactions were local in character, because in the shear-dominated case, only planetesimals on orbits separated by no more than several times r_H (corresponding to their encounter distance) were able to approach each other closely.

Rafikov (2003a) states that it is however more likely that planetesimal-planetesimal gravitational scattering in realistic protoplanetary disks proceeds in the dispersion-dominated (rather than the shear dominated) regime, at least in the later stages of disk evolution. In this case, the evolution of planetesimal random motion can strongly affect the growth rate of protoplanetary embryos. It is also tightly coupled to the evolution of the spatial distribution of planetesimals, because any change in the energy of epicyclic motion comes at the expense of the orbital energy of planetesimals.

The treatment of the dispersion-dominated case is complicated by the fact that planetesimals in this regime can explore different regions of the disk in the course of their epicyclic motion. This makes disk evolution a nonlocal process. On the other hand, as has been previously stated by many of the references listed, there are natural simplifications that are valid in the dispersion-dominated regime. These include the two-body scattering approximation (relative velocities are high), Fokker-Planck type expansions (scattering is weak), and so on.

4. Embryo - Planetesimal Dynamics

An "embryo" is a body of mass M_e much larger than the masses of individual planetesimals, m (Rafikov 2003c). There are several reasons for studying this important problem separately from the mutual gravitational scattering of planetesimals.

First, planetesimal-planetesimal encounters in realistic disks usually occur in the dispersion-dominated regime as stated in §3, which applies when the relative approach velocity of the two particles is greater than the differential shear in the disk across the Hill radius.

However, in the same protoplanetary disk, gravitational interaction between the embryo and planetesimals could be in the opposite velocity regime - shear dominated - when the planetesimal random motion is small compared with the shear across the Hill radius, simply because the embryo mass, and therefore the Hill radius, is much larger. Indeed, in the case of embryo-planetesimal interactions the Hill radius $R_H = a_e(\frac{M_e}{M_c})^{1/3} \gg r_H$ (here a_e is the semimajor axis of the embryo) because $M_e \gg m_i$. Thus, reduced (normalized in the Hill coordinates) values of random velocities in the embryo-planetesimal case are smaller by a factor $[(m_1 + m_2)/M_e]^{1/3} \ll 1$ than those corresponding to the planetesimal-planetesimal interactions.

Second, embryo-planetesimal interactions are complicated by the presence of a special type of orbit in a three-body problem - the so called horseshoe (or librating) orbits. Planetesimals on these orbits do not perform the usual circulating motion that is characteristic of passing orbits (the most important case for planetesimal-planetesimal scattering), but a librating one. This horseshoe motion can only occur when the difference in semimajor axes of the interacting bodies is smaller than their Hill radius (Parisi et al. 1999; Rafikov 2003b). For planetesimal-planetesimal interactions, r_H is negligable compared with the scale of surface density variations or the radial epicyclic excursion of an individual planetesimal. Thus, horseshoe orbits are unimportant in this case. However, the Hill radius of the embryo-planetesimal interaction R_H can be comparable to the length scale of the disk inhomogeneities caused by the embryo. Thus, the phenomenon of horseshoe motion can be crucial for the planetesimal dynamcis near the embryo.

Third, planetesimal-planetesimal scattering is described in terms of the disk properties *averaged* over some region of the disk, which diminishes the importance of the details of spatial distributions of disk properties. On the contrary, in the case of embryo-planetesimal interaction, the *spatial behavior* of various quantities characterizing the state of the disk is of primary interest.

Numerical orbit integrations and N-body simulations provide an interesting route to studying embryo-planetesimal interactions. The drawback is their intrinsically low speed as stated in §2. However, since the physics incorporated in them is on a very basic level with minimal additional assumptions, they can provide us with robust predictions. To use this advantage of numerical methods and to avoid their handicaps, a self-consistent *analytical* description of the embryo-planetesimal interaction in different velocity regimes can be obtained. To check this description and to verify the validity of the simplifying assumptions utilized in this development, Rafikov (2004) used numerical orbit integrations performed for several sets of typical planetesimal disk parameters.

The condition on the embryo's mass, $M_e \gg m$, has important dynamical implications. In many applications, it justifies neglect of the embryo's recoil resulting from planetesimal scattering. Also, dynamical friction between the embryo and planetesimals will tend to produce random-energy equipartition, which means that the embryo's eccentricity and inclination are zero. Rafikov (2004) also considered the embryo to be isolated from the gravitational effects of other massive bodies that may be growing nearby.

5. Discussion and Summary

The study of the dispersion-dominated regime relies on the methods of kinetic theory, and it uses many of the results obtained in (Rafikov 2003b). However, the present treatment used by Rafikov and others is more refined, since the description of embryo-planetesimal scattering requires clarifying many details that were not important for the planetesimal-planetesimal interactions. In particular, one needs to study not only passing but also horseshoe orbits of planetesimals to determine the spatial distribution of the disk properties. To do this, Rafikov (2003b) proposed a condition that separates the horseshoe and passing orbits and checked its viability using numerical orbit integrations. Angular momentum exchange between the embryo and planetesimal long before and after their closest approach turns out to be important for scattering on passing orbits near the horseshoe boundary. It was illustrated by Rafikov (2003a,b) by comparing the analytical scattering probability function with that obtained from numerical integrations. A simple method to account for this affect in their Fokker-Planck approach is proposed.

Taken together, all these refinements provide rather good agreement with the results of the numerical orbit integrations. Thus, it is hopeful that Rafikov has grasped the most important features of the embryo-planetesimal interaction with the help of Rafikov (2003b)'s theoretical approach.

This does not mean that the embryo-planetesimal dynamics are fully known by any means. He only focused on the most important, dominant effects, and there is certainly room for additional refinements, which would further improve the agreement with numerical results. On a somewhat deeper level, one could try to come up with a more sophisticated treatment of the separation of horseshoe and passing orbits (instead of the complete spatial separation of these two types of orbits assumed by Rafikov (2003a,b,c, 2004)). His purely deterministic treatment of the shear-dominated regime can also be improved.

REFERENCES

Furuya, I., Daisaka, H., & Nakagawa, Y. 2002, 8th Asian-Pacific Regional Meeting, Volume II, 41

Leinhardt, Z. M. & Richardson, D. C. 2002, Icarus, 159, 306

Parisi, M. G. & Brunini, A. 1999, Planet. Space Sci., 47, 607

Rafikov, R. 2001, Bulletin of the American Astronomical Society, 33, 1352

Rafikov, R. R. 2003, ApJ, 125, 906

Rafikov, R. R. 2003, ApJ, 125, 922

Rafikov, R. R. 2003, ApJ, 126, 2529

Rafikov, R. R. 2004, ApJ, 128, 1348

Rice, W. K. M., Lodato, G., Pringle, J. E., Armitage, P. J., & Bonnell, I. A. 2004, MNRAS, 452

Tanga, P., Weidenschilling, S. J., Michel, P., & Richardson, D. C. 2004, A&A, 427, 1105

Toomre, A. 1964, ApJ, 139, 1217

Wurm, G., Krauss, O., & Paraskov, G. 2003, Meteoritics & Planetary Science, vol. 38

This preprint was prepared with the AAS LaTeX macros v5.2.

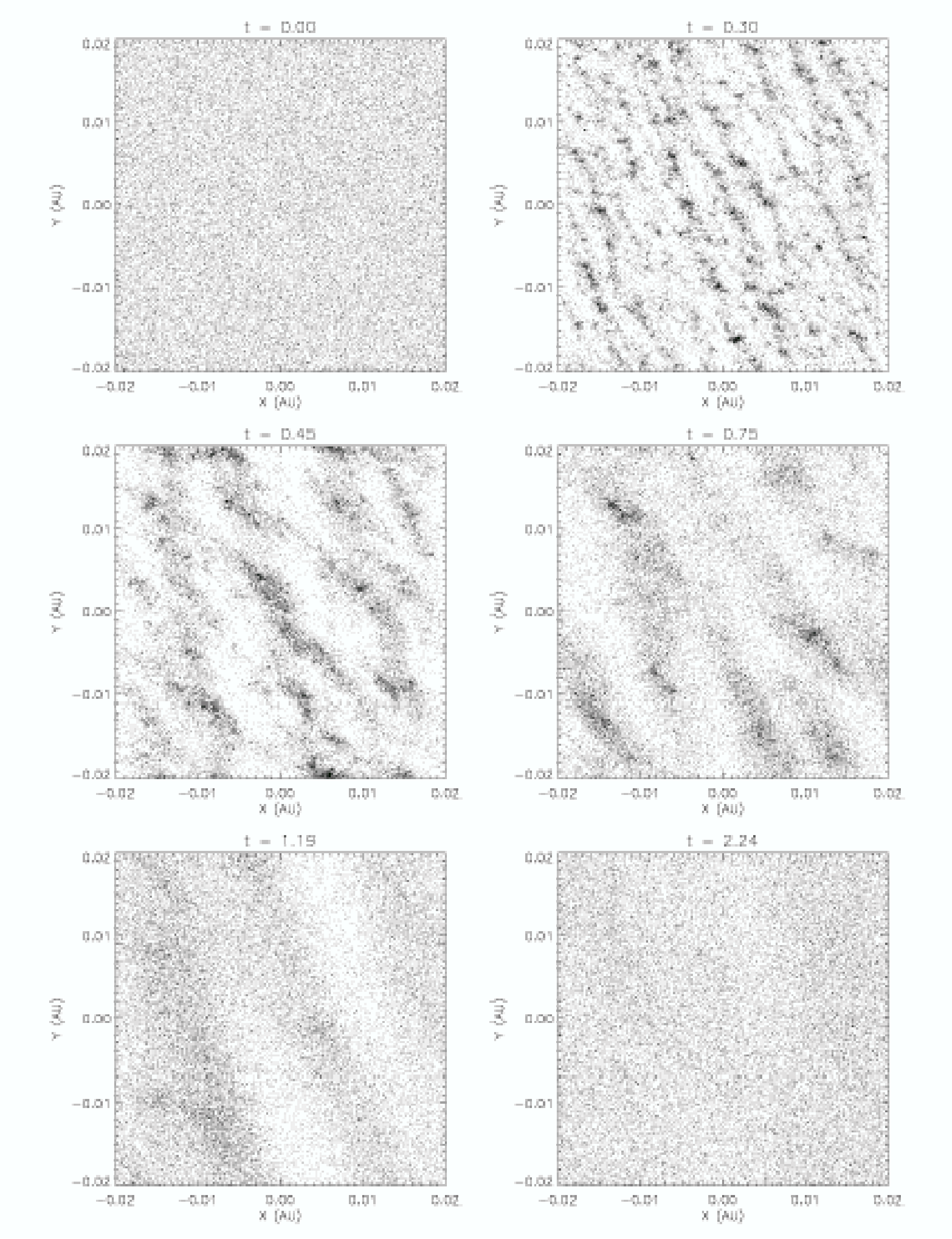

Fig. 1.— Particles positions on the (x,y) plane in the simulations done by the pkdgrav used by Tanga et al. (2004). The first panel represents the initial conditions. Time is expressed in orbital periods (at about 30 AU, about 164 years).

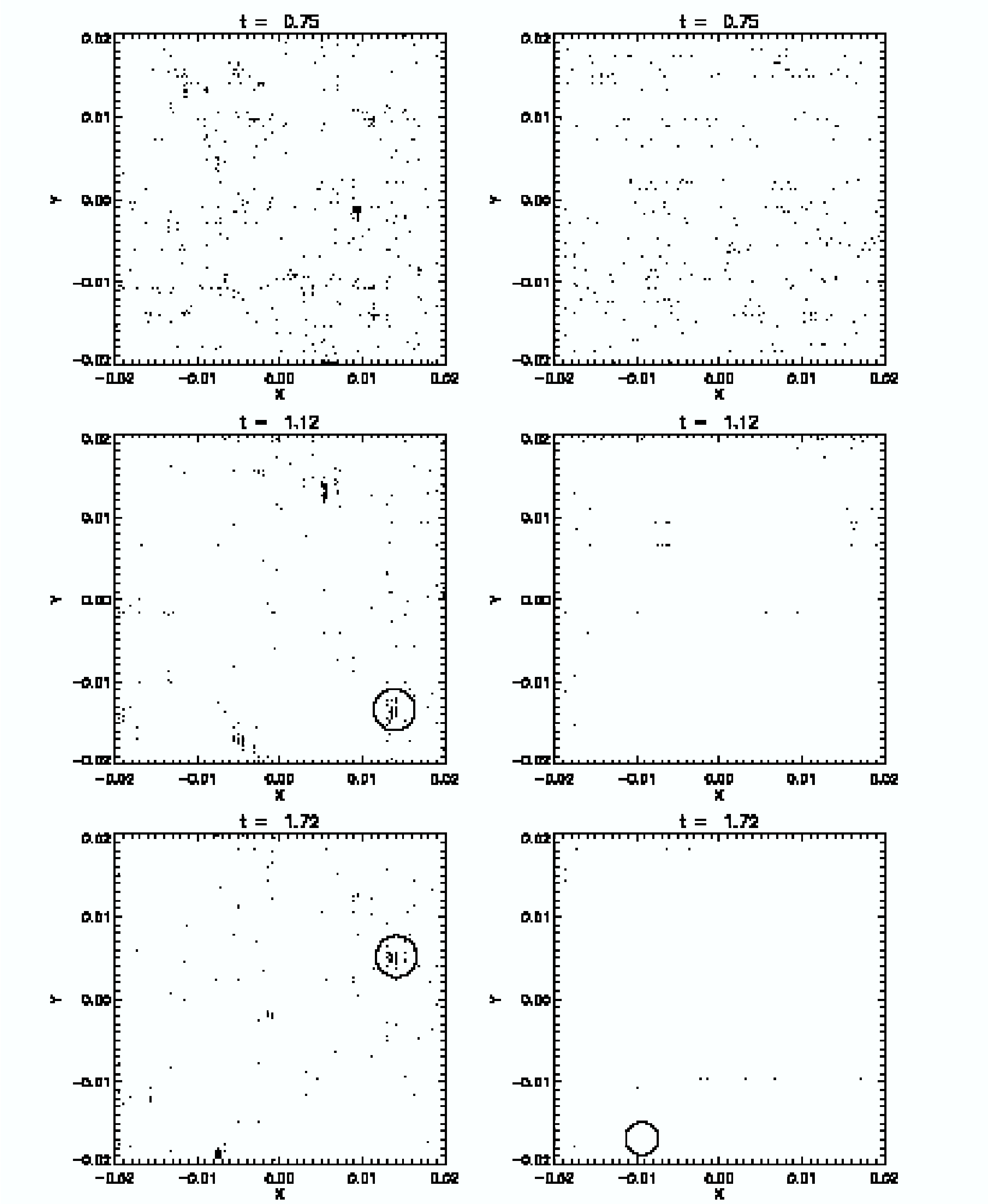

Fig. 2.— Results from another simulation done by Tanga et al. (2004). Large cluster merging is observed in the first case, while in the second one only a small spherical cluster survives at the end of the simulation. Circles in the left column snapshots represent the area used for computing detailed statistics.