

WIMP-nucleon cross sections and the prospects for detection

Padraic Finnerty^{1,2}

¹*The University of North Carolina at Chapel Hill, Chapel Hill, North Carolina 27599-3255, USA*

²*Triangle Universities Nuclear Laboratory, Durham, North Carolina, 27708-0308, USA*

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Theoretical calculations of neutralino-nucleon cross sections are of great interest to collaborations who wish to directly detect dark matter. These cross sections are generally computed with standard one- or two-parameter model-dependent nuclear form factors. In this paper we present calculations of nuclear form factors for the scalar (spin-independent) interaction using uniform and Helm charge distributions which lead to cross sections of $\sigma_\chi \simeq 10^{-43} - 10^{-46}$ cm² for $M_\chi \simeq 10$ GeV – 10 TeV. We then discuss the prospects for detection by next generation experiments.

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The discovery of dark matter is of fundamental importance to cosmology, astrophysics, and elementary particle physics. A broad range of observations from the rotation speed of galactic spiral arms to the gravitational lensing of superclusters tell us that 80-90% of the matter in our universe is in some new form, different from ordinary particles, that does not emit or absorb electromagnetic radiation. Cosmological observations, especially the Wilkinson Microwave Anisotropy Probe (WMAP) of the cosmic microwave background (CMB), have provided spectacular confirmation of the astrophysical evidence. The resulting picture, the so called *Standard Cosmology* finds that a quarter of the energy density of the universe is dark matter and most of the remainder dark energy. A basic foundation of the model, Big Bang Nucleosynthesis (BBN), tells us that at most 5% is made up of ordinary matter, or baryons. The term *dark* illustrates our ignorance on the subject. Modern physics is challenged by basic questions, for example: What is the dark matter made of? How much is there, and is it the same on all scales?

One possible and very likely solution to this dark matter problem are WIMPs (Weakly Interacting Massive Particles). A leading candidate for galactic WIMPs is the lightest neutralino (χ) - a heavy neutral supersymmetric fermion. The lightest neutralino is a linear combination of four mass eigenstates - two Higgsinos, Wino, and Bino. Neutralinos interact with matter and therefore they may be directly detected in the laboratory as they elastically recoil from target nuclei, depositing enough energy to give typical nuclear recoils of order ~ 10 keV. Therefore, it is imperative that we are able to calculate the cross section of this WIMP-nucleon interaction. This interaction can be split up into two separate terms, one for the axial (spin dependent) interaction and one for the scalar (spin independent) interaction. We present the calculation of the WIMP-nucleon cross section for the spin independent interaction. It should be noted that the spin independent interaction dominates for target nuclei with $A > 30$ [1].

In the elastic scattering of a neutralino of mass M_χ off a target of mass M_A , the maximum momentum transfer

is $q_{max} = 2M_R v$, where M_R is the reduced mass and v is the velocity of the neutralino. The velocity of the scattering neutralinos is taken to be $v \simeq 220$ km s⁻¹ $\simeq 10^{-3}c$ (non-relativistic). The justification for this is as follows, the dark matter halo at our location of the Milky Way is rotating at the same speed as our solar system about the galactic center. If $q_{max}R \ll 1$ then we need not concern ourselves with the structure of the target nucleus. However, if $q_{max}R \sim 1$, then the distribution of the nucleons becomes important. For heavy neutralinos, the maximum q is not necessarily small for $A > 30$, therefore we need to be concerned with the associated form factors which detail the nucleon distribution [1].

Assuming that the nucleus is a collection of nucleons interacting via static potentials, one can immediately write down the cross section for the spin independent WIMP-nucleon interaction in the impulse approximation [1]:

$$\frac{d\sigma_s}{dq^2}(q) = \frac{8G_F}{v^2} S(0) F^2(q) \quad (1)$$

where G_F is the Fermi constant, v is the neutralino velocity, and $S(0)$ is the *structure function* at zero momentum transfer for the target nucleus. The structure function is given by:

$$S(0) = \frac{1}{4\pi} c_0^2 A^2 \quad (2)$$

where c_0 is the isoscalar coefficient which couples the WIMPs to the nucleons; c_0 is the sum of the proton and neutron isoscalar coefficients, $c_0 = c_p + c_n$. Though c_0 is small, the scalar interaction is *constructively* coherent and the cross section for scattering from nuclei acquires a factor of A^2 , where A is the number of nucleons [2], as can be seen in Equation (2). Now we have the basic setup of the problem at hand and all that is left to do is calculate the appropriate nuclear form factors.

The nuclear form factor is the Fourier transform of a spherically symmetric ground state *mass* density, or more precisely

$$F(q) = \frac{1}{M_A} \int \rho_{mass}(r) e^{-i\mathbf{q}\cdot\mathbf{r}} d^3r \quad (3)$$

Assuming that the mass density is proportional to the charge density,

$$\rho_{mass}(r) = \frac{M}{Ze} \rho_{charge}(r) \quad (4)$$

(which is often a good assumption), we can then say that the nuclear form factor for mass is equal to the nuclear form factor for charge. Accurate data on charge distributions from elastic electron scattering is well known, however, mass distributions are not as well known. We now need some models for the charge distributions. The simplest example of a model for the charge density of a nucleus is the uniform model in which the charge density is constant up to some cut-off radius, $R = 1.2A^{1/3}$ fm, and zero elsewhere

$$\rho_U(r) = \frac{3Ze}{4\pi R^3}, \quad r < R \quad (5)$$

where the charge density is normalized so that the total charge in the nucleus is equal to Ze . The corresponding form factor for this charge distribution is

$$F_U(q) = \frac{3}{qR} j_1(qR) \quad (6)$$

where $j_1(qR)$ is a spherical Bessel function of the first kind,

$$j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x} \quad (7)$$

It is clear that this charge distribution is far from physically accurate due to the discontinuity in the charge distribution at the cut-off radius. Therefore we must come up with a more accurate description of the charge distribution. The Woods-Saxon distribution immediately comes to mind of course, however, the Fourier transform of a Woods-Saxon distribution cannot be computed in closed form. As a result we resort to a distribution that looks similar to a Woods-Saxon, but is fully analytic, namely the Helm distribution [3]. The Helm distribution solves the problem of the discontinuity by convoluting the uniform charge distribution with a Gaussian *surface smearing* density. The Helm distribution is given by

$$\rho_H(\mathbf{r}) = \int \rho_U(\mathbf{r}') \rho_G(\mathbf{r} - \mathbf{r}') d^3\mathbf{r}' \quad (8)$$

where $\rho_G(\mathbf{r})$ is taken to be

$$\rho_G(\mathbf{r}) = \frac{1}{(2\pi g^2)^{3/2}} e^{-r^2/2g^2} \quad (9)$$

where g is a parameter which is related to the radius of the Gaussian. The power of the convolution theorem allows us to compute the form factor with ease

$$F_H(q) = \frac{3}{qR} j_1(qR) e^{-r^2/2g^2} \quad (10)$$

There exist other models for the charge distribution, but we will end the discussion of form factors with these two distributions. Equations (6) and (10) are the calculated form factors for each distribution. We can then input these form factors into Equation (1) to obtain a cross section that only depends on the isoscalar coefficients, c_0 . We have derived an expression for the cross section as a function of momentum transfer for the scalar WIMP-nucleon interaction which has only one free parameter, c_0 , which depends on the particle physics of the interaction. In the Minimally Supersymmetric Model (MSSM), assuming $M_\chi \simeq 60$ GeV/ c^2 , $q \sim 0.3$, $R \simeq 6.1$ fm (Xenon), we calculate that the total cross section for the spin-independent interaction is given by

$$\sigma_\chi \simeq 10^{-43} \text{ cm}^2 \quad (11)$$

It can also be shown that the cross section can be as small as $\sigma_\chi \simeq 10^{-46} \text{ cm}^2$. As was shown, the cross section for the elastic scattering through the scalar term is fairly straightforward, and these simple methods can be applied in all nuclei.

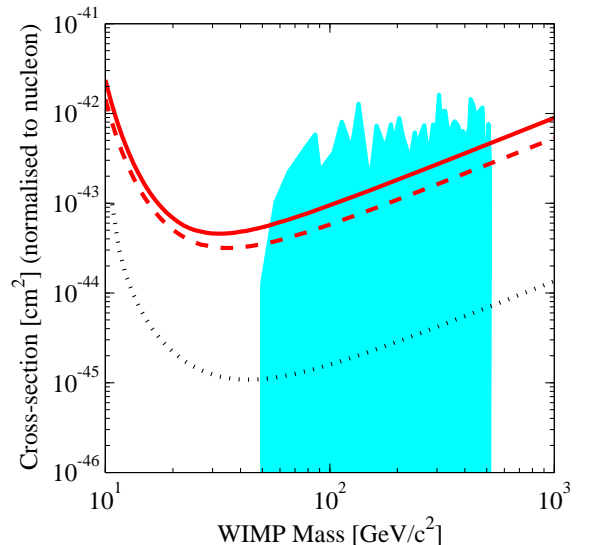


FIG. 1: The solid red line represents recent results from the XENON10 collaboration. The cyan shading indicates the phase space in which the MSSM spin independent cross section is permitted [4]. The red dashed line indicates the projected BG subtracted results from XENON10. The black dashed line indicates the projected sensitivity for the XENON100 experiment.

The detection of dark matter is an experimental challenge that requires the development of sophisticated detectors, suppression of radioactive contamination, and siting in deep underground laboratories such as Gran Sasso or DUSEL to shield from cosmic-ray induced backgrounds. The fundamental challenge of the direct detection of WIMPs is based on elastic scattering between WIMPs in the halo and atomic nuclei in a terrestrial

detector as stated above. The energy range of WIMP-nuclear recoils is of order 10 keV, an energy range in which the rate from electromagnetic backgrounds are dominant by many orders of magnitude. It is therefore essential that WIMP detectors combine low-radioactivity materials and environments with background rejection of electron recoil events to keep spurious signals at bay. The dark matter community has developed a broad range of techniques to address these challenges. At present the experimental upper limit on the WIMP-nucleon cross section is below 10^{-43} cm², which is well into the region of parameter space where SUSY particles could account for the dark matter, as illustrated in Figure 1.

Many of the promising ideas for the direct detection of WIMP dark matter have been developed using techniques in experimental nuclear physics and include a strong synergy with other parts of the nuclear physics community such as the search for neutrinoless double beta decay ($0\nu\beta\beta$) and the detection of low-energy solar neutrinos. Collaborations that are currently developing detector technology for direct detection of dark matter include the XENON, LUX, DEAP, CLEAN, and

WARP collaborations. Figure 1 illustrates results from the XENON collaboration in addition to their projected sensitivity in a large scale version of their current experiment.

We have only begun to experimentally probe the parameter space where supersymmetric neutralinos are thought to exist - with the advancement of current technologies, in addition to the large scale experiments being developed, we are confident that WIMP dark matter will be detected, if it in fact exists, in next generation experiments.

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